



## Current issues in coherence for small laser sources

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with the support of :









### **Coworkers**

## At the INLN

Experiments Tao Wang (Ph.D. Student)



B. Benzimoun (Intern, Univ. Clermont-Ferrand)

At CNR (Italy)

Numerics <u>G.P. Puccioni</u> – Firenze (Italy)



Consiglio Nazionale delle Ricerche

Institute for Complex Systems Istituto dei Sistemi Complessi







## <u>Outline of talk</u>

- **0.** General Introduction and Overview of the Problem
- I. Coherence in Macroscopic Lasers
- **II.** Coherence in Micro- and Nanolasers
- **III.** Statistical Mixture of Thermal and Coherent light
- **IV. Oscillations in Coherence Function**
- V. Additional Remarks















## **Brief reminder**









## **Cavity modes**











## **Cavity modes**



















## **I. Coherence in Macroscopic Lasers**

## **Experiments (and theory)**

He-Ne  $\lambda$  = 6328 Å

Cavity volume ~ 1 cm<sup>3</sup>  $\beta$  < 10<sup>-8</sup>





**Statistics of Light** 

G

S



Estratto da Rendiconti della Scuola Internazionale di Fisica « E. Fermi» - XLII Corso 19

Photocount Distributions and Field Statistics.

F. T. ARECCHI

CISE Laboratories - Segrate (Milano) Istituto di Fisica dell'Università - Milano

Thermal light photocount statistics: Laser light photocount statistics: Superposition of thermal and laser light:



Fig. 2. – Photocount distributions of a Gaussian field, a laser field, and a field made by the superposition of the two (nonlinear method); G, gaussian light; L, laser light; and S, superposed light. The three curves are not normalized.





**Statistics of Light** 

1967

#### Photocount Distributions and Field Statistics.

Estratto da Rendiconti della Scuola Internazionale di Fisica « E. Fermi» - XLII Corso

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# Evolution of the statistical photon mixture



Fig. 23. – Evolution of the mean value  $\langle n \rangle$ of the statistical distribution  $p(n, T, \tau)$ . The solid lines represent the theoretical curves which best fit the experimental points  $(o, \bullet)$ . The experimental points related to curve I) have been obtained by the statistical distributions of Fig. 22.



Fig. 22. - Experimental statistical distributions with different time delays obtained on a laser transient. The solid lines connect the experimental points which are not shown to make clearer the figure. All distributions are normalized to the same area a) 2.6  $\mu$ s; b) 3.7  $\mu$ s; c) 4.3  $\mu$ s; d) 5  $\mu$ s; e) 5.6  $\mu$ s; f) 8.8  $\mu$ s.





**Moment distributions** 



Estratto da Rendiconti della Scuola Internazionale di Fisica « E. Fermi» - XLII Corso 1967

#### Photocount Distributions and Field Statistics.

F. T. ARECCHI

CISE Laboratories - Segrate (Milano) Istituto di Fisica dell'Università - Milano

(6.28) 
$$H_2 = \frac{\langle n(n-1) \rangle}{\langle n \rangle^2} - 1,$$



Fig. 17. – Measured and theoretical values of the reduced second-order factorial moment  $H_2$  as a function of the normalized intensity  $M_1/M_{10}$  in the threshold region. —— Theory; • experimental point.





**Moment distributions** 



Estratto da Rendiconti della Scuola Internazionale di Fisica « E. Fermi» - XLII Corso 1967

Photocount Distributions and Field Statistics.

F. T. ARECCHI

CISE Laboratories - Segrate (Milano) Istituto di Fisica dell'Università - Milano

(6.29) 
$$H_3 = \frac{\langle n(n-1)(n-2) \rangle}{\langle n \rangle^3} - 1,$$









## **II. Coherence in Micro- and Nanolasers**

**Experiments and theory** 

 $0.8 \ \mu m < \lambda < 1.5 \ \mu m$ 

Cavity volume < 10  $\mu$ m<sup>3</sup>  $\beta$  > 10<sup>-4</sup>









j M





Vol 460 9 July 2009 doi:10.1038/nature08126

### Direct observation of correlations between individual photon emission events of a microcavity laser

J. Wiersig<sup>1</sup><sup>†</sup>, C. Gies<sup>1</sup>, F. Jahnke<sup>1</sup>, M. Aßmann<sup>2</sup>, T. Berstermann<sup>2</sup>, M. Bayer<sup>2</sup>, C. Kistner<sup>3</sup>, S. Reitzenstein<sup>3</sup>, C. Schneider<sup>3</sup>, S. Höfling<sup>3</sup>, A. Forchel<sup>3</sup>, C. Kruse<sup>1</sup>, J. Kalden<sup>1</sup> & D. Hommel<sup>1</sup>





Q





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Q



#### For the larger laser the g<sup>(3)</sup>(0) function does not match the theoretically expected values

**Coherence?** 

Figure 5 | Measured third-order correlation function  $g^{(3)}(0)$ . Characterization of the simultaneous arrival of three photons as a function of excitation power, for the three different micropillar samples studied in this paper. For the II-VI and low-Q resonators, the time resolution was increased to about 20 ps. These two samples show values close to the theoretical prediction,  $g^{(3)}(0) = 6$ , at lowest excitation powers. The *k*th-order correlation functions for a thermal source of radiation are readily obtained from the factorial moments of the Planck distribution,  $\langle n(n-1)\cdots(n-k+1)\rangle = k!\langle n\rangle^k$ , to give  $g^{(k)}(0) = k!$ , corresponding to the number of permutations of indistinguishable photons.

Wiersig et al., Nature **460**, 245 (2009)



**n**ice



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PHYSICAL REVIEW A 84, 061802(R) (2011)

#### Higher-order photon correlations in pulsed photonic crystal nanolasers







PRL 98, 043906 (2007)

PHYSICAL REVIEW LETTERS

week ending 26 JANUARY 2007

#### Photon Statistics of Semiconductor Microcavity Lasers



FIG. 3 (color online). (a),(b) Integrated intensities (bottom) for the 3 (4  $\mu$ m) pillars under nonresonant pulsed excitation. Strong photon bunching  $\tilde{g}^{(2)}(0) > 1$  is found from corresponding correlation measurements (top) over a broadened regime around the threshold.



FIG. 4 (color online). (a)  $\tilde{g}^{(2)}(\tau = 0)$  autocorrelation results derived from the 3  $\mu$ m pillar fundamental mode under cw excitation [see series in Fig. 2(a)], together with the corresponding mode emission intensity (b). (c) First-order  $g^{(1)}(\tau)$  series, revealing a strong  $\tau_c$  coherence decrease in the low excitation limit.







## <u>Summary</u>

- g<sup>(1)</sup>(t) shows very short coherence time
- g<sup>(2)</sup>(0) remains above 1 drops below 1 shows strong photon bunching (g(2) > 1)
- g<sup>(3)</sup>(0) for larger, high Q, laser inconsistent with theoretical values for thermal light
   1.5 μm nanolaser: unsatisfactory convergence
- g<sup>(4)</sup>(0) "constant" value (1.5 2)







## **III. Statistical mixture**

## of thermal and coherent light

## **Does it hold for nano- and microlasers?**







#### PHYSICAL REVIEW B 81, 165314 (2010)

## Ultrafast tracking of second-order photon correlations in the emission of quantum-dot microresonator lasers



ration effects become apparent. Filled squares mark the data sets shown in Fig. 2.

Hanbury-Brown & Twiss Workshop: OCA, May 2014

t-t<sub>peak</sub> (ps)





#### PHYSICAL REVIEW B 81, 165314 (2010)

## Ultrafast tracking of second-order photon correlations in the emission of quantum-dot microresonator lasers

Marc Aßmann, Franziska Veit, and Manfred Bayer Stephan Reitzenstein, Sven Höfling, Lukas Worschech, and Alfred Forchel



FIG. 3. (Color online) Upper panel: relative fractions of coherent and thermal emission at a fixed  $g^{(2)}(t,0)$  as given by a twomode model (solid lines) compared to the ideal case for infinitely small sampling time (dashed lines). Small deviations occur at high thermal fractions. Lower panel: effect of jitter on the measured  $g^{(2)}(t,0)$  for a coherent light pulse depending on the ratio r of the mean photon count rates at the pulse positions connected by the jitter and the relative frequency p of jitter occurrence. While frequent jitter (solid red line) has the same effects for pulse positions with high and low intensity, rare events (dotted black line) only affect regions with low mean photon count rate (high r).







#### PHYSICAL REVIEW B 81, 165314 (2010)

## Ultrafast tracking of second-order photon correlations in the emission of quantum-dot microresonator lasers



-20

0

-40

20

t-t<sub>peak</sub> (ps)

40

mode under nonresonant pulsed excitation. Above threshold, saturation effects become apparent. Filled squares mark the data sets shown in Fig. 2.

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80

60



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PHYSICAL REVIEW A 88, 013801 (2013)

#### Theory of interferometric photon-correlation measurements: Differentiating coherent from chaotic light

A. Lebreton, I. Abram, R. Braive, I. Sagnes, I. Robert-Philip,\* and A. Beveratos

#### Measure average correlation in a scanning Michelson interferometer

$$\hat{E}_{A}^{*}(t) = E_{0}^{*} \frac{a^{\dagger}(t + d/c) - a^{\dagger}(t)}{\sqrt{2}}, + \text{h.c. (4)}$$

$$\hat{E}_{B}^{*}(t) = E_{0}^{*} \frac{a^{\dagger}(t + d/c) + a^{\dagger}(t)}{\sqrt{2}}, + \text{h.c. (4)}$$

$$\langle \hat{E}_{A}^{*}(t)E_{A}(t) \rangle = \frac{I_{1}}{2} \{1 - 2\operatorname{Re}[g^{(1)}(d/c)]\}, \quad (5)$$

$$\langle \hat{E}_{B}^{*}(t)E_{B}(t) \rangle = \frac{I_{1}}{2} \{1 + 2\operatorname{Re}[g^{(1)}(d/c)]\}, \quad (5)$$

#### Compute normalized correlation:

 $g^{(2X)}(\tau, d/c) = \langle \hat{E}_A^*(0) \hat{E}_B^*(\tau) \hat{E}_B(\tau) \hat{E}_A(0) \rangle, \qquad (6)$ 



Technique borrowed from Photon-correlation Fourier Spectroscopy X. Brockmann *et al.*, Opt. Expr. **14**, 6333 (2006)





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PHYSICAL REVIEW A 88, 013801 (2013)

#### Theory of interferometric photon-correlation measurements: Differentiating coherent from chaotic light







FIG. 2. (Color online) Interferometric second-order intensity cross-correlation function for a chaotic field  $g_{\text{chaotic}}^{(2X)}(\tau, d/c)$  at the output ports of an unbalanced Michelson (or Mach-Zehnder) interferometer with pathlength difference  $d = 5c\tau_c$  (continuous red curve) and  $d = 10c\tau_c$  (dashed green curve).

FIG. 3. (Color online) Interferometric second-order intensity cross-correlation function for a stable coherent wave  $g_{\rm coh}^{(2X)}(\tau, d/c)$  at the output ports of an unbalanced Michelson (or Mach-Zehnder) interferometer with pathlength difference  $d = 0.5c\tau_{\phi}$  (dotted red curve),  $d = c\tau_{\phi}$  (dashed green curve), and  $d \gg c\tau_c$  (continuous black curve).





PHYSICAL REVIEW A 88, 013801 (2013)

#### Theory of interferometric photon-correlation measurements: Differentiating coherent from chaotic light

A. Lebreton, I. Abram, R. Braive, I. Sagnes, I. Robert-Philip,\* and A. Beveratos







FIG. 5. (Color online) Interferometric second-order intensity cross-correlation function for a 50:50 "mixture" of coherent and chaotic light  $g_{\text{mix}}^{(2X)}(\tau, d/c)$  at the output ports of an unbalanced Michelson (or Mach-Zehnder) interferometer with pathlength difference  $d = 5c\tau_c$  (continuous red curve),  $d = 10c\tau_c$  (dashed green curve), and  $\tau_{\phi} = \tau_c$ .





PRL 110, 163603 (2013)

#### PHYSICAL REVIEW LETTERS



#### Unequivocal Differentiation of Coherent and Chaotic Light through Interferometric Photon Correlation Measurements

A. Lebreton, I. Abram, R. Braive, I. Sagnes, I. Robert-Philip,\* and A. Beveratos

1.5



1.4 1.4 1.3 1.2 1.1 1.0 0.9 -10 -8 -6 -4 -2 0 2 4 6 8 10Time difference r (ns)

FIG. 2 (color online). Second order autocorrelation function  $g^{(2)}(\tau)$  of a nanoscale laser pumped at 1.1  $P_{\rm th}$  [green (gray) dots and curve] and a filtered chaotic source [red (black) dots and curve]. The points are experimental data and the continuous curves are fits to Eqs. (6) and (9).

#### **Photon-correlation Fourier spectroscopy**

FIG. 1 (color online). Experimental setup. Light is sent into an unbalanced Michelson interferometer made of optical fibers. Two single photon detectors (SSPDs) measure the cross correlations between the two output ports of the interferometer.

μ



j M





## <u>Summary</u>

Contradicting information coming from two different experiments

- In the "intermediate threshold region" a statistical superposition of chaotic light (spontaneous emission) and coherent light (laser) is emitted by the device
- 2. The light emitted by the device is coherent, but exhibits (strong) amplitude fluctuations

<u>Comparison between the two experiments difficult:</u> Pulsed regime Entirely different devices Reproducibility of samples







## **IV. Oscillations in coherence function**

## **Experiments and theory in**

## **Microcavities and nanocavities**





**Coherent oscillations** 



<u>Nanolaser</u>

Vol 460 9 July 2009 doi:10.1038/nature08126

## Direct observation of correlations between individual photon emission events of a microcavity laser

J. Wiersig<sup>1</sup><sup>†</sup>, C. Gies<sup>1</sup>, F. Jahnke<sup>1</sup>, M. Aßmann<sup>2</sup>, T. Berstermann<sup>2</sup>, M. Bayer<sup>2</sup>, C. Kistner<sup>3</sup>, S. Reitzenstein<sup>3</sup>, C. Schneider<sup>3</sup>, S. Höfling<sup>3</sup>, A. Forchel<sup>3</sup>, C. Kruse<sup>1</sup>, J. Kalden<sup>1</sup> & D. Hommel<sup>1</sup>



in)





#### PHYSICAL REVIEW A 85, 053811 (2012)

## Fast periodic modulations in the photon correlation of single-mode vertical-cavity surface-emitting lasers







### **Coherent oscillations**



PHYSICAL REVIEW A 85, 053811 (2012)

#### Fast periodic modulations in the photon correlation of single-mode vertical-cavity surface-emitting lasers

Naotomo Takemura,1 Junko Omachi,2 and Makoto Kuwata-Gonokami2,3,\*

#### **Microlaser**





### **Oscillations in coherence**

Hanbury-Brown & Twiss Workshop: OCA, May 2014

0.8 0.6

0

2

3

τ (ns)

5

6

7



### **Coherent oscillations**



New Journal of Physics

2013 New J. Phys. 15 033039

## Stochastically sustained population oscillations in high- $\beta$ nanolasers

A Lebreton<sup>1</sup>, I Abram<sup>1</sup>, N Takemura<sup>2</sup>, M Kuwata-Gonokami<sup>2,3</sup>, I Robert-Philip<sup>1,4</sup> and A Beveratos<sup>1</sup>



**Figure 3.** Second-order photon correlation function  $g^{(2)}(\tau)$  of a nanolaser  $(\beta = 0.25)$  undergoing photon population cycles, for  $P = 0.5 P_{\text{th}}$  (red line) and  $P = 4.5 P_{\text{th}}$  (black line). The dashed horizontal line corresponds to  $g^{(2)}(\tau) = 1$ , the value in continuous-wave lasers.

 $q^{(2)}(\tau) = 1 + |q^{(1)}(\tau)|^2$ 

Violation of Siegert relation (through population oscillations)

Numerical: Rate Equations (discrete variables)







New Journal of Physics

2013 New J. Phys. 15 033039

## Stochastically sustained population oscillations in high- $\beta$ nanolasers









## <u>Summary</u>

## Oscillations in coherence g<sup>(2)</sup>(τ) appear both in (some) nanolasers in microlasers

## Result from coherent oscillations between population and e.m. field

## Can give rise to misinterpretation of $g^{(2)}(\tau=0)$ as imperfect coherence







## **V. Additional remarks**









#### **Atom/carrier number**









#### **Atom/carrier number**









#### **Atom/carrier number**









#### **Atom/carrier number**









#### **Atom/carrier number**









#### **Atom/carrier number**









#### Atom/carrier number





**Modeling small lasers** 

PRL 102, 053902 (2009)

PHYSICAL REVIEW LETTERS



**Quantum Fluctuations in Small Lasers** 

Kaushik Roy-Choudhury and Stephan Haas, A.F.J. Levi







### **Modeling small lasers**



Phys. Rev. Lett. 102, 053902 (2009)

Uniform random walk on the grid defined by the Master Equation

Pulsing regime



FIG. 4 (color online). Time evolution of electrons and photons calculated by the random walk method. (a) Current, I = 9.6 nA. (b) I = 48 nA. (c) I = 72 nA. (d) I = 192 nA. The inset shows discrete step changes in photon number with time. Parameters as in Fig. 2.







### **Preliminary results**







Autocorrelations





j M







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V.10



Autocorrelations









### Autocorrelations











## <u>Summary</u>

From First Principles, Granular Modeling:

- Strong spiking regime
- g<sup>(2)</sup>(0) grows and then decreases
- Strong fluctuations in g<sup>(2)</sup>(0) before reaching coherent value
- g<sup>(3)</sup>(0) and g<sup>(4)</sup>(0) converge "later"
- Increased sensitivity to fluctuations in higher order correlations (especially g<sup>(4)</sup>(0))









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V.13



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## <u>Summary</u>

From First Principles, Granular Modeling:

- Oscillations present in signal mimicking the physical process
- Power spectrum shows peak features
- Autocorrelation > 1 (coherent oscillations)
- Oscillations in correlation
- Higher order correlations more sensitive (in amplitude and shape)







## **Conclusions**

Correlations widely used (and necessary) for characterizing coherence in micro- and nanolasers

Results strongly dependent on experimental system (reproducibility of samples, intrinsic features ...)

Most small lasers are pulsed: influence on correlations?

**Problems with higher correlations** 

**Coherent oscillations, statistical mixture of light ...** 

Many open questions







## Thank you for your attention!

















Nature **460** Wiersig et al.











Nature **460** Wiersig et al.

FIG. 5 (color online). (a),(b) Calculated output curves (bottom) and  $g^{(2)}(0)$  (top) for various  $\beta$ . (c),(d) Calculated results vs experimental data (pulsed) from Fig. 3(a) for the 3  $\mu$ m pillar. In (c), a convolution (solid line) with the experimental temporal resolution  $\tau_c$  is shown. (d) Corresponding I/O curves, explicitly indicating the effects of pump saturation.







RAPID COMMUNICATIONS

PHYSICAL REVIEW A 84, 061802(R) (2011)

#### Higher-order photon correlations in pulsed photonic crystal nanolasers

D. Elvira,<sup>1</sup> X. Hachair,<sup>1</sup> V. B. Verma,<sup>2</sup> R. Braive,<sup>1</sup> G. Beaudoin,<sup>1</sup> I. Robert-Philip,<sup>1</sup> I. Sagnes,<sup>1</sup> B. Baek,<sup>2</sup> S. W. Nam,<sup>2</sup> E. A. Dauler,<sup>3</sup> I. Abram,<sup>1</sup> M. J. Stevens,<sup>2</sup> and A. Beveratos<sup>1,\*</sup>



FIG. 4. (Color online) Dashed lines: Four different realizations of the stochastic rate equations at  $P = 1.8P_{\rm th}$ . Solid line: The mean pulse averaged over 1500 realizations. It can be clearly seen that the longer it takes for the pulse to emerge, the smaller is its amplitude. Inset: The dots show the statistical distribution of the total number of photons per pulse at  $P = 1.8P_{\rm th}$ , and the line shows the expected distribution for Poisson statistics of the same mean.







 $\beta = 0.01$ 

















