Diffraction modelling for solar coronagraphy

Application to ASPIICS

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Outline

1) Proba-3 mission and ASPIICS
2) Diffraction from external occulters
3) Penumbra profile
4) Light propagation in the coronagraph
5) Conclusion
Proba-3 mission and ASPIIICS
Proba-3 mission

• In-orbit demonstration of precise Formation Flying

• Two spacecraft flying 144m apart, controlled with a millimeter accuracy

• The Occulter Spacecraft will carry a 1,42m diameter occulter disk

• The Coronagraph Spacecraft will fly the solar coronagraph
Proba-3 mission

- The formation will be **co-aligned with the Sun** during the **6-hours** apogee phase
Solar coronagraph ASPIICS

- Associtation de Satellites Pour l’Imagerie et l’Interférométrie de la Couronne Solaire

- ASPIICS in a nutshell:
  - white light [540nm ; 570nm]
  - 2.81 arcsec/pixel
  - 3 polarizers
  - high cadence

- Observation of the K-corona:
  - Findings on the heating process
  - Alven’s waves, dynamics of the plasma
  - Coronal Mass Ejections
Solar coronagraph ASPIICS

• Hybrid externally occulted Lyot-style solar coronagraph

Diagram:
- External occulter
- Pupil
- Internal Occulter
- Lyot stop
- Focal plane

Dimensions:
- External occulter: 710mm
- Pupil: 25mm
- Internal Occulter: 1,66mm
- Lyot stop: 24,25mm
- Focal plane: 1,66mm

Distances:
- O to A: 144,348m
- A to B: 330,348mm
- B to O': 1,66mm
- C to D: 24,25mm

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Diffraction by an external occulter
Diffraction by an external occulter

Planar wave front

Point source at $\infty$

$\lambda = 550\text{nm}$

$R = 710\text{mm}$

$N_f = \frac{R^2}{\lambda z} \approx 6400$

Plane of observation

Fresnel diffraction

$z = 144.348\text{m}$
Diffraction by an external occulter

Major point in solar coronagraphy: the Sun is an extended source!

We must:
- know the diffraction pattern over a large extent
- perform a convolution with the solar disk
- what happens in the centre (few $\lambda / D$) is not sufficient!
**Diffraction by an external occulter**

We investigated several numerical methods to compute diffraction (solar case):

- **Analytical Hankel transformation:**
  + Exact calculation
  - Axis-symmetry required (only radial apodisation), computational time

- **Brute force 2D FFT to compute the two dimension Fresnel integrals:**
  + Any type of occulters
  - Strong sampling requirements, very large size of arrays (order $10^5$ to $10^6$)

- **Vanderbei et al. (2007) approach:**
  + Expands the Fresnel integral into a series
  - Not suitable for our solar case

- **Maggi-Rubinowicz representation, the boundary diffraction integral:**
  + Fast and accurate
  - Requires a 1-or-0 occulter (no apodization)
Diffraction by an external occulter

- The sharp-edged occulting disk

Occulting ratio of 1.05 solar radius at $z_0=144\text{m}$

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Diffraction by an external occulter

• The sharp-edged occulting disk

The bright spot of Arago (or Poisson... demonstrated by Fresnel)

Credit: Minerva.union.edu

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Diffraction by an external occulter

• The sharp-edged occulting disk

(a)

(b)

(c)

Bright spot of Arago

Transition shadow/light

Intensity $|\psi|^2$ (log)

Intensity $|\psi|^2$

Radius (mm)

Radius (μm)

Radius (mm)
Diffraction by an external occulter

• The apodized occulting disk

Variable radial transmission
Diffraction by an external occulter

• The apodized occulting disk
Diffraction by an external occulter

- The serrated (or petalized) occulter

In stellar coronagraphy, the reasoning starts from the ideal apodized occulter

$$\tau_{apod}(r) = \int \tau_{petal}(r, \theta) d\theta$$

The petalized occulter is the discrete substitute

Cady, 2006
Vanderbei et al., 2007
Diffraction by an external occulter

- The serrated (or saw-toothed) occulter

In solar coronagraphy, the reasoning is well different!

The diffraction occurs perpendicularly to the edge
A toothed disc rejects the light outside the central region

Boivin (1978) predicted the radius of the dark inner region of the diffraction pattern based on geometrical considerations
Diffraction by an external occulter

- The serrated (or saw-toothed) occulter

\[ N_t = 1024 \; ; \; \Delta = 20\text{mm} \]
Diffraction by an external occulter

- The serrated (or saw-toothed) occulter
Diffraction by an external occulter

- The serrated (or saw-toothed) occulter

![Diagram showing diffraction patterns](image-url)
Diffraction by an external occulter

- The serrated (or saw-toothed) occulter
Diffraction by an external occulter

- The serrated (or saw-toothed) occulter
Diffraction by an external occulter

• The serrated (or saw-toothed) occulter

![Diagram showing diffraction patterns with annotated regions A and B]

- A: Dark inner region
- B: Intermediate region
Diffraction by an external occulter

- The serrated (or saw-toothed) occulter

![Diagram showing diffraction patterns with regions labeled A, B, and C: Dark inner region, Intermediate region, Fully illuminated region.](image)
Diffraction by an external occulter

• We numerically verified the geometrical predictions of Boivin (1978)
Diffraction by an external occulter

- The serrated (or saw-toothed) occulter
Penumbra profiles
Penumbra profiles

Major point in solar coronagraphy: the Sun is an extended source!

To compute the penumbra, we must:
- know the diffraction pattern over a large extent
- perform a convolution with the solar disk
Penumbra profiles

Major point in solar coronagraphy: the Sun is an extended source!

\[ R_{\text{sun}} = 16.2' \]

Penumbra: \[ \int \text{Diffraction} \times \text{Solar image} \]
Penumbra profiles

- The sharp-edged occulting disk

![Graph showing Penumbra profiles with labels for Diffraction and Purely geometrical.](image)
Penumbra profiles

• The serrated (or saw-toothed) occulter

$\Delta = 20\text{mm}$

$N_t = 464$

$N_t$ increases

$\Delta$ increases
Penumbra profiles

- The serrated (or saw-toothed) occulter

Integrated illumination over the pupil, normalized to the sharp-edged disk case

Boivin radius \( (N_t, \Delta) > r_{sun} = 671 \text{mm} \)
Propagation inside the coronagraph
Propagation inside the coronagraph

- The hybrid externally occulted Lyot solar coronagraph
Propagation inside the coronagraph

• Propagation of the diffracted wave front from one plane to the next one
  - Fourier optics formalism, Fresnel free-space propagation
  - Ideal optics
  - Perfect axis-symmetric geometry

• Integration over the solar disk

• Numerical implementation: successive FFT2 with arrays of large size

• Objective:
  - estimate the level and spatial distribution of the residual diffracted sunlight
  - address the rejection performance of the coronagraph
Propagation inside the coronagraph

- Intensity in plane $O'$, with the internal occulter

Without external occulter

With external occulter
Propagation inside the coronagraph

- Intensity in plane O’, with the internal occulter

![Graph](image)

- With external occulter
- Without external occulter

Solar disk image (out-of-focused)
Propagation inside the coronagraph

- Intensity in plane O’, with the internal occulter

![Graph showing intensity in the plane O’ with and without external occulter.](Image)
Propagation inside the coronagraph

- Intensity in plane C, with the Lyot stop

Without external occulter

With external occulter
Propagation inside the coronagraph

- Intensity in plane C, with the Lyot stop

![Graph showing intensity with and without external occulter](chart.png)

- With external occulter
- Without external occulter (Lyot coronagraph)
Propagation inside the coronagraph

- Intensity in plane C, with the Lyot stop

![Graph showing the intensity in plane C with the Lyot stop, comparing with and without an external occulter.](image-url)
Propagation inside the coronagraph

- Intensity in plane D, final focal plane with the detector
Propagation inside the coronagraph

- Intensity in plane D, final focal plane with the detector

- No occulter and stop
  - Solar disk image

- Just the external occulter
  - No internal occulter
  - No Lyot stop

- Without external occulter
  - (Lyot coronagraph)

- With external/internal occulators and Lyot stop
Propagation inside the coronagraph

- Impact of sizing the internal occulter and the Lyot stop
  Intensity on plane D, the final focal plane

Fixed Lyot stop

Fixed internal occulter
Propagation inside the coronagraph

- Impact of sizing the internal occulter and the Lyot stop

Residual diffracted sunlight @ $1.3R_\odot$

Better rejection

Closer to solar edge

PSF in the vignette zone
Conclusion
Conclusion

• Headlines of the presentation:
  - the different types of external occulter (in solar coronagraphy)
  - the penumbra profiles
  - propagation of diffracted light to understand rejection performance

• Reference:
  - Aime C., 2013, A&A
Conclusion

- On-going/future works:
  - deviation from ideal optics: scattering, optical aberrations...
  - end-to-end performance for the serrated occulters
Questions?

Why a 150m long coronagraph?

Thank you for your attention!
Annex
**Diffraction by an external occulter**

How to model diffraction? We looked at (when applicable):

<table>
<thead>
<tr>
<th>Method</th>
<th>Radial apodisation</th>
<th>No axis-symmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytical Hankel transformation</td>
<td>✓</td>
<td>✗</td>
</tr>
<tr>
<td>Lommel series*</td>
<td>✗</td>
<td>✗</td>
</tr>
<tr>
<td>Vanderbei et al. (2017) approach**</td>
<td>✓</td>
<td>✓ (periodic)</td>
</tr>
<tr>
<td>Brute force 2D FFT</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Rubinowicz representation</td>
<td>✗</td>
<td>✓</td>
</tr>
</tbody>
</table>

* Not introduced in this presentation
** Not suitable for the solar case
The Hankel transformation

Fourier wave optics formalism

Fresnel free-space propagation

Axis-symmetric (apodized) occulter

\[
\Psi_z(x, y) = (1 - f(r)) \otimes \frac{1}{i \lambda z} \exp \left( \frac{i \pi}{\lambda z} (x^2 + y^2) \right)
\]

\[
\Psi_z(r) = \frac{\varphi_z(r)}{i \lambda z} \int_0^R 2\pi \rho \times f(\rho) \times \exp \left( \frac{i \pi \rho^2}{\lambda z} \right) \times J_0 \left( \frac{2\pi \rho r}{\lambda z} \right) d\rho
\]

Diffraction at z
Radial function

Lommel series – decomposition into series (Aime, 2013)
2D FFT technique

Fourier wave optics formalism

Fresnel free-space propagation

Occulter of any shape and any transmission (ideally)

\[ \Psi_z(x, y) = \Psi_0(x, y) \bigotimes \frac{1}{i\lambda z} \exp \left( \frac{i\pi}{\lambda z} (x^2 + y^2) \right) \]

\[ \Psi_z(x, y) = \mathcal{F}^{-1} \left[ \mathcal{F} \left[ \Psi_0(x, y) \right] \times \exp(-i\pi\lambda z (u^2 + v^2)) \right] \]

Diffraction at \( z \)

2D function

Occulter

2D shape + apodisation

Fresnel filter
2D FFT technique

The occulter $\Psi_0(x, y)$ is padded in an array $K \times K$ with sampling $\sigma$

- Usually, for FFT routines:
  - The bigger $K$, the better (padding)
  - The smaller $\sigma$, more accurate computation (high-frequency)
2D FFT technique

An additional condition!

The Fresnel filter \( \exp(i\pi\lambda z u^2) \) has its phase varying as \( u^2 \)

At the edge of the array, \( u_c = 1/2\sigma \)

We impose that the (maximum) phase variation at the edge of the array is \( <\pi \)

\[
\sigma > \sqrt{\frac{\lambda z}{K}}
\]

Consequence: \( \sigma \downarrow \implies K \uparrow \)
2D FFT technique

Very sensitive to numerical sampling: impact of the size of the array
2D FFT technique

Very sensitive to numerical sampling: impact of sampling

- Sampling too small regarding Fresnel filter’s condition
- Sampling $S$ meeting the condition
- Sampling too large to correctly sample the occulter
The use of serrated external occulters in stellar and solar coronagraphy comes from very different reasoning, but the diffraction principle is the same.

Vanderbei et al. (2007) introduces another method to compute Fresnel diffraction.

For serrated or petal-shaped occulter, i.e. a periodic pattern by rotation

\[
\Psi_z(r, \theta) = \Psi_z^{apod}(r) + \sum_{j=1}^{\infty} f_1(j, N_t) \times \int_0^{R+\Delta} f_2(j, \rho) \times J_{jN_t} \left( \frac{2\pi r \rho}{\lambda z} \right) \rho d\rho
\]

Diffraction from related apodized occulter

Sum up to infinity

High-orders Bessel functions (jN_t)

In stellar coronagraphy:

\( N_t \approx 20 \), and very small working angles: \( j=1 \) dominates

In solar coronagraphy:

\( N_t \approx 100 - 1000 \), and large region (671mm): the computation is very heavy
Rubinowicz representation

Based on Kirchhoff integral theorem (Born & Wolf; Cady, 2012)
Requires a “1 or 0” occulter: no apodization
The diffraction is written as a boundary integral along the edge of the occulter

\[
\Psi_z(x, y) = -\Psi_z^{(d)}(x, y) \text{ in the geometrical shadow}
\]
\[
\Psi_z(x, y) = \Psi_0 \quad \Psi_z^{(d)}(x, y) \text{ otherwise}
\]

\[
\Psi_z^{(d)} = \int_{\partial \Omega} W \, dl
\]

Geometrical wave

Diffraction disturbance
= boundary diffraction wave integral

Edge of the occulter
Penumbra for serrated occulters

Convolution of the diffraction intensity $|\Psi_z(x, y)|^2$ with the solar stenope image
Includes limb darkening function

$R_{\text{sun}} = 16.2'$

Penumbra: $\int \text{Diffraction} \times \text{Solar image}$
Penumbra for serrated occulters

\[ I(x = 0) \]

Boivin’s radius

R=710mm  X=0  R_{\text{sun}}=671mm  X-axis

Diffraction pattern

Solar image
Penumbra for serrated occulters

\[ I(x_1) \approx I(0) \]

R = 710mm

Boivin’s radius

Intensity

Diffraction pattern

Solar image

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Penumbra for serrated occulters

\[ I(x_2) > I(0) \]
Penumbra for serrated occulters

Boivin’s radius

\[ I(x = 0) \downarrow \]

Diffraction pattern
Solar image

\[ R_{\text{sun}} = 671 \text{mm} \]
Penumbra for serrated occulters

We can predict the penumbra depth for serrated occulters:

The deepest umbra is achieved when:

\[ \text{Boivin radius}(N_t, \Delta) > r_{\text{sun}} \]

The second parameter is the intensity level of the diffraction pattern

→ Large number of teeth preferred!