In Honor or Annick Pouquet

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A few issues in turbulence and how to cope with them using computers

Annick Pouquet, NCAR

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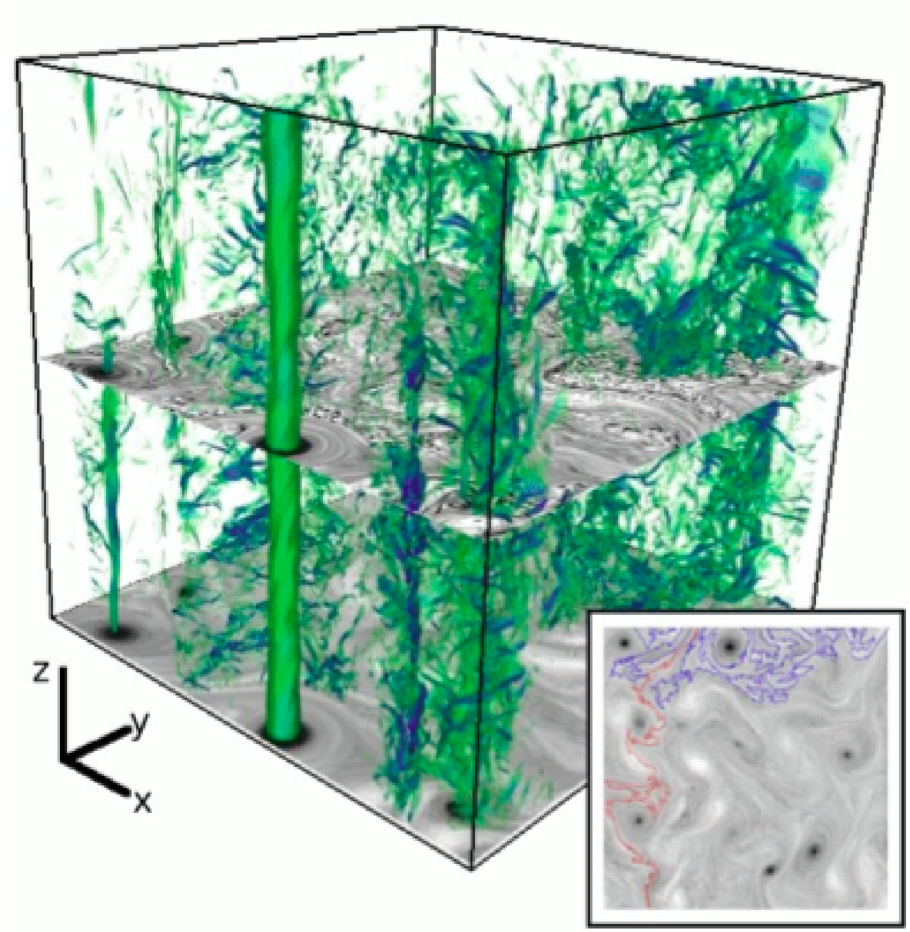

On being co-PI with Annick

You learn a lot and she makes you look good!

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Other papers are in preparation.

Minimal Models for Precipitating Turbulent Convection

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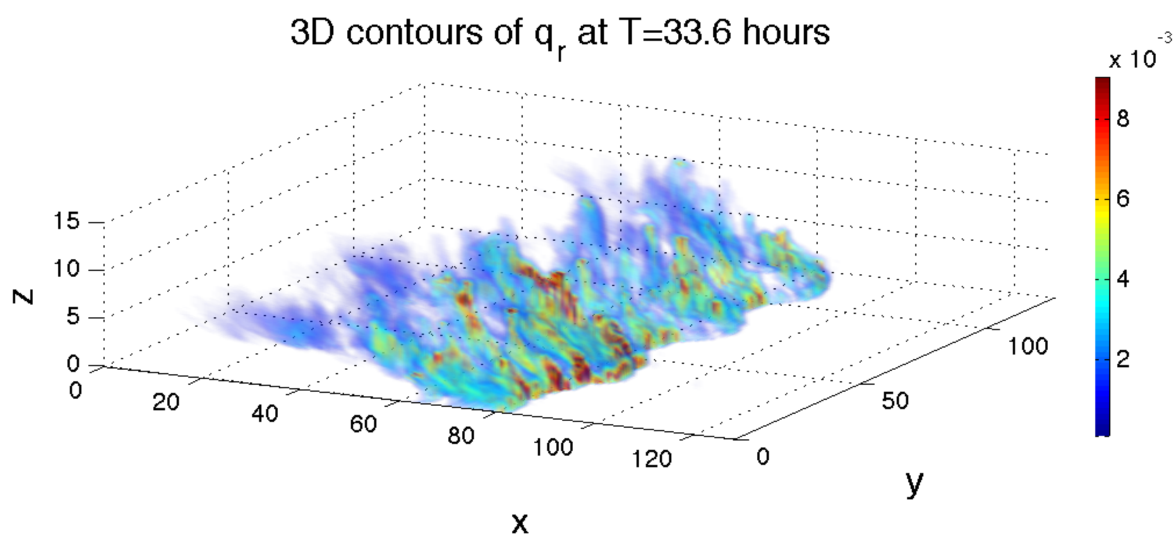

Contours of rain water in a simulated squall line

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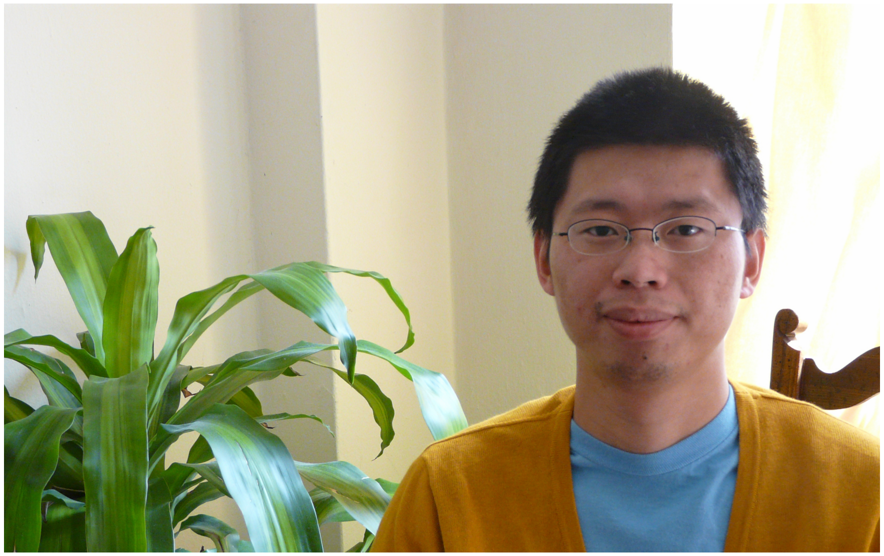
People



Goal

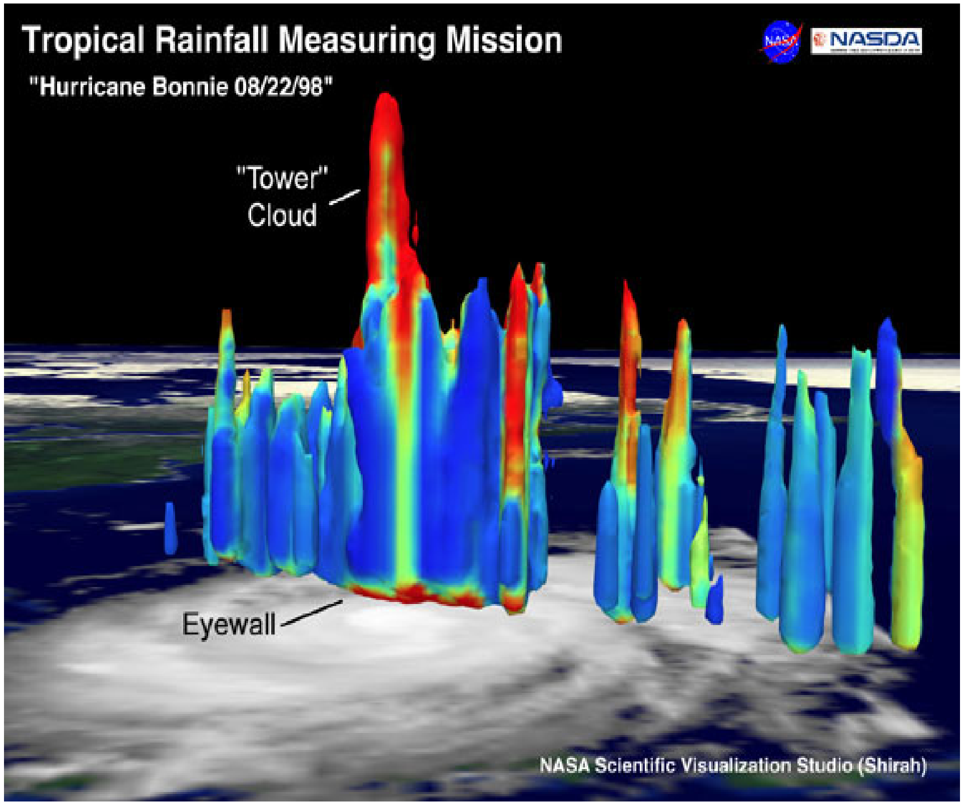
Find the simplest possible model that captures convective organization at many different length scales, for example

vortical hot towers: tall cumulonimbus clouds roughly 10 km (horizontal) by 15-20 km (vertical)

squall lines: horizontal scales $\approx$ 200 km

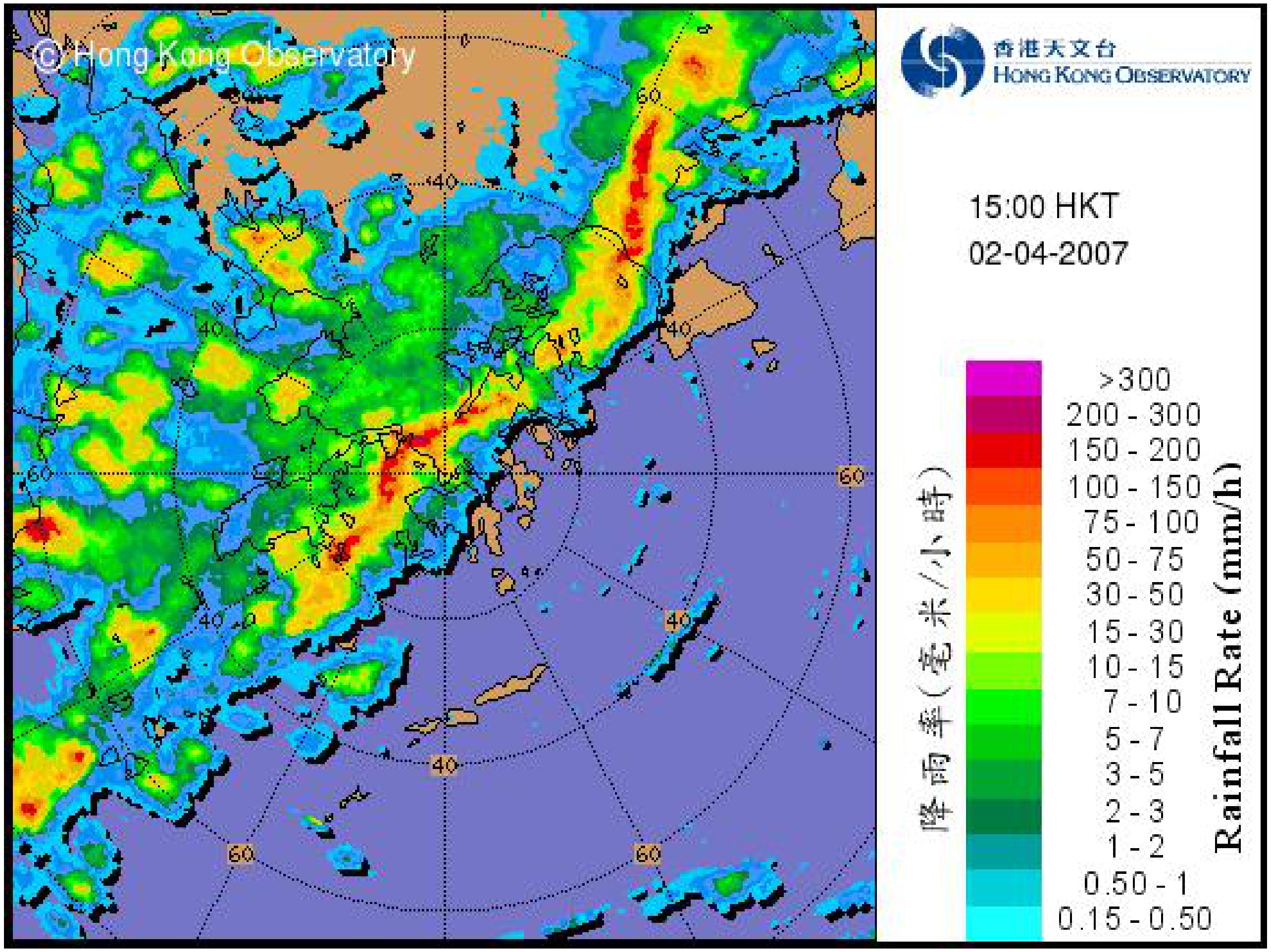
merger of the polar and subtropical jets: scales larger than 1,000 km

Examples from Observations



Satellite data of “hot towers” during Hurricane Bonnie 1998: colors correspond to surface precipitation; 18 km high; $\approx$ 10 km wide.

Examples from Observations



Radar image of a squall line near Hong Kong, roughly 200 km long, propagates normal to the line

Balancing competing factors...

The model should be simple enough to separate the essential from the non-essential

e.g. by removing turbulence models, sponge layers, detailed cloud microphysics...

but not too simple; keep key ingredients of latent heat release due to vapor/liquid phase change and precipitation

Hope is for “value-added.”

Outline

- Structure of a more comprehensive Cloud Resolving Model (itself already simplified)

  Anelastic dynamical core; linearized thermodynamics; warm-rain bulk cloud physics

- Minimal modeling

  Boussinesq dynamical core; linearized thermodynamics; asymptotically fast cloud microphysics

- Test case: Tropical squall lines

- Stability analysis

Anelastic; Warm-Rain Bulk Cloud Physics; Simplified Thermo

$$\frac{D\mathbf{u}}{Dt} + f\sin(\phi)\mathbf{u}_h^{\perp} = -\nabla\left(\frac{p'}{\tilde{\rho}(z)}\right)$$

$$+\mathbf{k}\, g\left(\frac{\theta'}{\tilde{\theta}(z)} + \epsilon_o q_v - q_c - q_r\right)$$

$$\nabla\cdot(\tilde{\rho}(z)\mathbf{u}) = 0, \qquad \frac{D\theta'}{Dt} + w\frac{d\tilde{\theta}(z)}{dz} = \frac{L\tilde{\theta}(z)}{c_p\tilde{T}(z)}(C_d - E_r)$$

$\rho(\mathbf{x},t) = \tilde{\rho}(z) + \rho'(\mathbf{x},t)$, with $\tilde{\rho}(z)$ prescribed, etc;
valid for $H \approx -\tilde{\rho}(d\tilde{\rho}/dz)^{-1}$ (the density scale height)

Bulk Cloud Physics

$$\frac{Dq_v}{Dt} = -C_d + E_r, \qquad \frac{Dq_c}{Dt} = C_d - A_r - C_r$$

$$\frac{Dq_r}{Dt} - \frac{1}{\tilde{\rho}(z)}\frac{\partial}{\partial z}(\tilde{\rho}(z)V_T q_r) = A_r + C_r - E_r$$

C_d: Condensation $q_v \to q_c$, E_r: Evaporation $q_r \to q_v$

A_r, C_r: Auto-conversion and Collection $q_c \to q_r$

V_T: Rainfall velocity

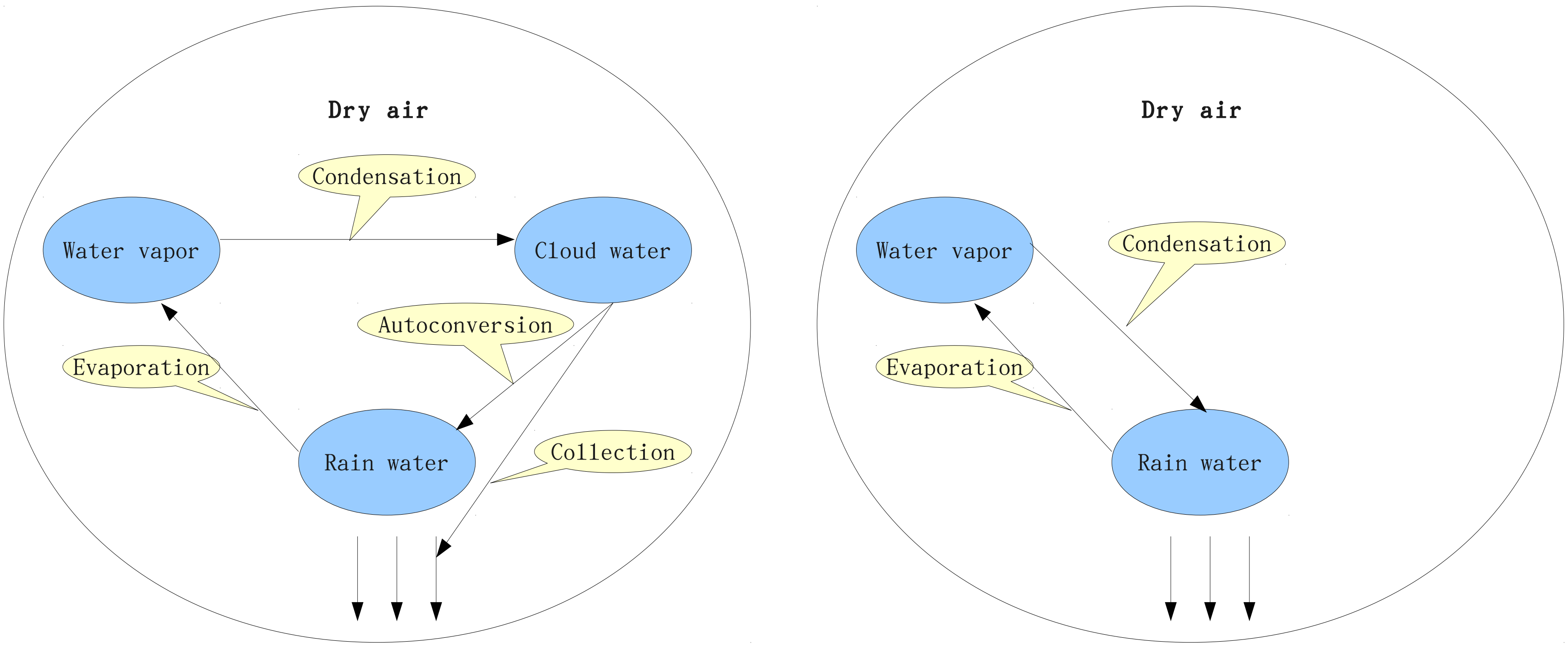
Fast Auto-Conversion of cloud to rain water

Seitter & Kuo (1983), Emanuel (1986),

Majda, Xing & Mohammadian (2010),

Deng, S & Madja (2012)

Infinitely fast auto-conversion and collection



right: excess water vapor above saturation (supersaturation) interpreted as cloud water.

The Boussinesq Approximation for $H \ll -\tilde{\rho}(d\tilde{\rho}/dz)^{-1}$ (not true here!)

un-differentiated background replaced by constants

$$\frac{D\mathbf{u}}{Dt} + f\sin(\phi)\mathbf{u}_h^{\perp} = -\nabla p' + \mathbf{k}\, g\left(\frac{\theta'}{\theta_o} + \epsilon_o q_v - q_r\right)$$

$$\nabla \cdot \mathbf{u} = 0, \qquad \frac{D\theta'}{Dt} + \frac{d\tilde{\theta}(z)}{dz}w = \frac{L}{c_p}(C_d - E_r)$$

$\theta = T(p_o/p)^{R/c_p}$

R, R_v gas constant for air, vapor; $R_v/R = \epsilon_o + 1$

Equations for water vapor and rain

$$\frac{Dq_v}{Dt} = -C_d + E_r, \qquad \frac{Dq_r}{Dt} - V_T \frac{\partial q_r}{\partial z} = C_d - E_r$$

$$C_d = \alpha_d^{-1}(q_v - q_{vs}(z))^+, \qquad E_r = \alpha_r^{-1}(q_{vs}(z) - q_v)^+ q_r$$

$q_{vs}(T, p) \approx q_{vs}(z)$ is the saturation profile

$\alpha_d = \alpha_r = \alpha$ is the cond/evap time scale

Reduction thus far

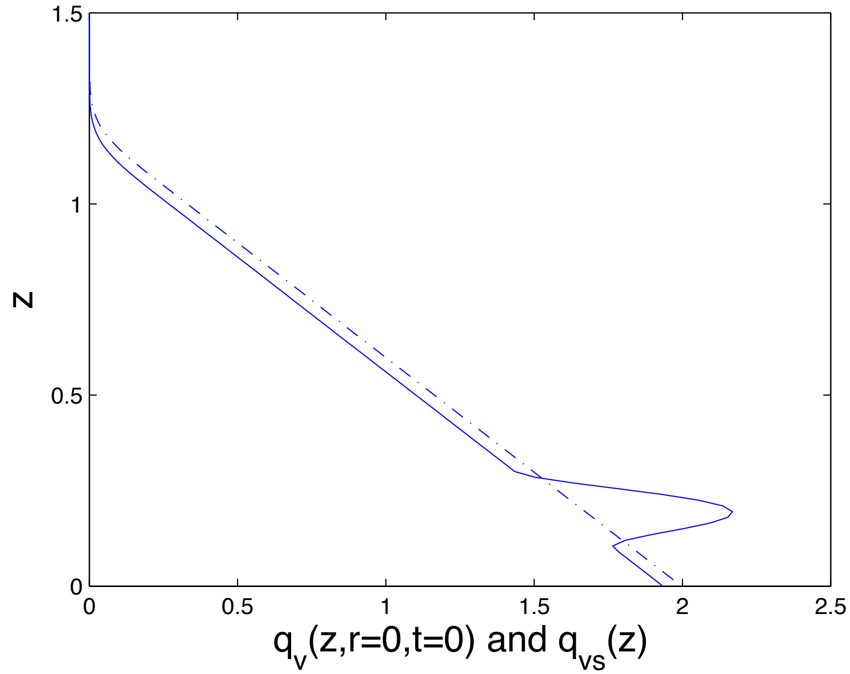
Eliminated: $\tilde{\rho}(z)$, $\tilde{p}(z)$, A_r, C_r, q_c

Still need: C_d, E_r, $q_{vs}(z)$, V_T, $d\tilde{\theta}/dz$

Standard values: $L^d \approx 2.5 \times 10^6 \text{ J kg}^{-1}$, $c_p \approx 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$, $\theta_o = T_o = 300 \text{ K}$, $d\tilde{\theta}(z)/dz \equiv B = 3 \text{ K km}^{-1}$, etc.

Moisture Profiles

$$C_d = \alpha^{-1}(q_v - q_{vs}(z))^+, \quad E_r = \alpha^{-1}(q_{vs}(z) - q_v)^+ q_r$$



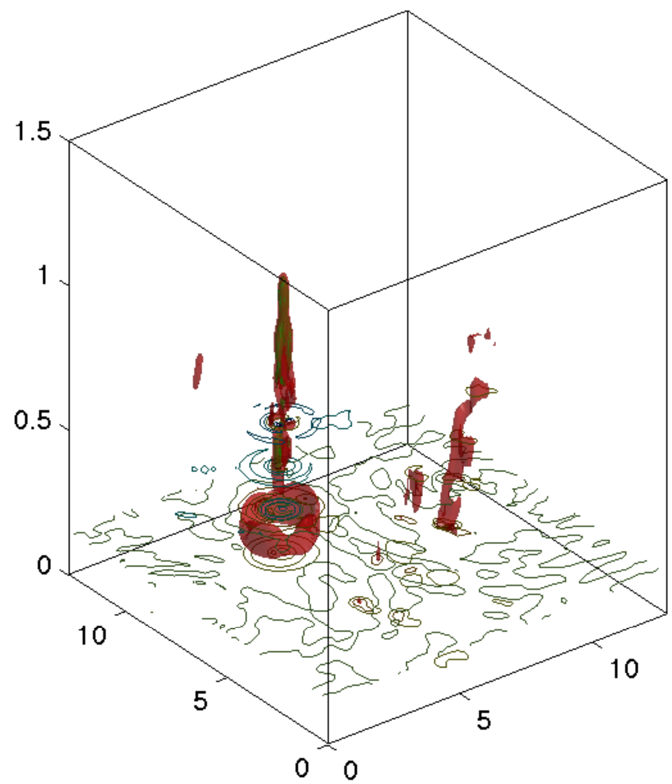
Initial water vapor profile $q_v(z, r = 0, t = 0)$ (solid) and saturation profile $q_{vs}(z)$ (dash); realistic surface saturation value $q_{vs,o} = 20 \text{ g kg}^{-1}$

With $q_v > q_{vs}$:

$$\frac{D\theta'}{Dt} + Bw = \frac{L}{c_p}(C_d - E_r)$$

$$\frac{Dw}{Dt} = -\frac{\partial p}{\partial z} + g\left(\frac{\theta'}{\theta_o} + \epsilon_o q_v - q_r\right)$$

Contours of Water Vapor Anomaly ($128 \times 128 \times 15$ km)



Moisture anomaly contour (1.2×10^{-3} kg/kg) in a Vortical Hot Tower; 30 min after moisture bubble injection at low altitude

Observations of VHTs in pre-hurricane Ophelia

Houze et al (2009) measured a VHT:

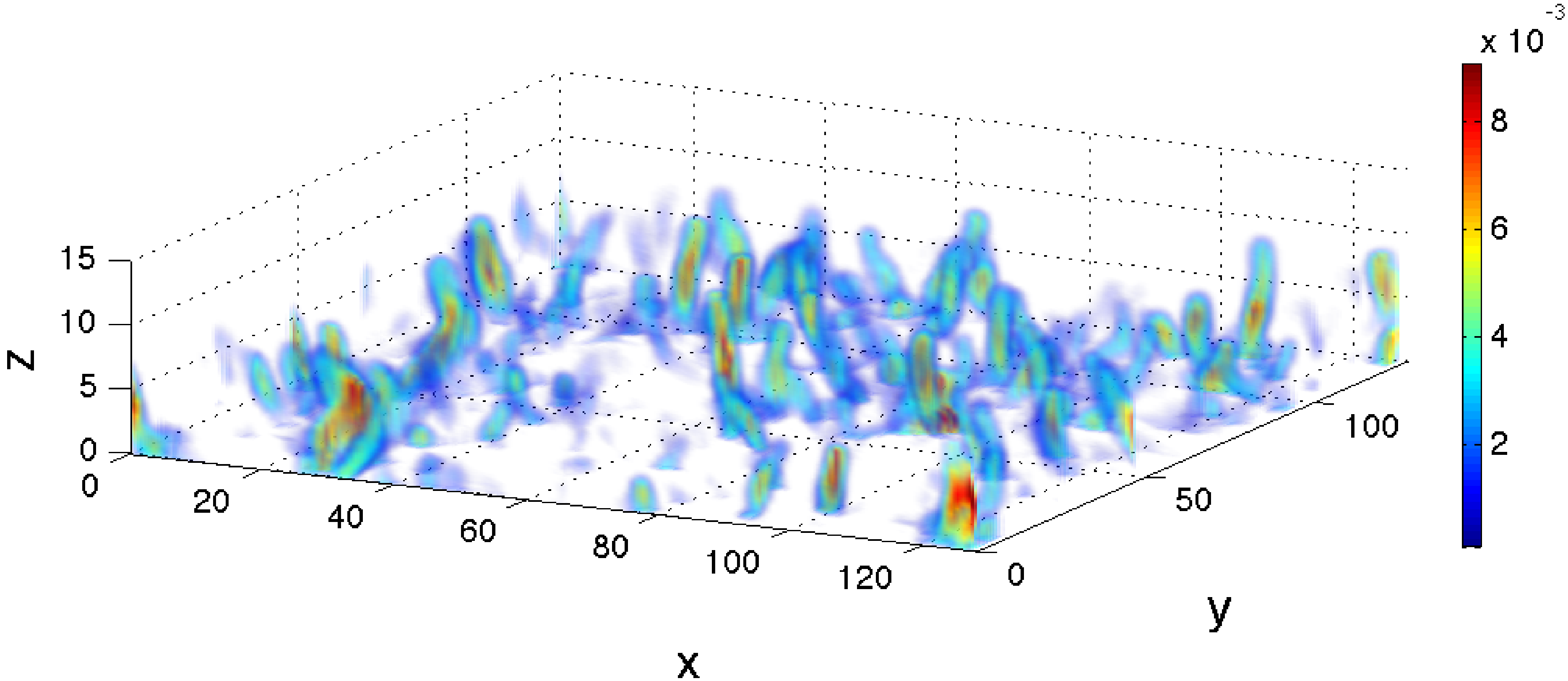
- 10 km wide
- 17 km high
- vertical velocities 10-25 m/s
- max values of vertical vorticity $5 - 10 \times 10^{-4}\ \mathrm{s}^{-1}$

  Consistency check!

  Merger into a larger-scale moist vortex; Deng, S & Madja (2012).

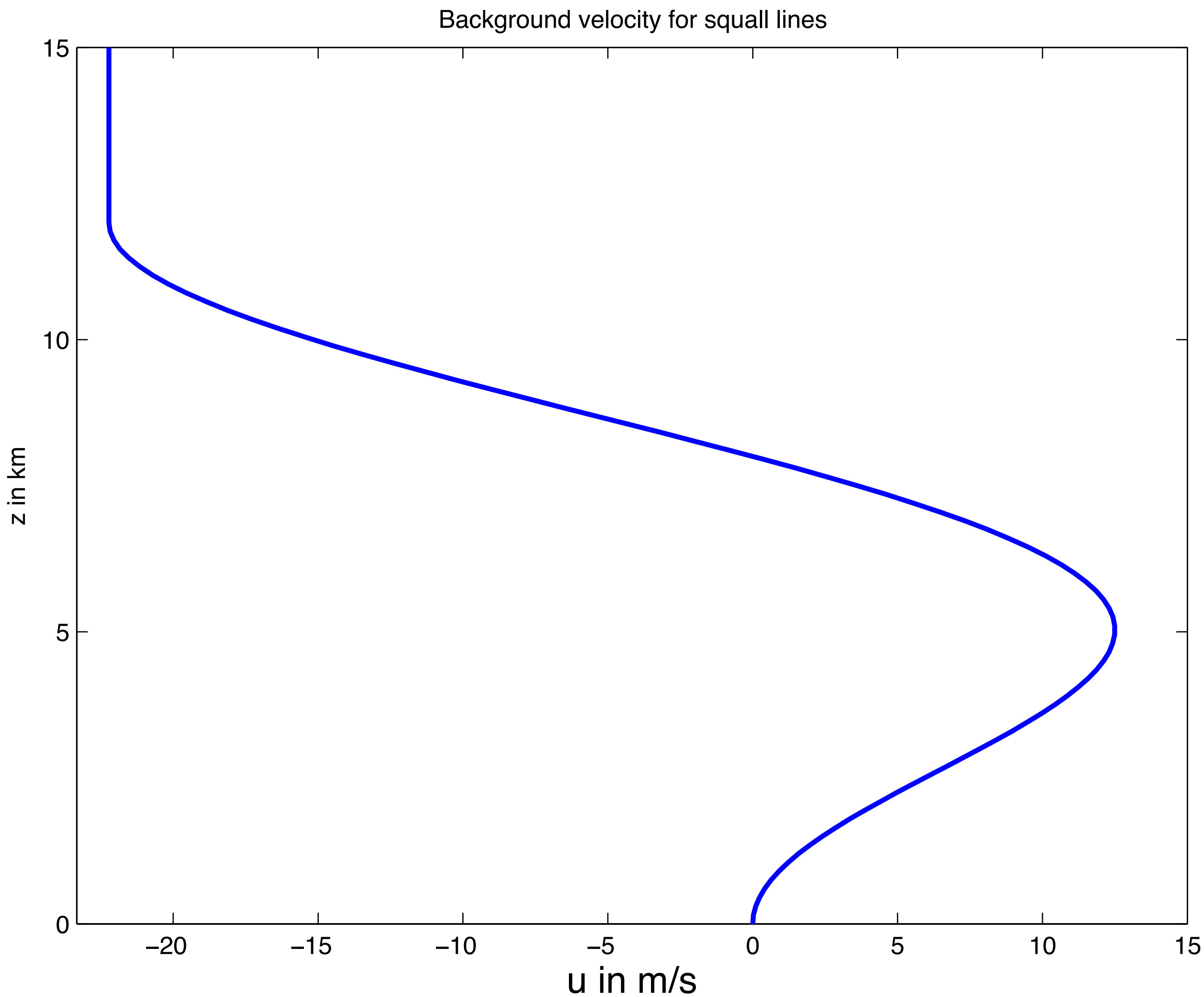
Environmental conditions (non-rotating from now on)

- Low-altitude moistening, e.g. over a warm ocean



Scattered convection; 3D contours of rain water

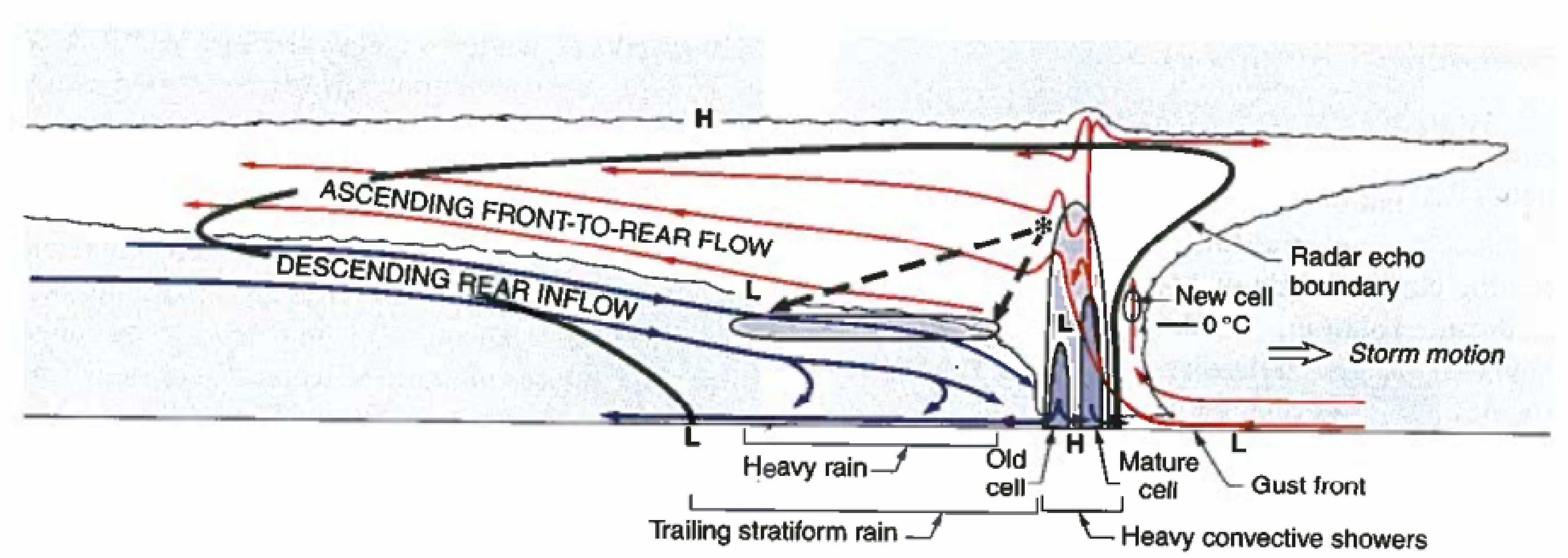
A background wind will organize the convection:



A squall line

Fred Roswald & Judy Jensen, http://wingssail.blogspot.com, 0337-SquallApproachesSumatra.jpg

More Characteristics of Squall Lines (Wallace & Hobbs, 2006)



- Long lasting multi-cell storm
- Tilted profile
- Propagate long distances (Houze 2004)
- Low-altitude cold pool

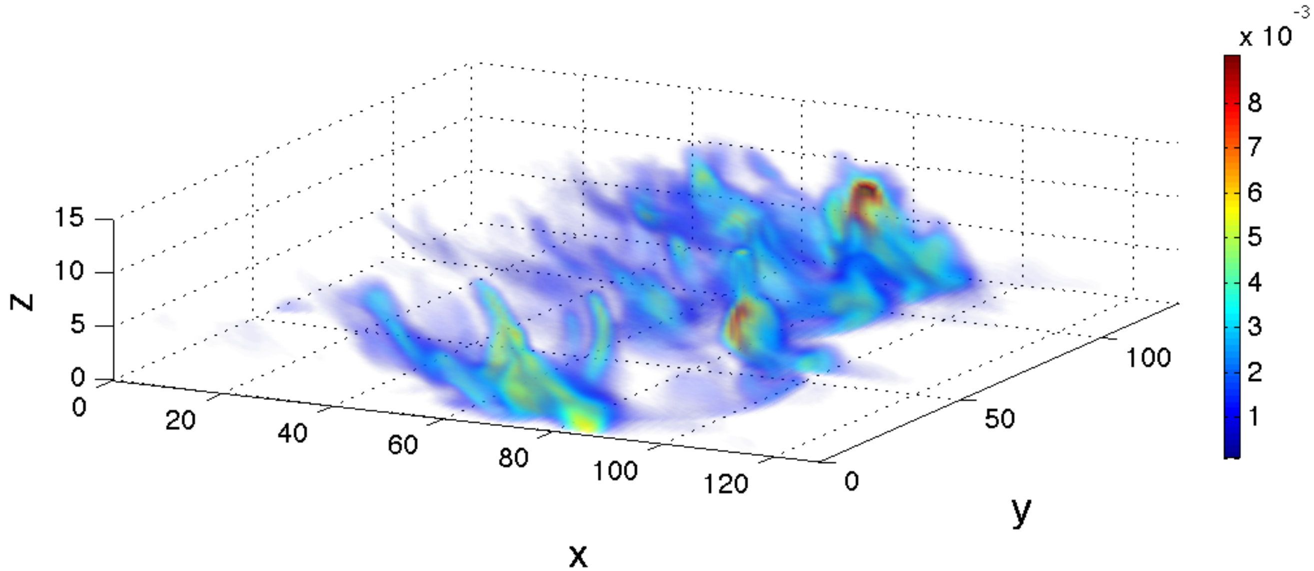
References

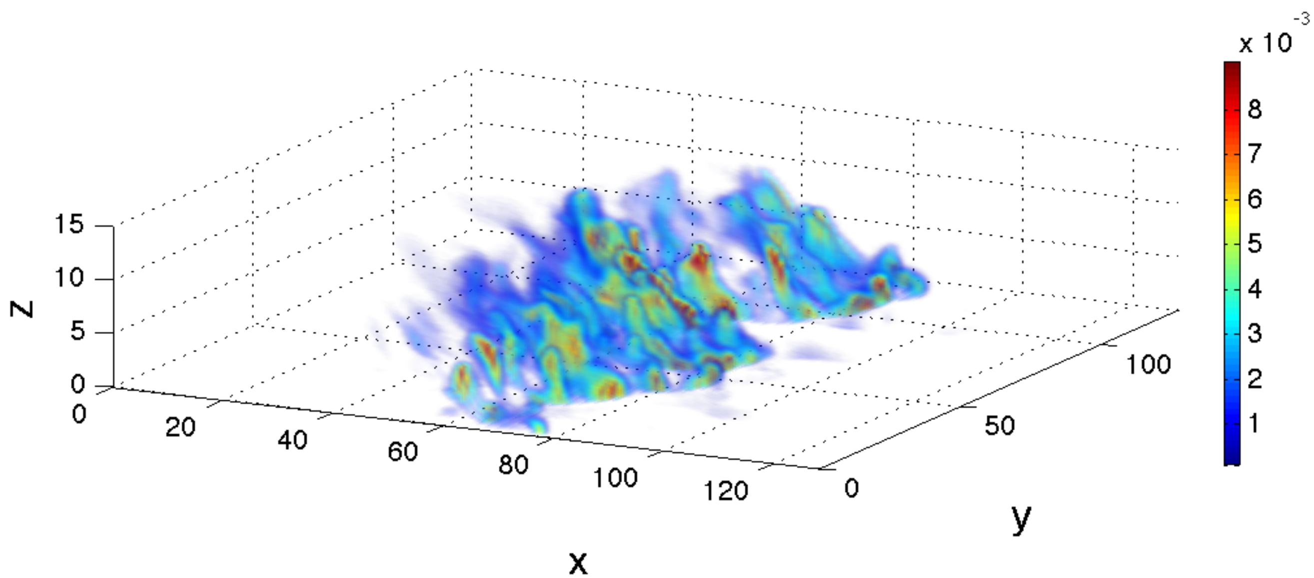
Observations, e.g. during GATE: Barnes & Sieckman 1984; during TOGA COARE: Jorgensen, LeMone & Trier 1997; LeMone, Zipser & Trier 1998; Review: Houze 2004.

CRMs: Fovell & Ogura 1988; Lafore & Moncrieff 1989; Grabowski, Wu & Moncrieff 1996, 1998; Xu & Randall 1996; Lucas, Zipser & Ferrier 2000; Lu & Moncrieff 2001

Conceptual & Minimal Models: Moncrieff & Green 1972; Moncrieff & Miller 1976; Moncrieff 1981; Emanuel 1986; Rotunno, Klemp, Weisman 1988; Majda & Xing 2010

A squall line ?





How did we get the "better" squall line?

In reality condensation time scale α_d is seconds while the auto-conversion time scale is about 15 minutes.

FARE=$\lim_{\alpha\to 0}$ FA

either:

unsaturated: $q_v < q_{vs}, \quad q_r = 0, \quad q_{\text{tot}} = q_v$

or:

saturated: $q_v = q_{vs}, \quad q_{\text{tot}} = q_r + q_{vs}$

The source terms maintain the constraints
$q_v = \min(q_{\mathrm{tot}}, q_{vs}), \quad q_r = \max(0, q_{\mathrm{tot}} - q_{vs})$:

$$C_d - E_r = \begin{cases} 0, & \text{if } q_{\mathrm{tot}} \leq q_{vs} \\ -w\, dq_{vs}(z)/dz, & \text{if } q_{\mathrm{tot}} > q_{vs}. \end{cases}$$

No closures for C_d, E_r necessary!

FARE Conservation Laws:

$\theta_e = \theta + \frac{L}{c_p} q_v$ (equivalent pot. temp.)

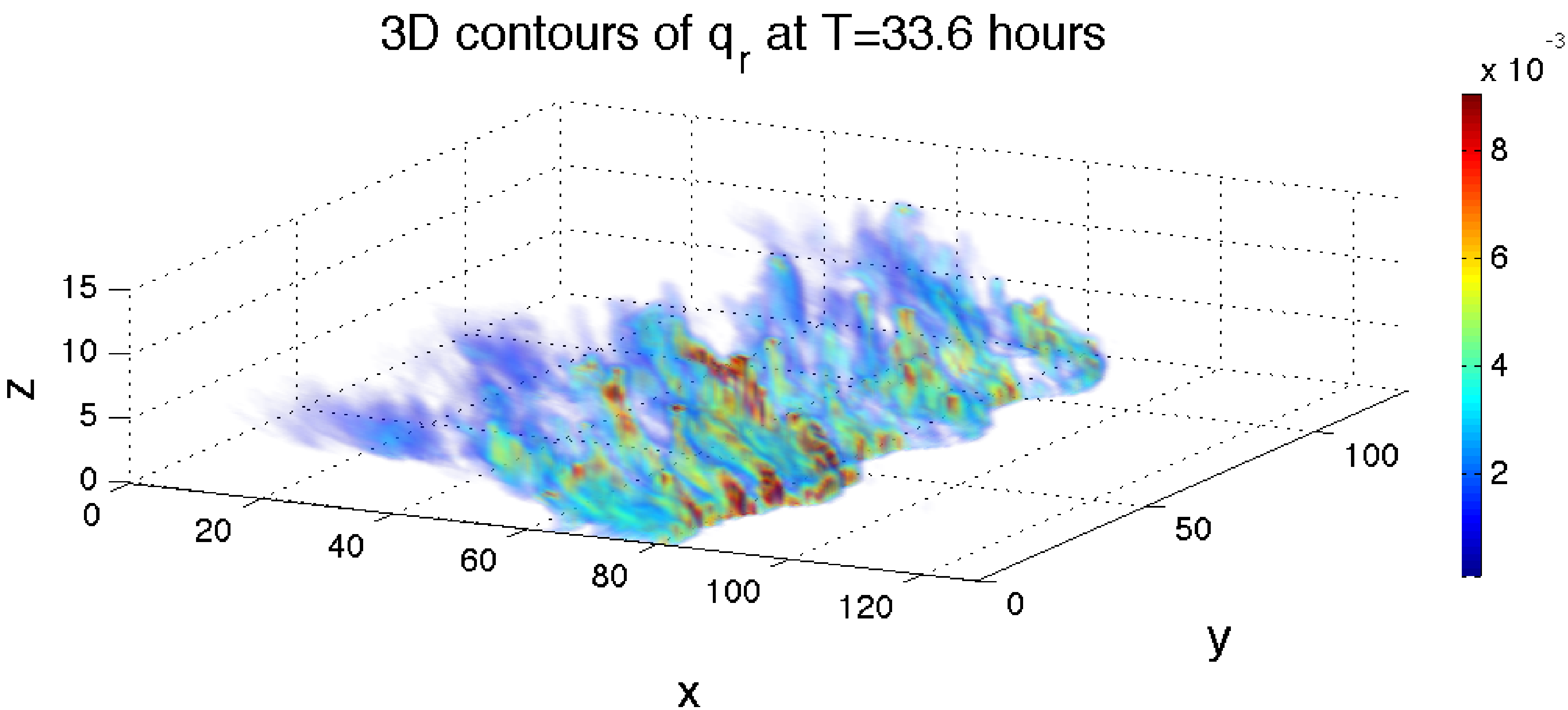
some algebra $\longrightarrow$

$$\frac{D\theta_e}{Dt} = 0, \qquad \frac{Dq_{\mathrm{tot}}}{Dt} - V_T \frac{\partial q_r}{\partial z} = 0$$

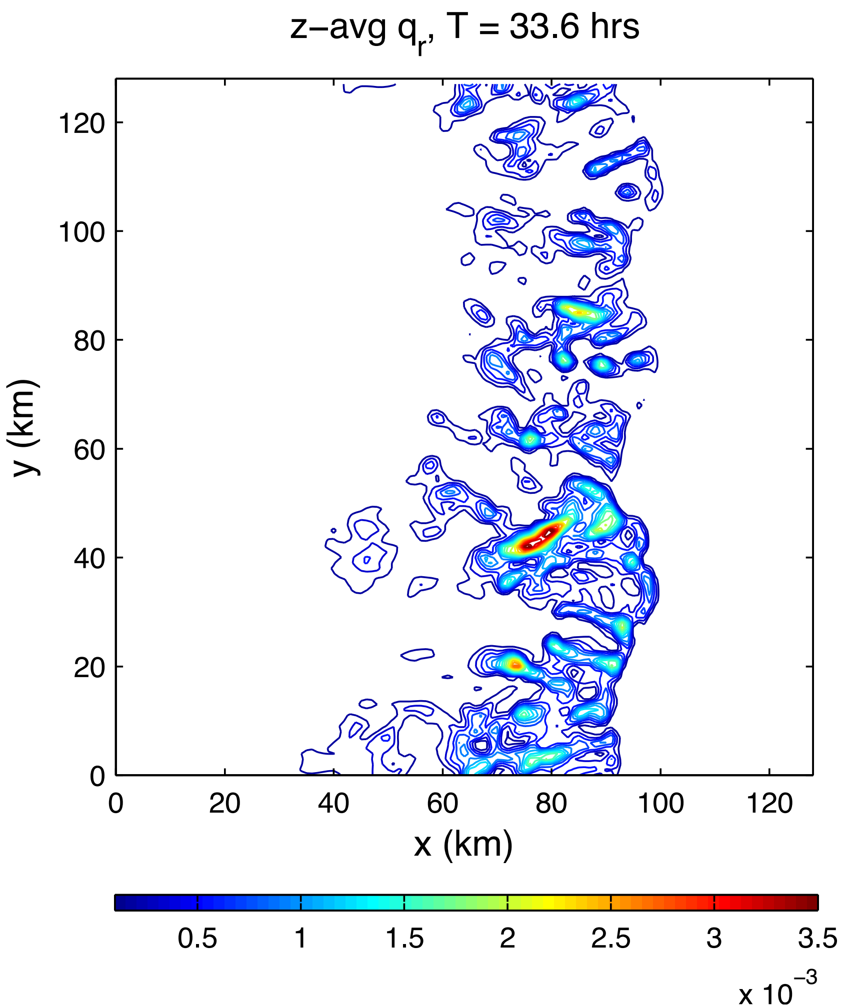
$$\text{no explicit source terms}$$

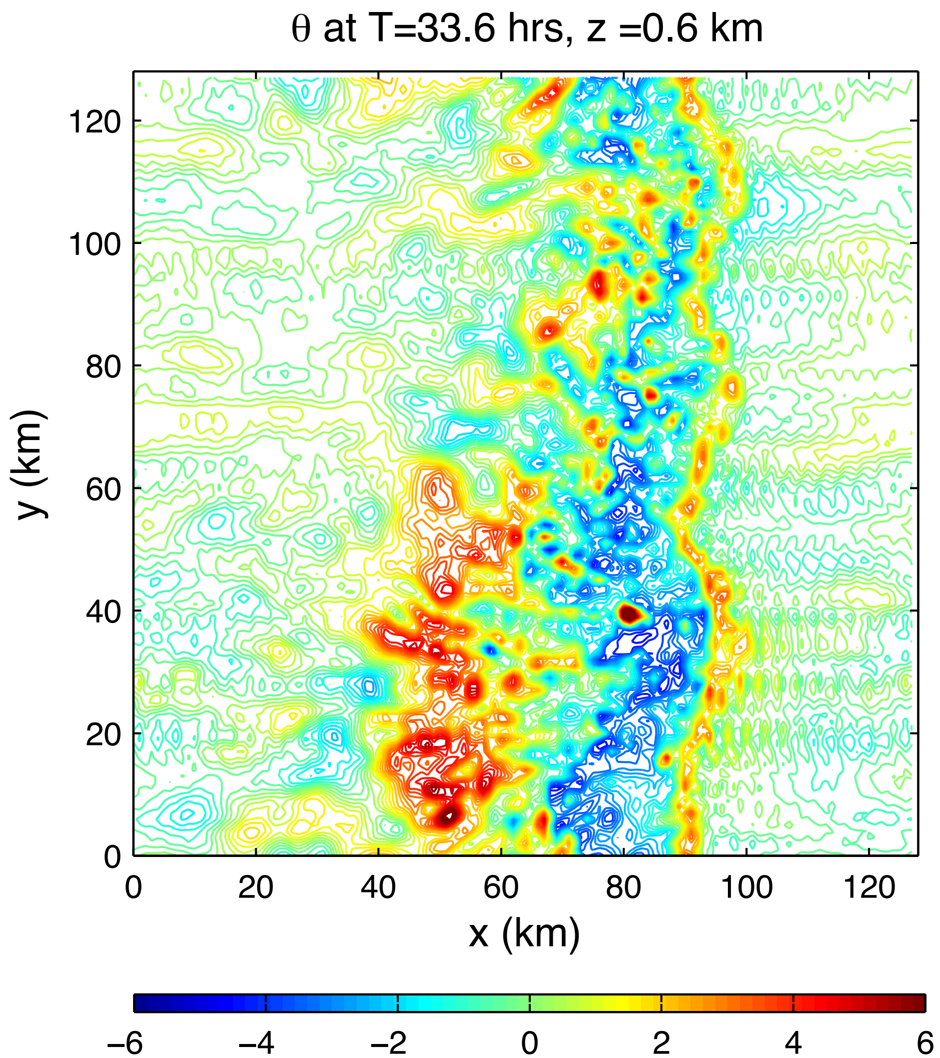
Similar models for non-precipitating, shallow convection; q_r replaced by q_c: Grabowski & Clark 1993; Spyksma, Bartello & Yao 2006; Pauluis & Schumacher 2010

3D Contours of Rain Water

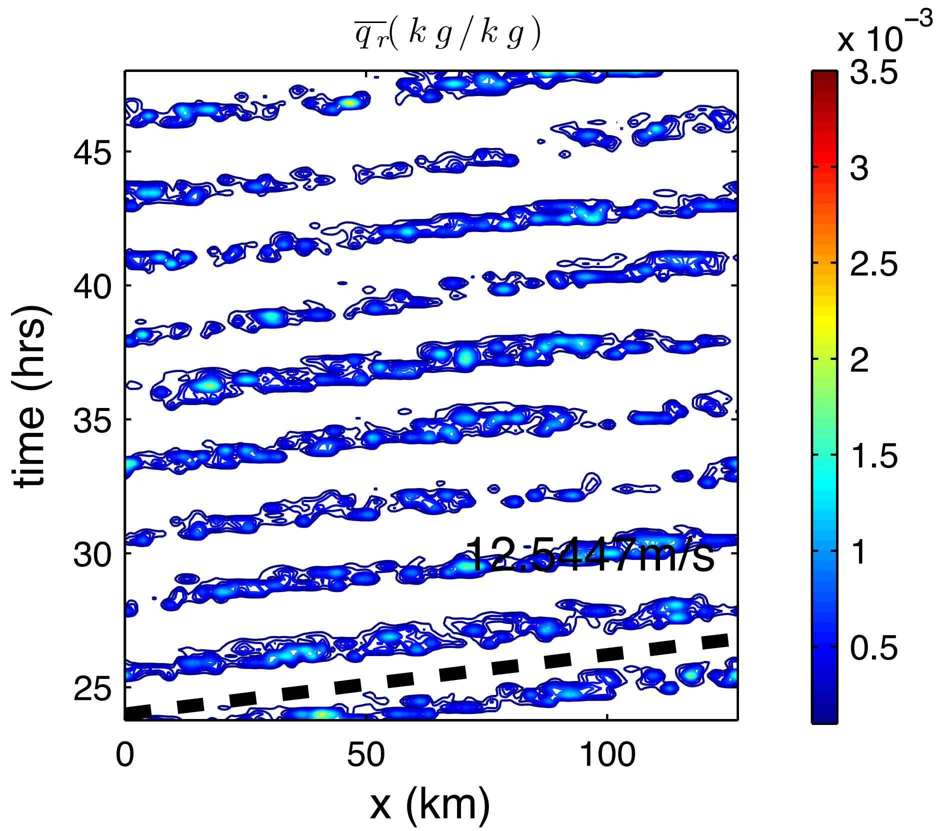


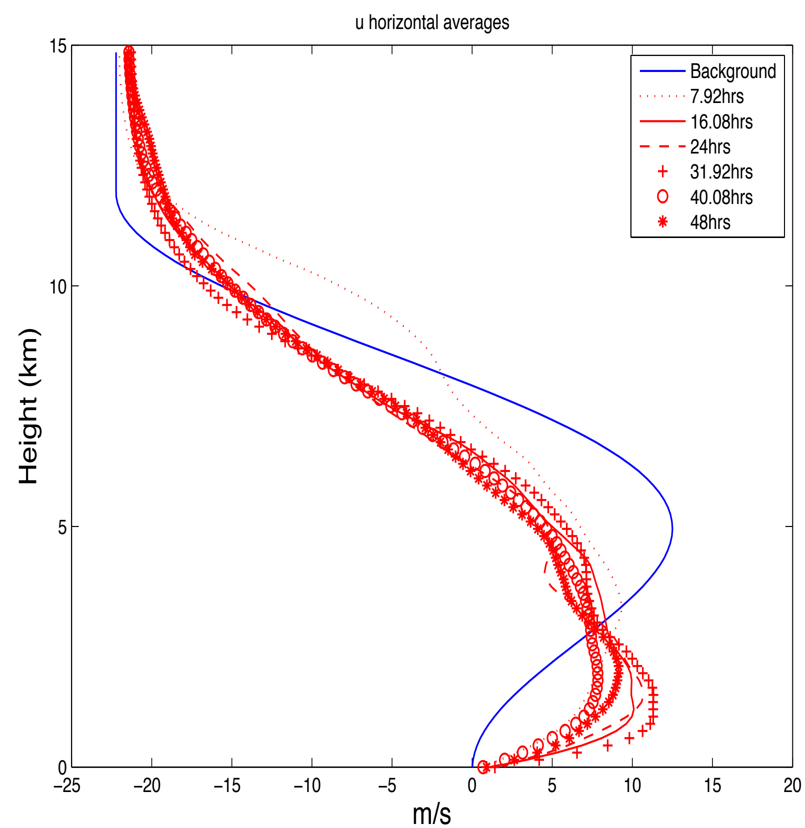
Vertically averaged rain, Low-level cold pool





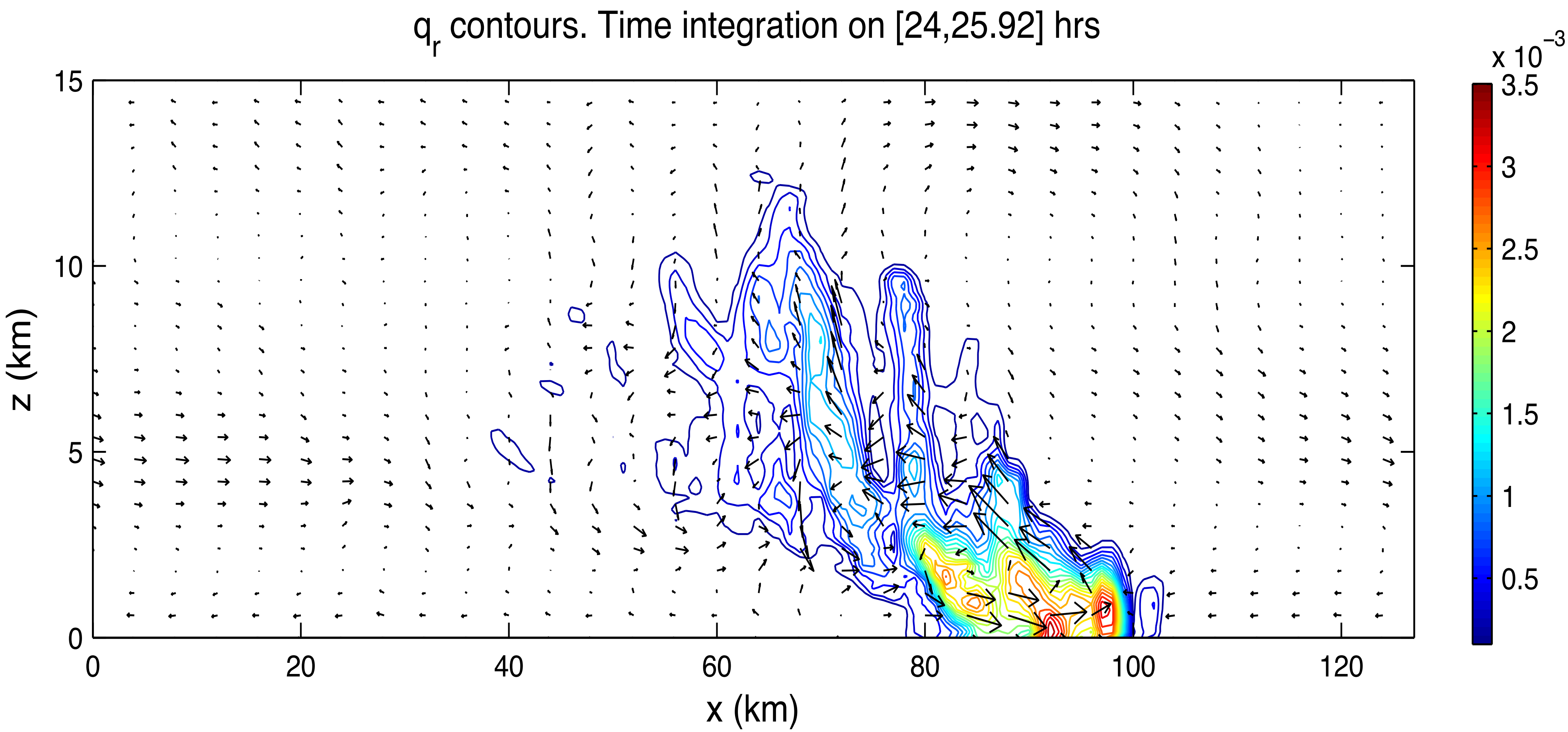
Time Evolution





Propagates at the speed of the jet max (black dash)

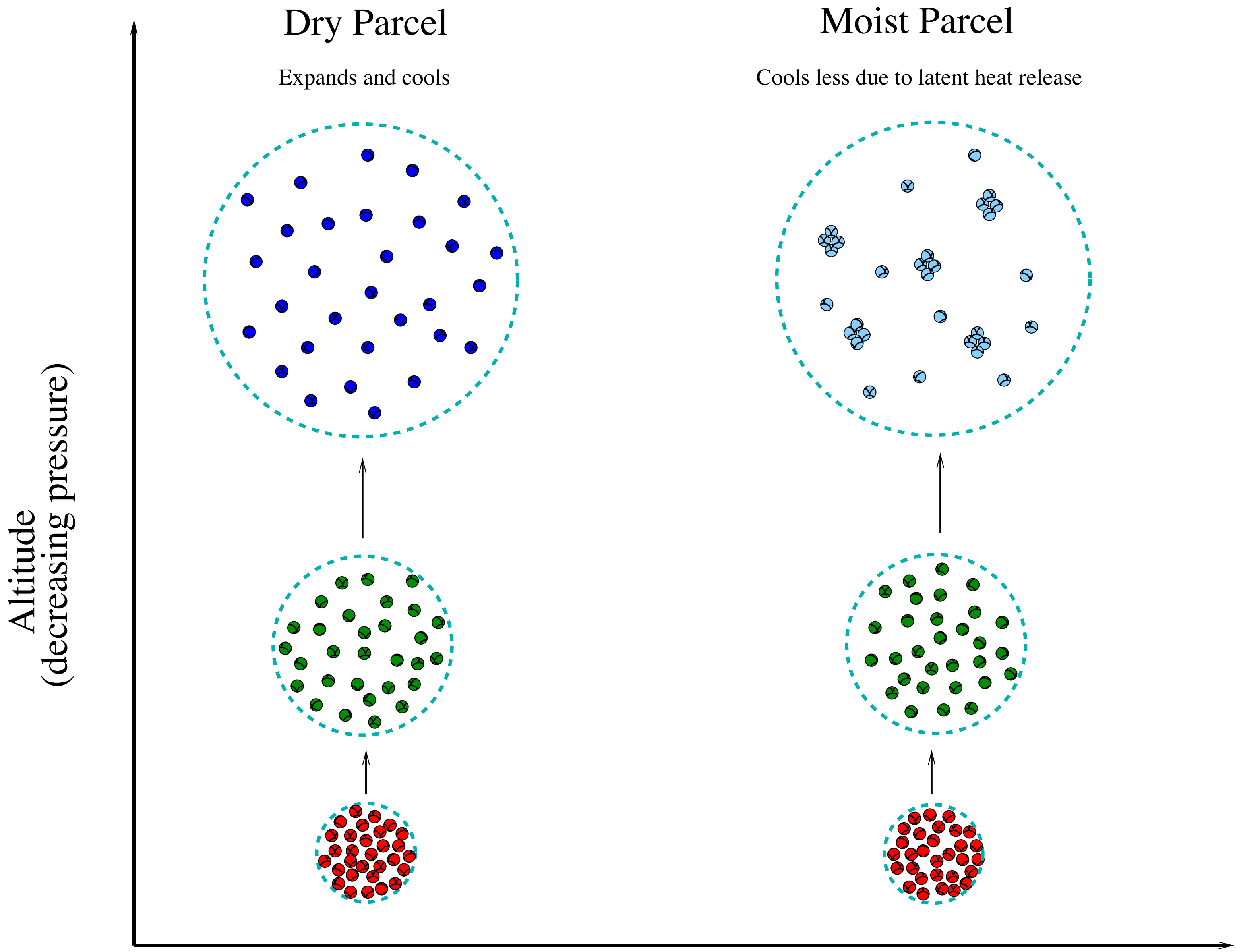
Multi-cloud structure



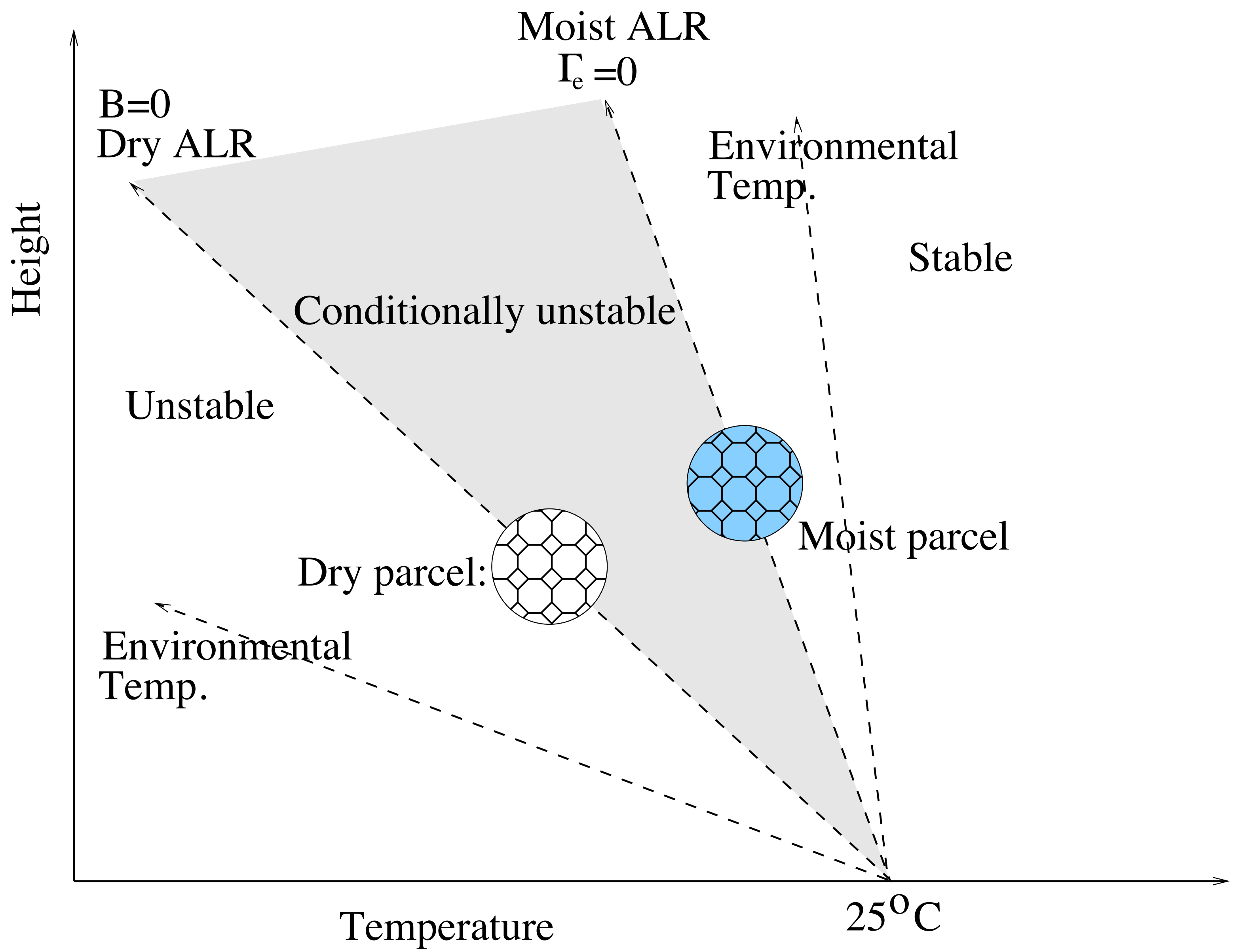
Thermodynamic stability: parcels rising adiabatically, equilibrium thermodynamics

Dynamic stability: linear/nonlinear stability analysis of the dynamical equations

Rising Dry vs Moist Parcels



Thermodynamic Concept of Conditional Stability



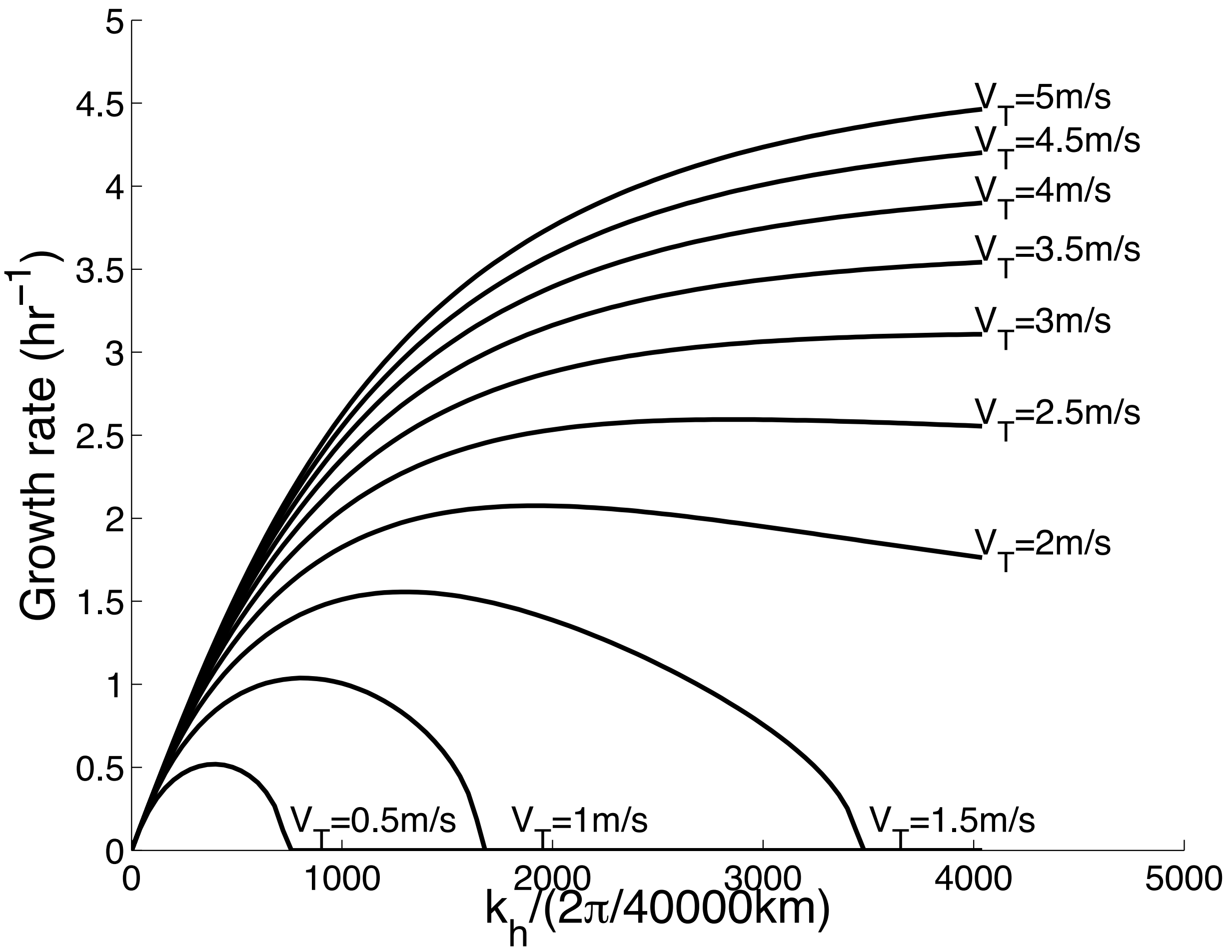
FARE with a saturated background gives the same right stability boundary:

$$\Gamma_e = (g/\theta_o)\, d\tilde{\theta}_e/dz > 0, \quad \theta_e = \theta + (L/c_p)q_{vs}$$

sufficient for stability;

as well as a left boundary sufficient for instability in the saturated case.

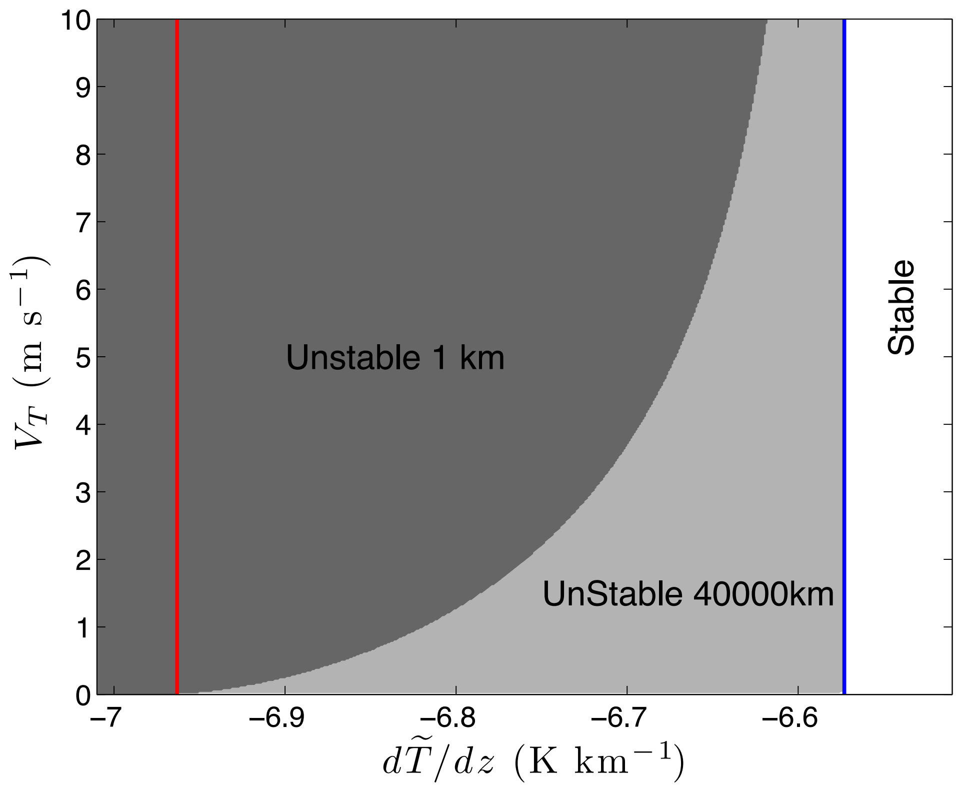
Growth rate of the unstable eigenmode



Fixed background temp/saturation gradients;

$k_z = 2\pi/15\ \mathrm{km}^{-1}$

Stability Diagram; fixed $q_{vs}(z)$ profile



Grey denotes unstable scales;

left of red line guaranteed unstable ($\Gamma < 0$);

right of blue line guaranteed stable ($\Gamma_e > 0$), [Emanuel 1986].

Full FARE and asymptotic limits

- Numerically find $\Gamma < 0$, $\Gamma_e > 0$ as sufficient conditions

- Find $\Gamma < 0$, $\Gamma_e > 0$

as the necessary and sufficient conditions

for $V_T = 0$ and $V_T \to \infty$, respectively.

Equation Structure: Saturated with $V_T = 0$ or $V_T \to \infty$

$$\frac{D\mathbf{u}}{Dt} = -\nabla p + \hat{\mathbf{k}} b', \qquad \frac{1}{w}\frac{Db'}{Dt} = -\Gamma \;\; (-\Gamma_e)$$

with positive definite energy for $\Gamma > 0$:

$$E = \frac{1}{2}\left(||\mathbf{u}||^2 + \frac{(b')^2}{\Gamma}\right)$$

and eigenvalues $\sigma^{\pm} = \pm(k_h/k)\Gamma^{1/2}$

Stability Parameter

- Saturated, $V_T = 0$:

$$\Gamma = \frac{g}{\theta_o}\frac{d}{dz}\left(\tilde{\theta}_e - \theta_o \tilde{q}_{\mathrm{tot}}\right)$$

- Saturated, $V_T \to \infty$:

$$\Gamma = \frac{g}{\theta_o}\frac{d\tilde{\theta}_e}{dz}$$

Energy Conservation in Saturated Regimes, $V_T > 0$ finite

$$E = \frac{1}{2}\left(||\mathbf{u}||^2 + \frac{(g\theta'_e/\theta_o)^2}{\Gamma_e} + \frac{(gq'_r/\theta_o)^2}{\Gamma - \Gamma_e} \right)$$

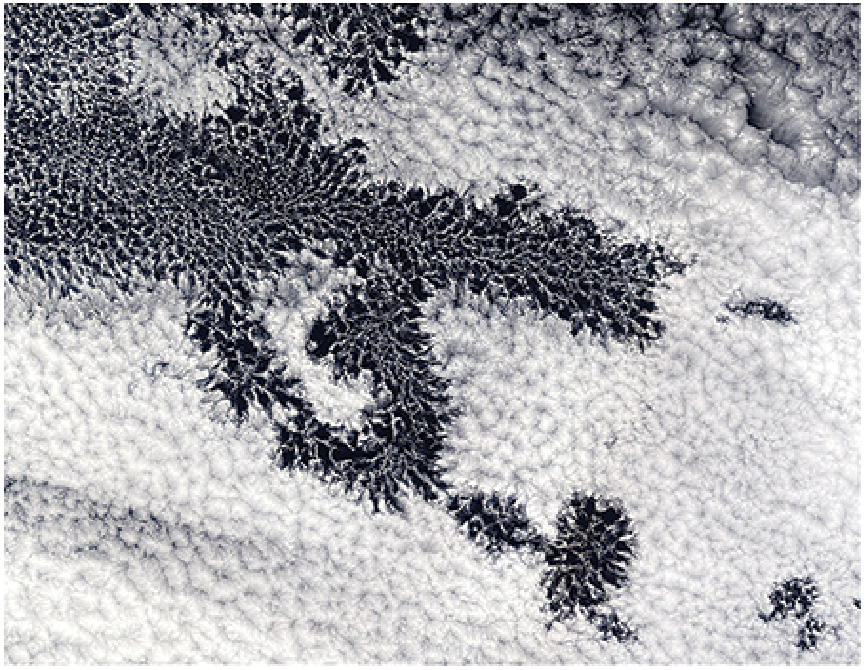
positive definite for

$$\Gamma_e = \frac{g}{\theta_o}\frac{d\tilde{\theta}_e}{dz} = \frac{g}{\theta_o}\left(\frac{d\tilde{\theta}}{dz} + \frac{L}{c_p}\frac{d\tilde{q}_{vs}}{dz} \right) > 0$$

$\Gamma - \Gamma_e$ strictly greater than zero

The wavelength dependence of the instability for $V_T > 0$

Formation of structures in broad areas of precipitating clouds, such as mesoscale convective systems in deep convection (Houze 2004);
POCs in boundary layer stratocumulus (Stevens et al. 2005; self-organization of cloud fields due to rain; northeast & southeast pacific oceans; NCAR picture)



Conclusions thus far on the FARE model

- Ignores cloud micro-physics, but retains conservations laws for

$$q_{\mathrm{tot}}, \quad \theta_e = \theta + \frac{L}{c_p} q_v$$

(why it "works")

- A framework for stability (and other) analysis