

# **Multi-Scale Modeling of Turbulence and Microphysics in Clouds**

Steven K. Krueger<sup>1</sup> and Alan R. Kerstein<sup>2</sup>

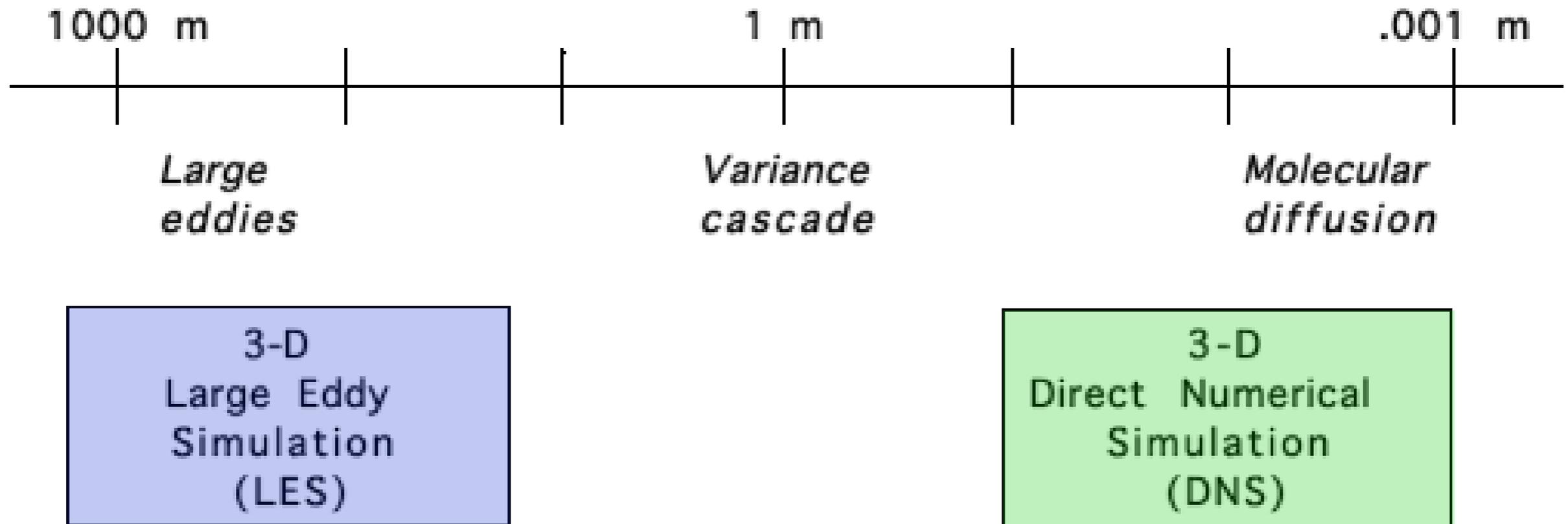
1. University of Utah

2. Sandia National Laboratories

**Fluctuations and Turbulence in the  
Microphysics and Dynamics of Clouds**

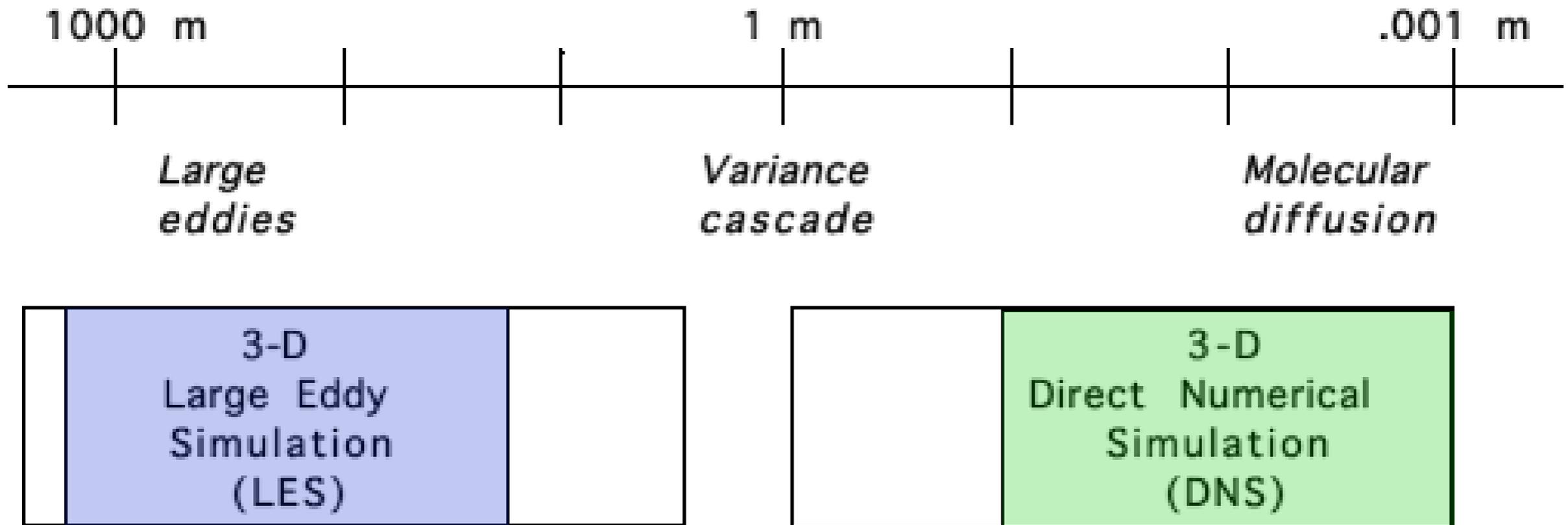
2-10 Sep 2010, Porquerolles, France

# *Scales of Atmospheric Turbulence*

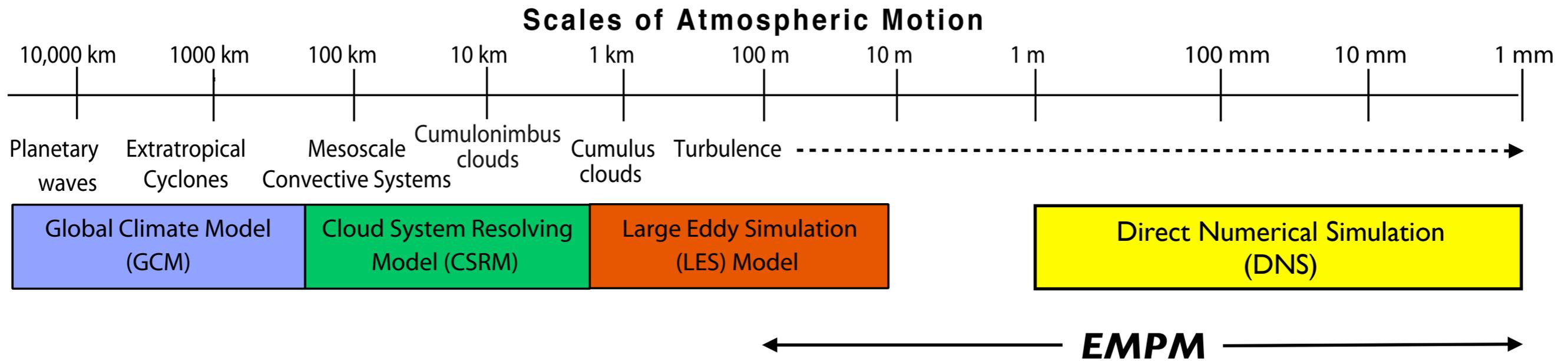


**1993**

# *Scales of Atmospheric Turbulence*



**2010**



The smallest scale of turbulence is the Kolmogorov scale:

$$\eta \equiv (\nu^3 / \epsilon)^{1/4}$$

For  $\epsilon = 10^{-2} \text{ m}^2 \text{ s}^{-3}$  and  $\nu = 1.5 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$ ,  $\eta = 0.7 \text{ mm}$ .

- Bridging the LES-DNS gap
  - Large-eddy simulation (LES)
  - Parcel model
  - Linear Eddy Model (LEM)
  - One-Dimensional Turbulence (ODT)
- 
- Explicit Mixing Parcel Model (EMPM)
  - ClusColl (Clustering and Collision Model)

- **Bridging the LES-DNS gap**

- Large-eddy simulation (LES)

- Parcel model

- Linear Eddy Model (LEM)

- One-Dimensional Turbulence (ODT)

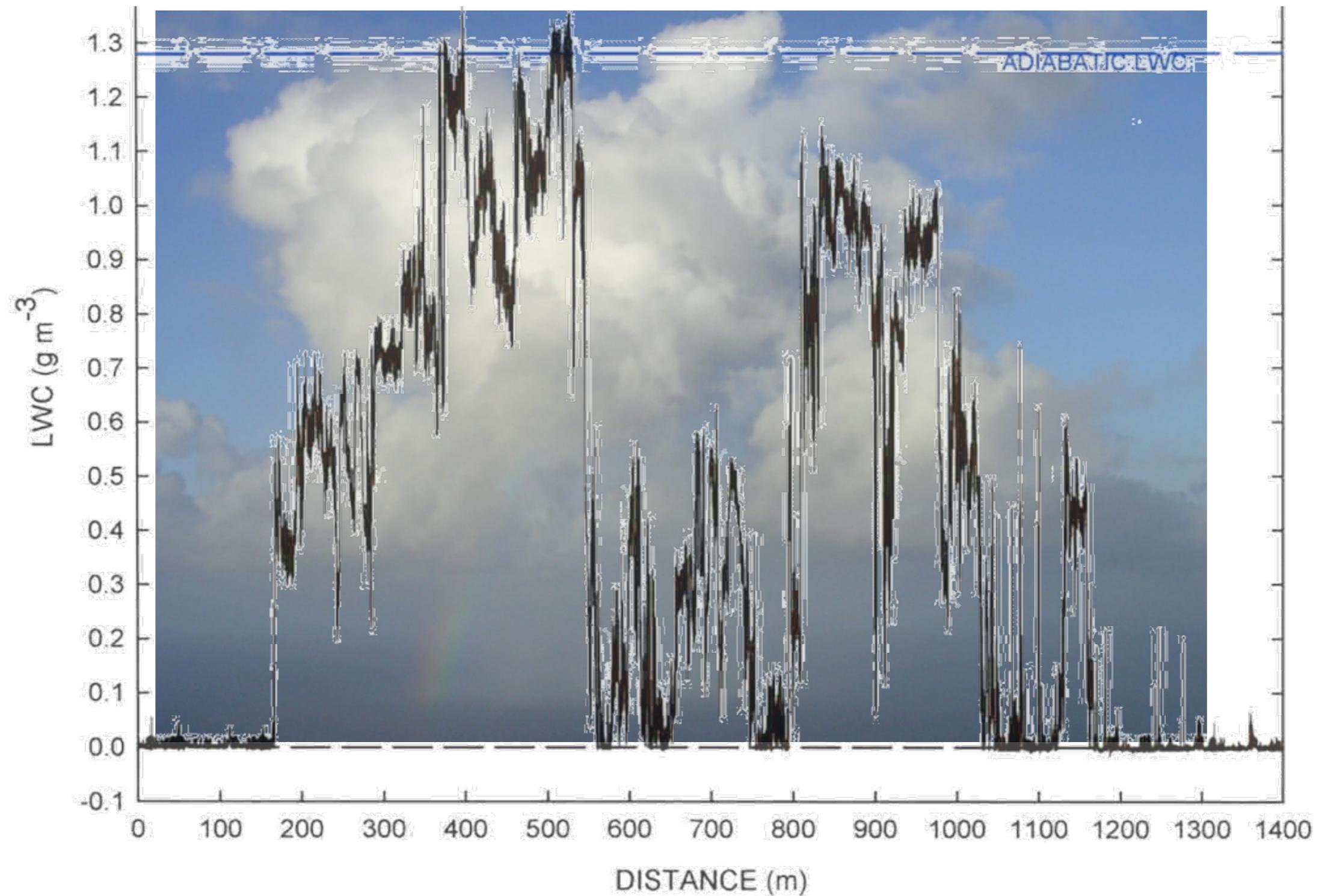
- Explicit Mixing Parcel Model (EMPM)

- ClusColl (Clustering and Collision Model)

- **Bridging the LES-DNS gap**

- Difficulty depends on process of interest.
- Higher resolution or improved conventional parameterization may work for some processes.
- For investigating how turbulence affects cloud droplet growth, **multi-scale modeling** (super-parameterization) is a promising solution.

# Small-scale variability in Cumulus mediocris



overlay is for illustration only

# Small-scale variability in Cumulus fractus

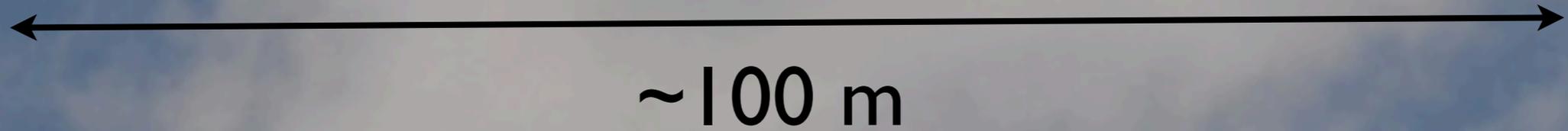
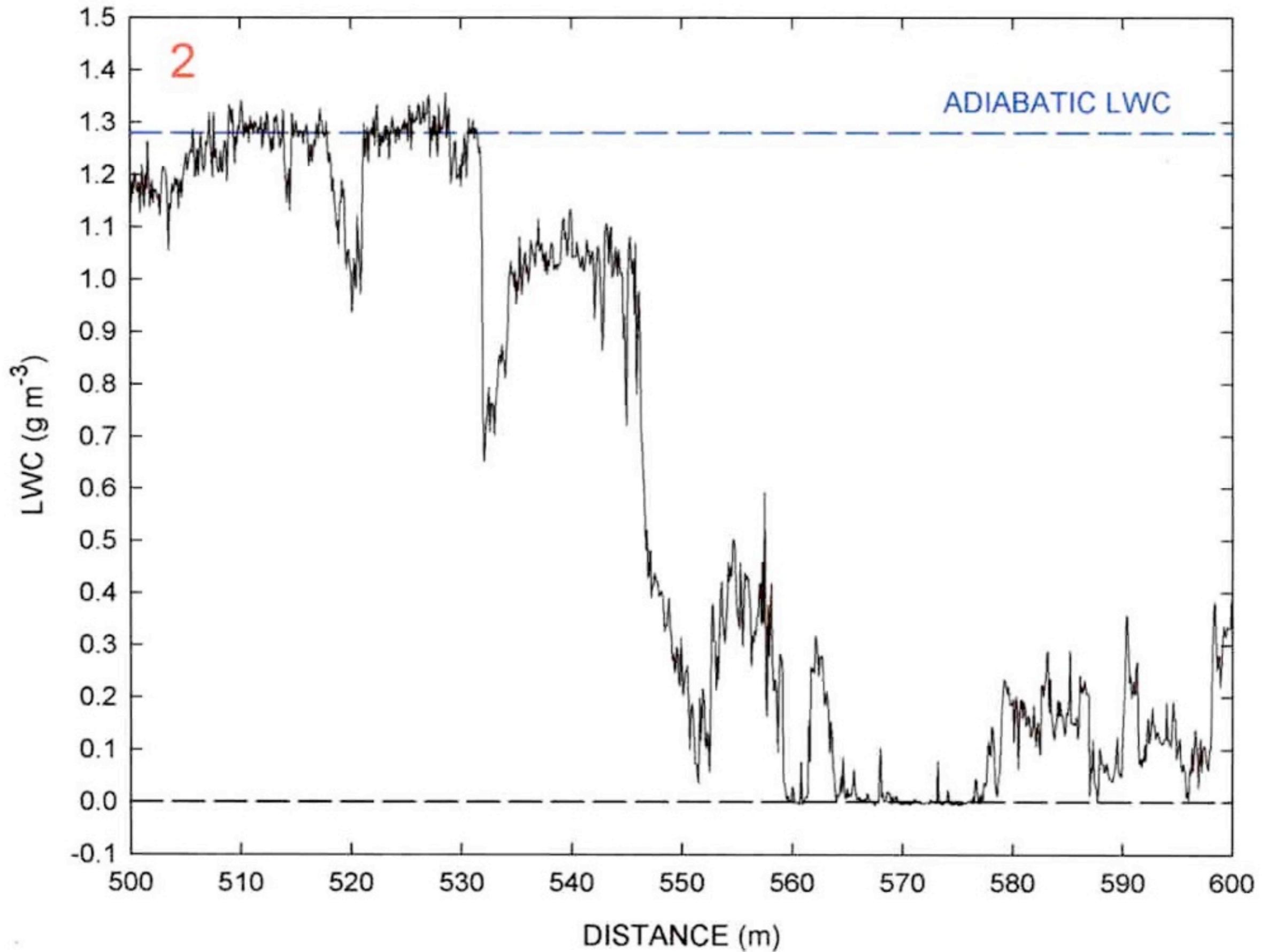
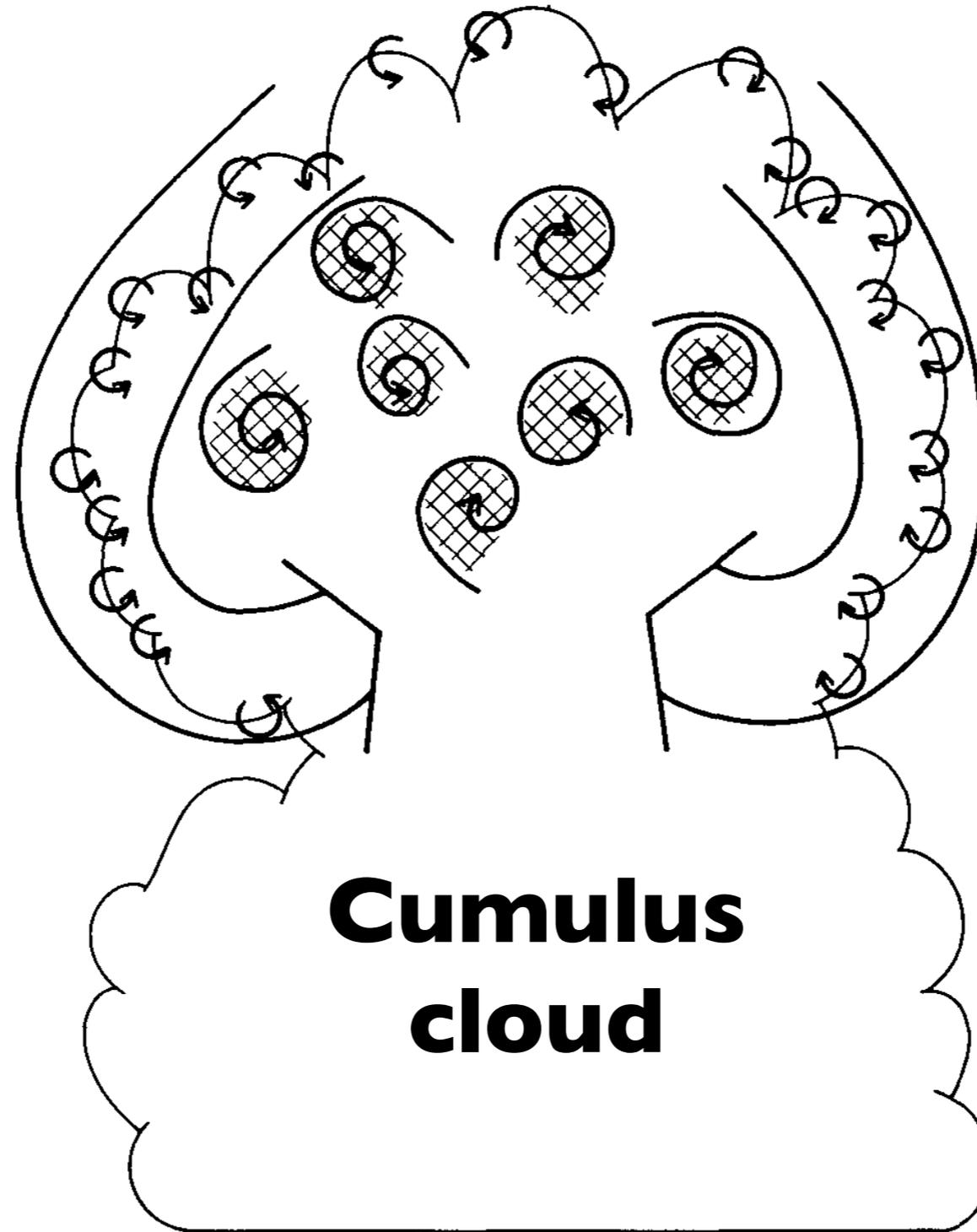


photo by Jan Paegle

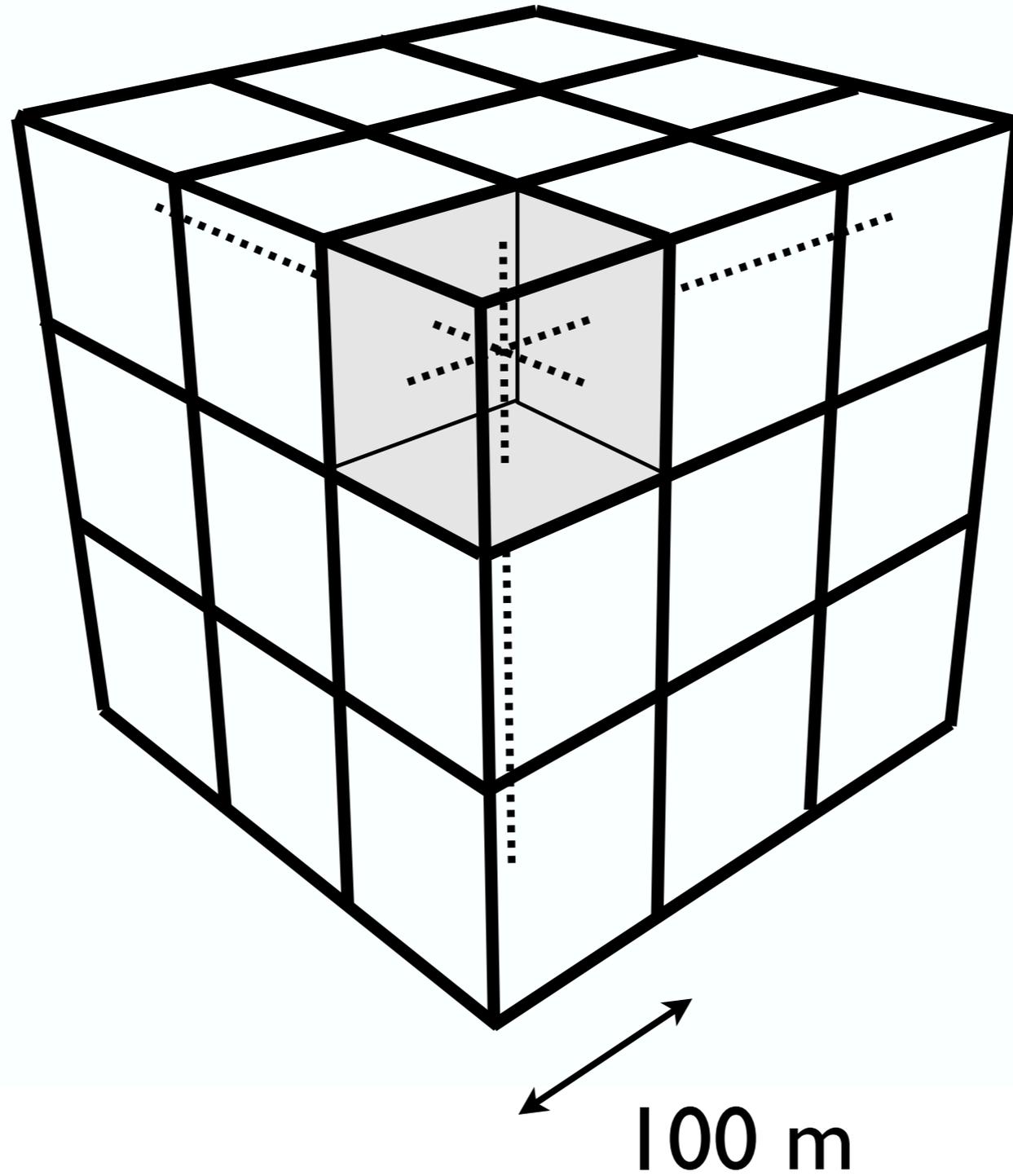
# Aircraft Measurements of Liquid Water Content



# Entrainment and Mixing



# Large-Eddy Simulation (LES) model



**no subgrid-scale variability**

# LES Limitations

- The premise of LES is that only the large eddies need to be resolved.
- Why resolve any finer scales? Why resolve the finest scales?
- LES is appropriate if the important small-scale processes can be parameterized.
- Many cloud processes are subgrid-scale, yet can't (yet) be adequately parameterized.

# Subgrid-scale Cloud Processes

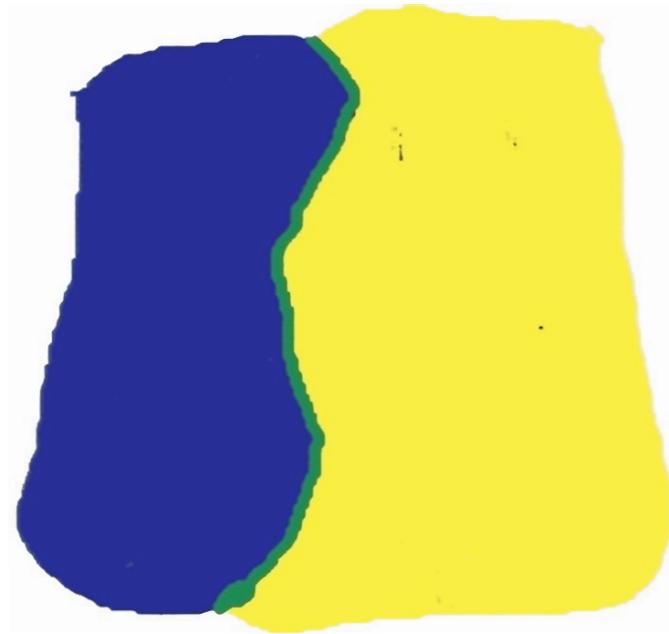
- Small-scale finite-rate **mixing** of clear and cloudy air determines evaporative cooling rate and affects buoyancy and cloud dynamics.
- Small-scale variability of water vapor due to entrainment and **mixing** broadens droplet size distribution (DSD) and increases droplet collision rates.
- Small-scale **turbulence** increases droplet collision rates.



~100 m

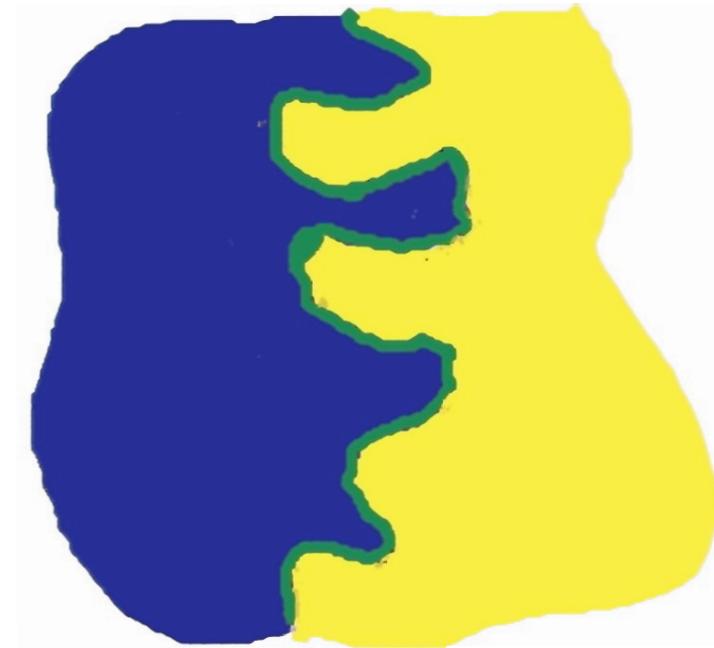
**Turbulent Mixing**

# Turbulent Mixing: Process by which a fluid with two initially segregated scalar properties mix at the molecular level



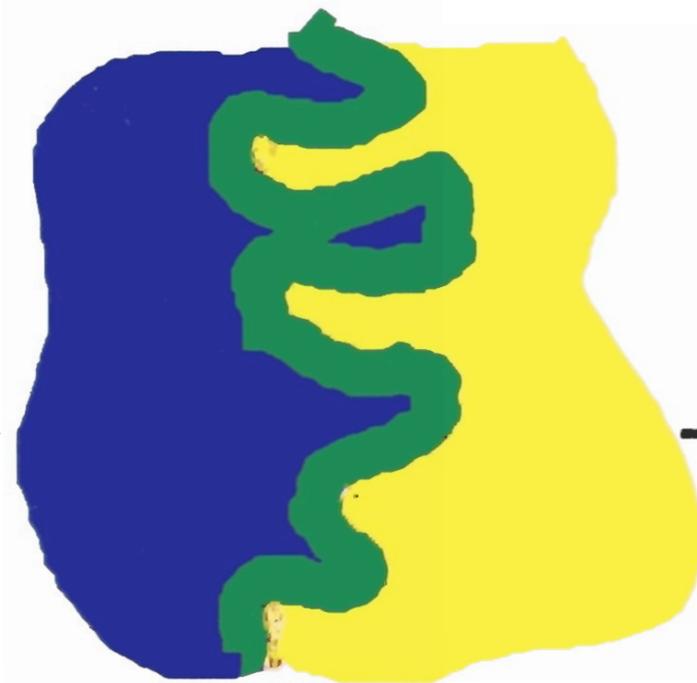
$$t_D = L^2 / D_m$$

Stirring

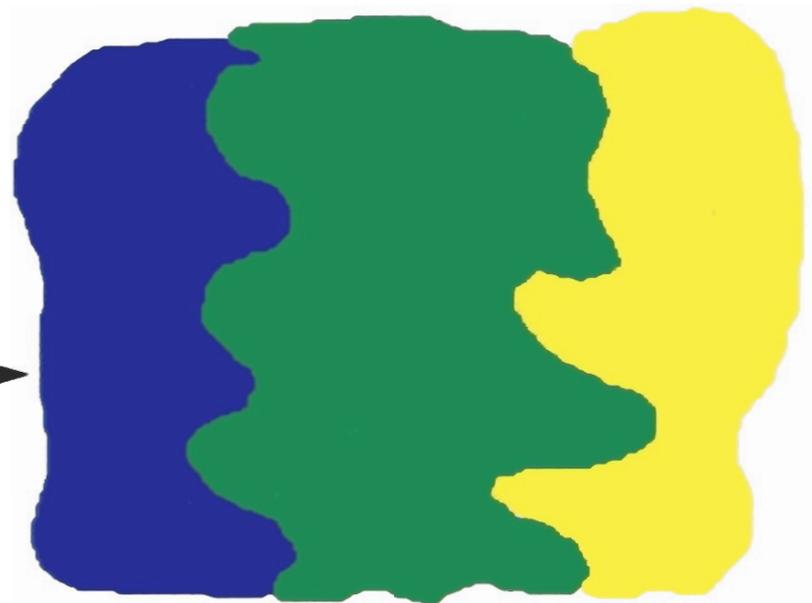


$$t_T = L / U$$

Stirring +  
Diffusion



Final Mixed  
State



# Kelvin-Helmholtz Instability: $Re=900$ and $1400$



# LES of passive scalar in a convective boundary layer (grid size = 20 m)

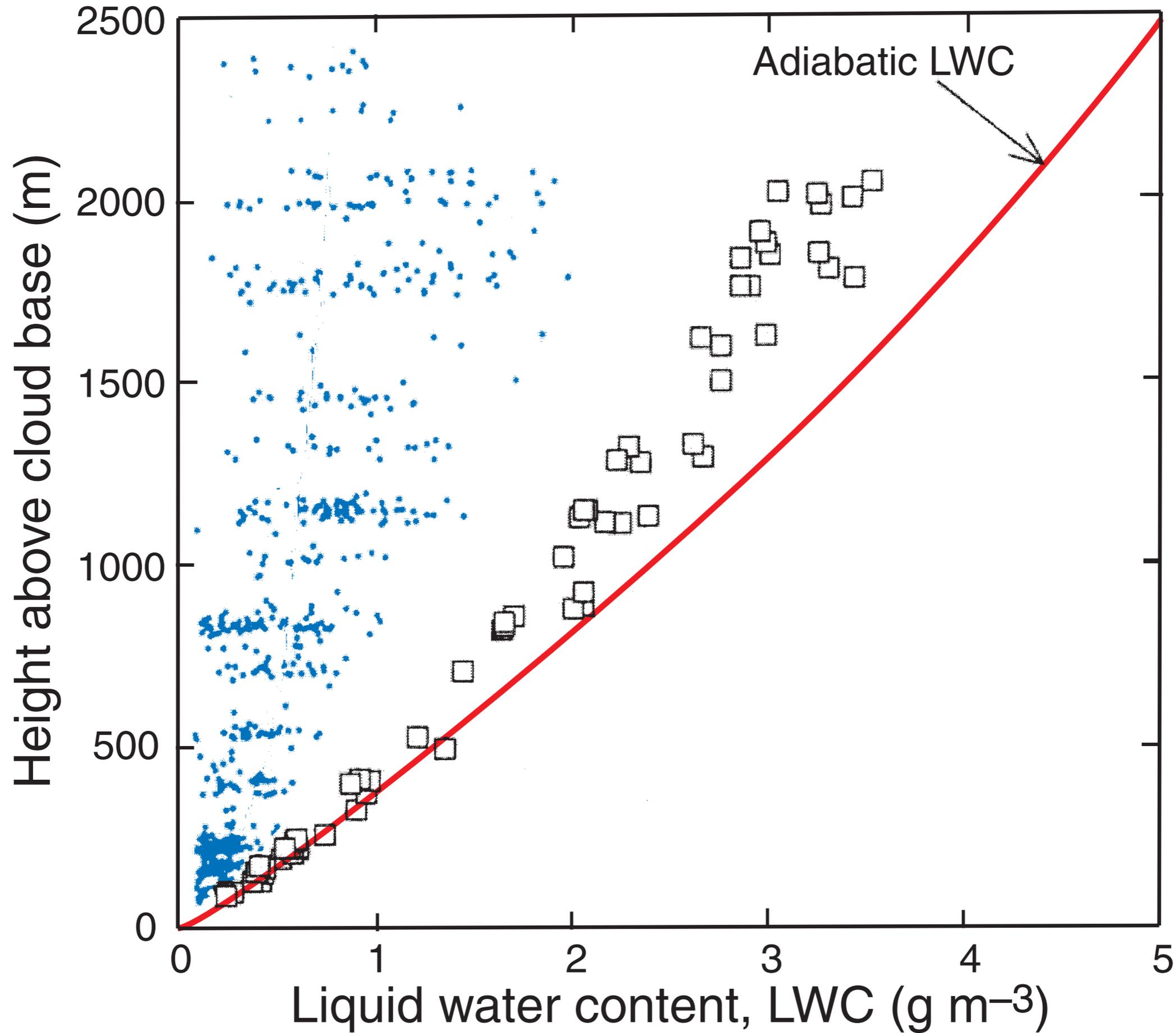


# Mixing Time Scale

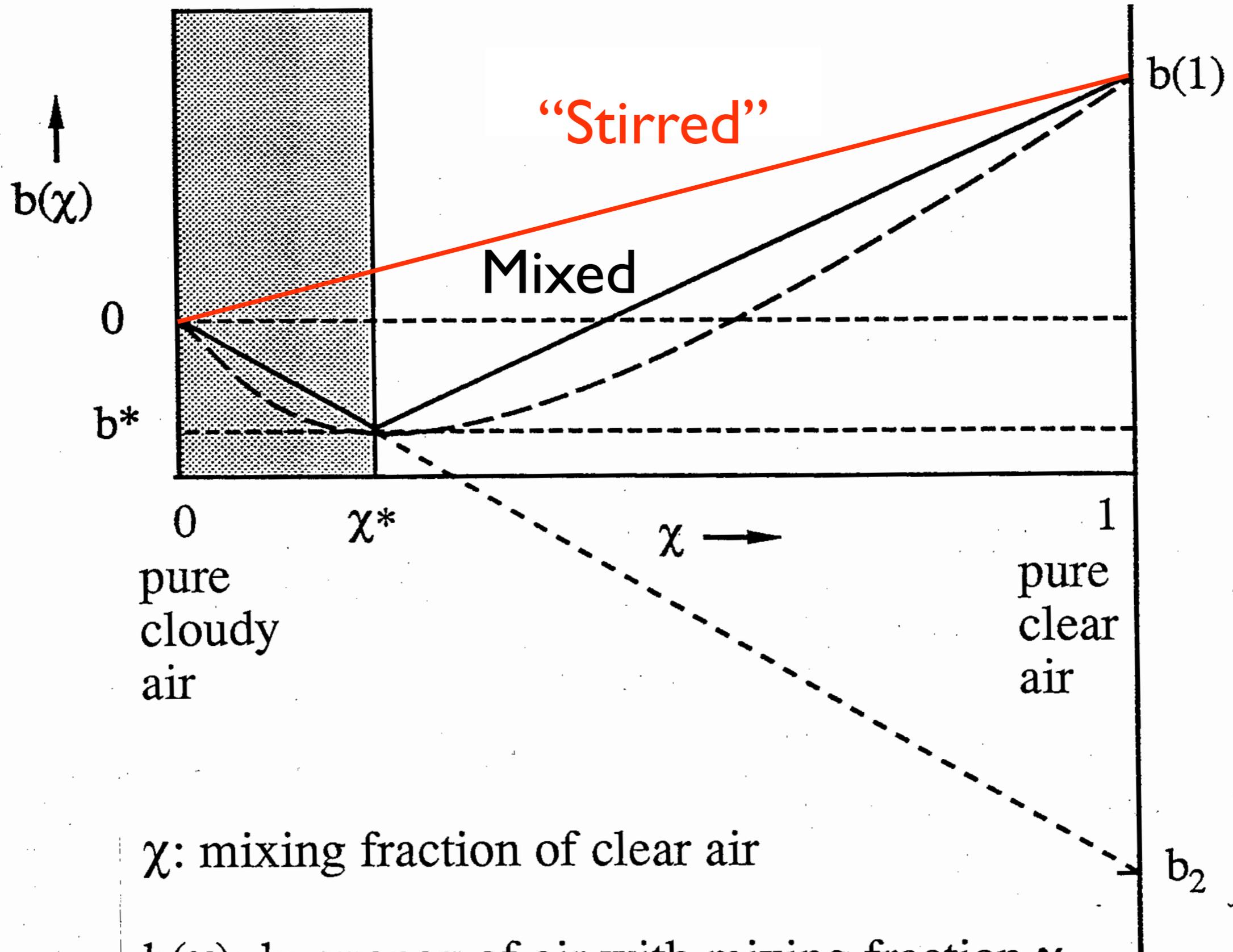
$$\tau = \left( \frac{d^2}{\epsilon} \right)^{1/3},$$

$d$  is entrained blob size,  $\epsilon$  is dissipation rate of turbulence kinetic energy.

For a **cumulus cloud**,  $U \sim 2$  m/s,  $L \sim 1000$  m, so  $\epsilon \sim U^3/L = 10^{-2}$  m<sup>2</sup>/s<sup>3</sup>. For  $d = 100$  m,  $\tau \sim 100$  s.



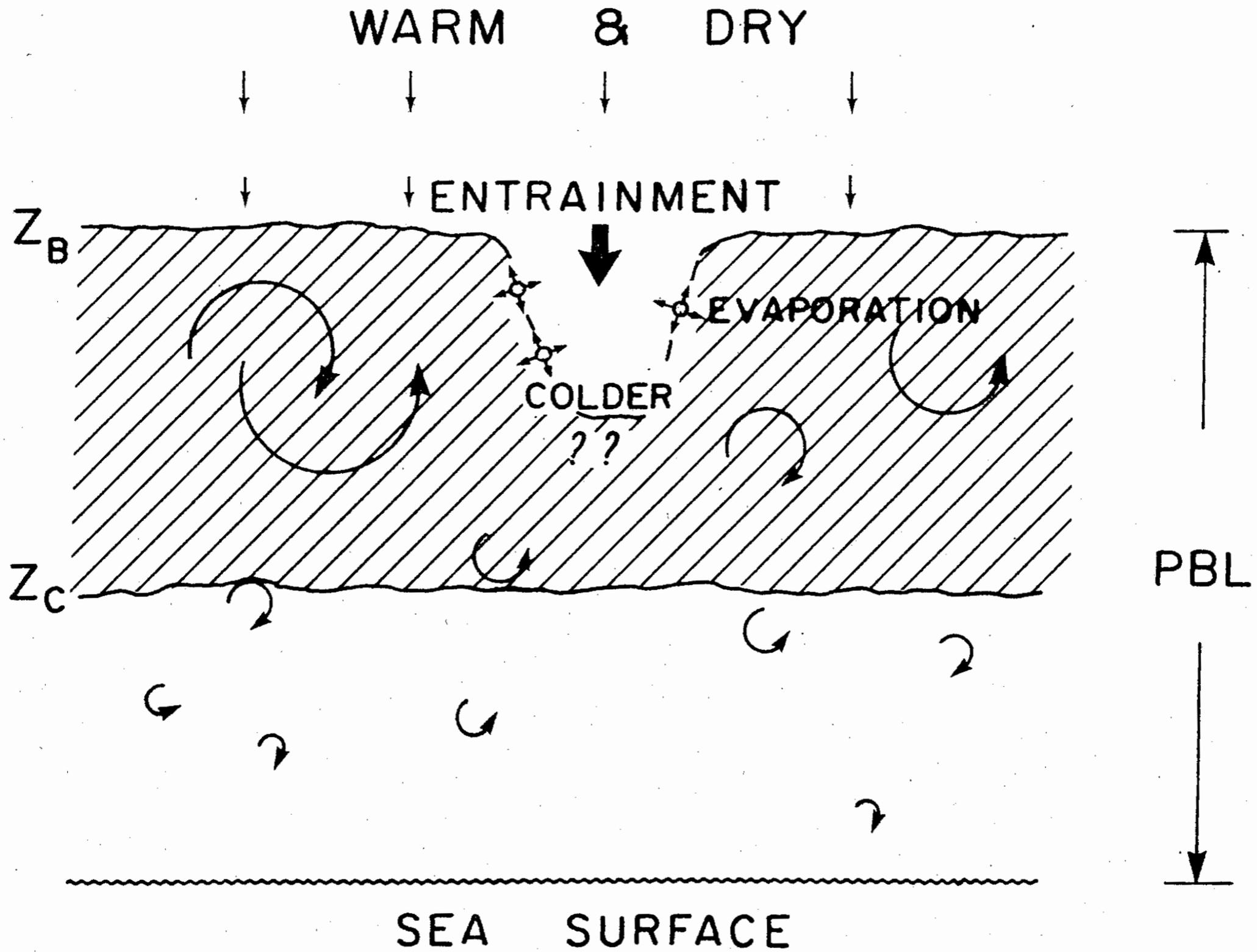
# Buoyancy vs Mixture Fraction



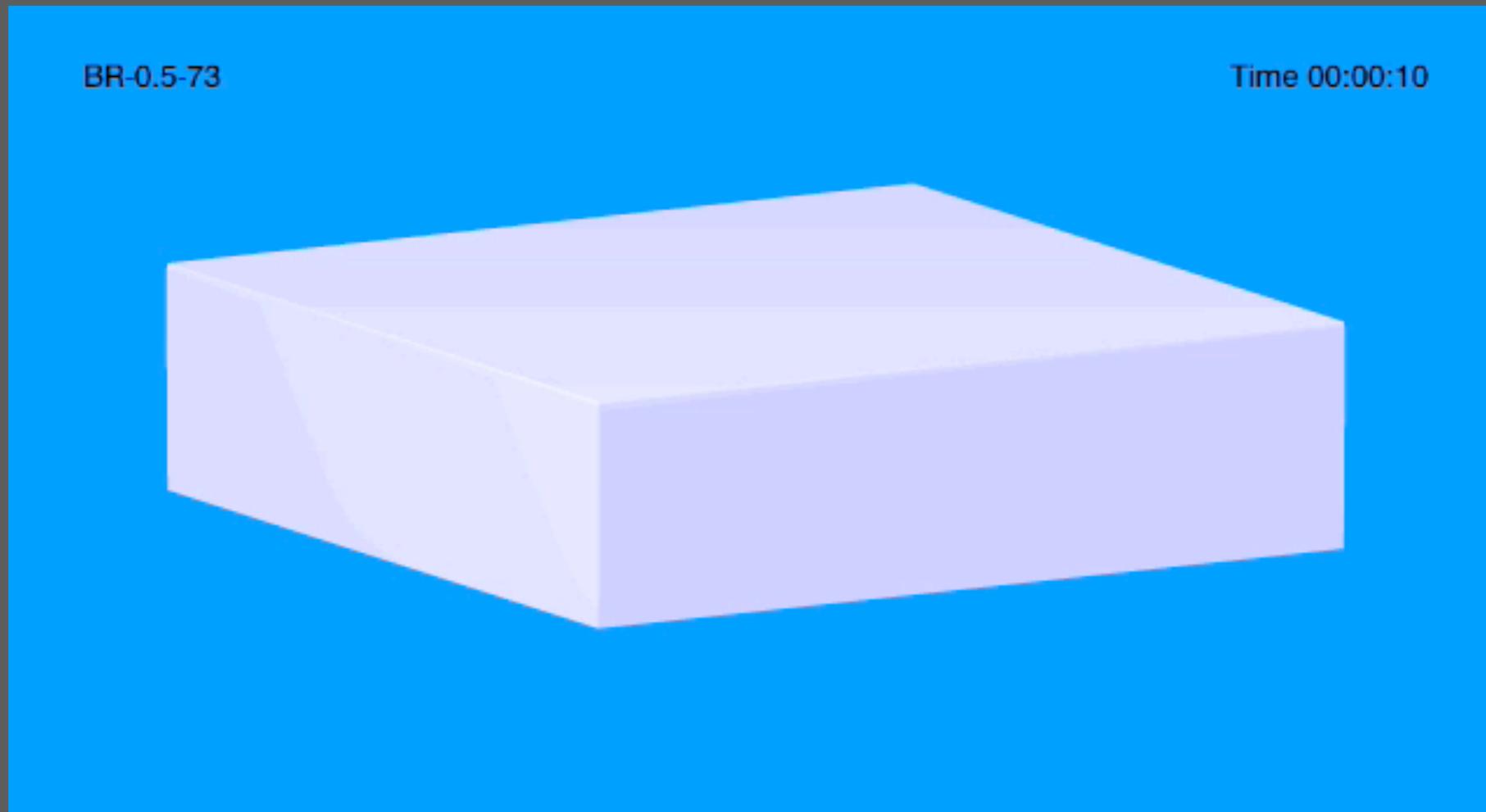
$\chi$ : mixing fraction of clear air

$b(\chi)$ : buoyancy of air with mixing fraction  $\chi$

# Cloud-top Entrainment Instability (CEI)



# 5 m isotropic grid



- Newly entrained thermals tend to follow the dry paths of earlier thermals.
- The dry paths become wider.

# Entrainment and mixing affect cloud droplet size distributions

An unsaturated blob is entrained at 375 s

some individual droplet radii

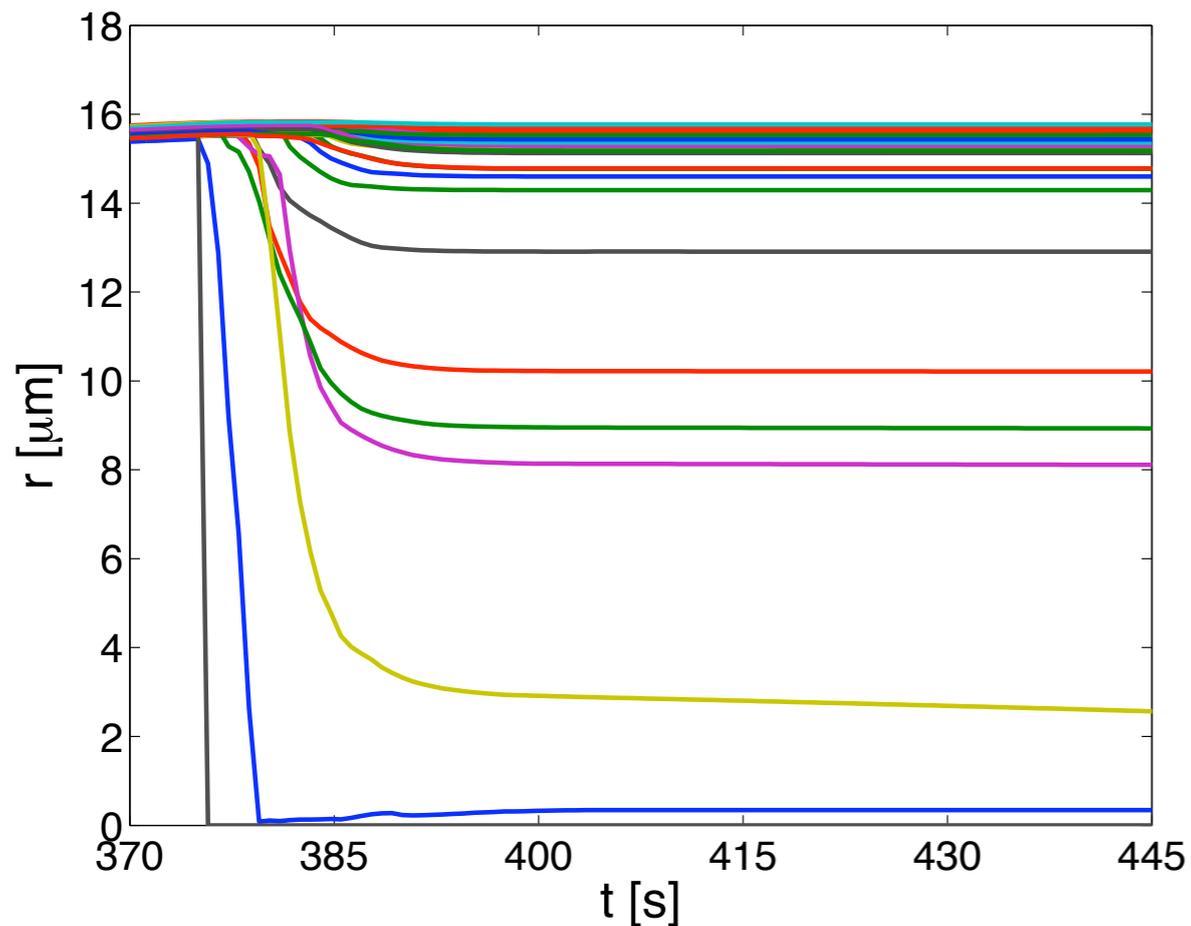


Figure 4.10: Radius histories of 30 droplets for  $f = 0.1$  and  $RH_e = 0.219$ .

width of droplet size distribution

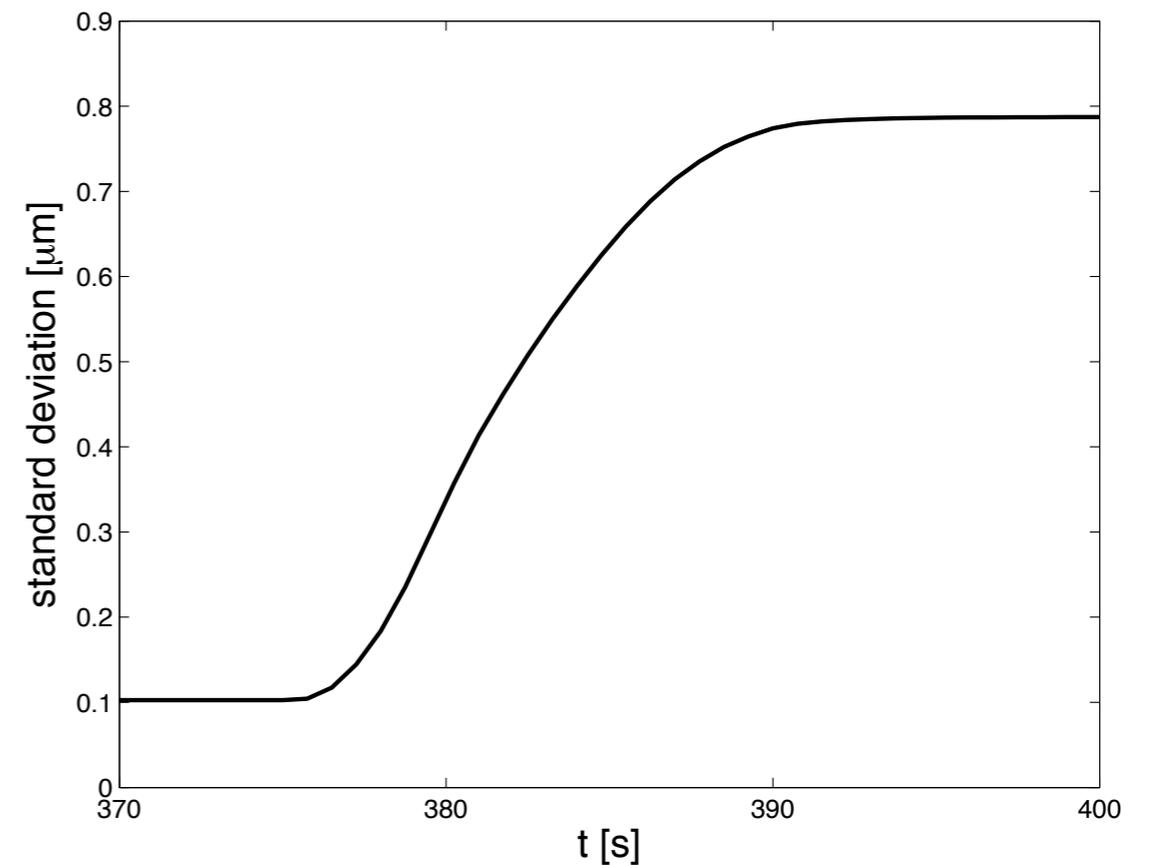
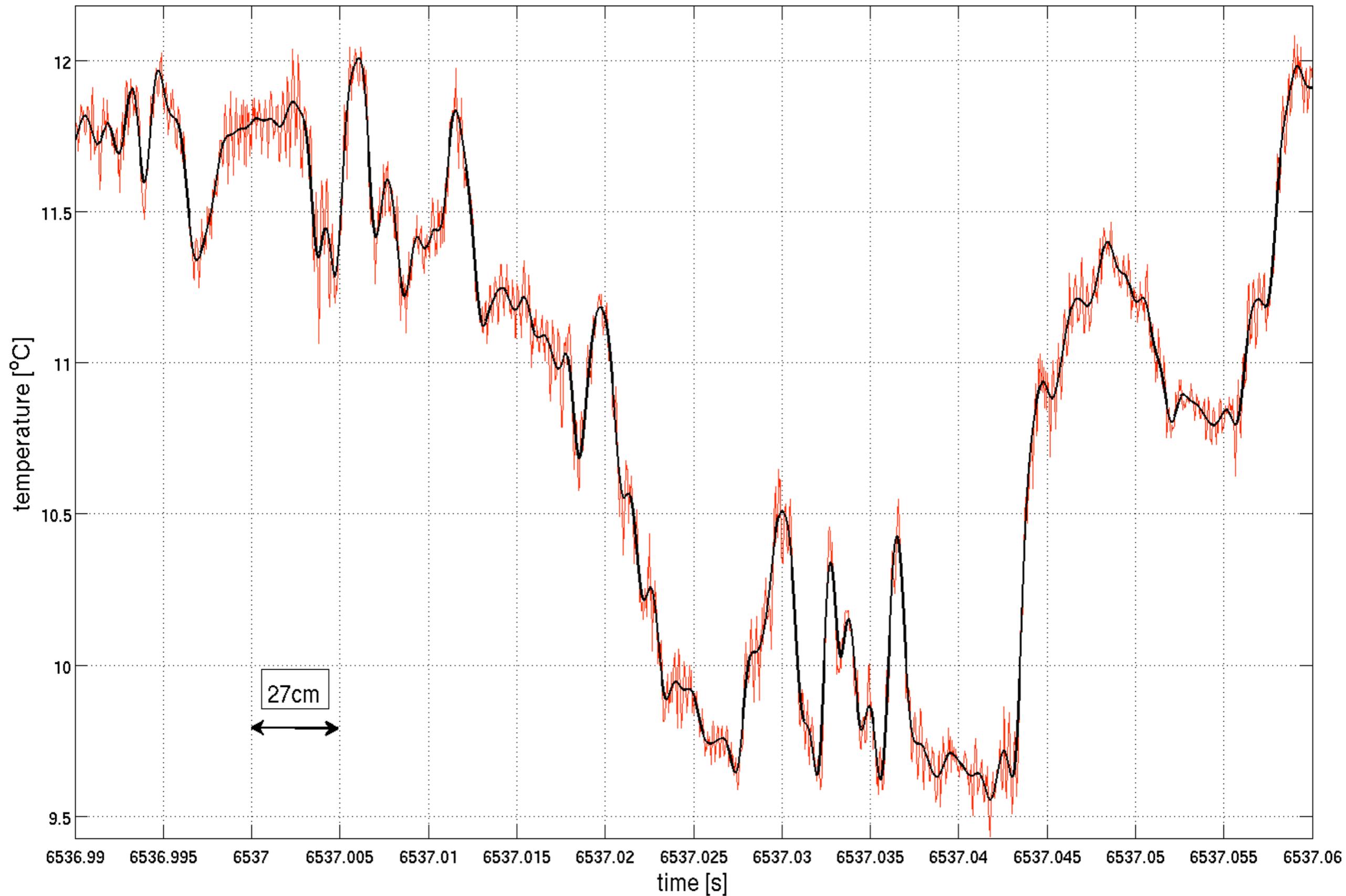


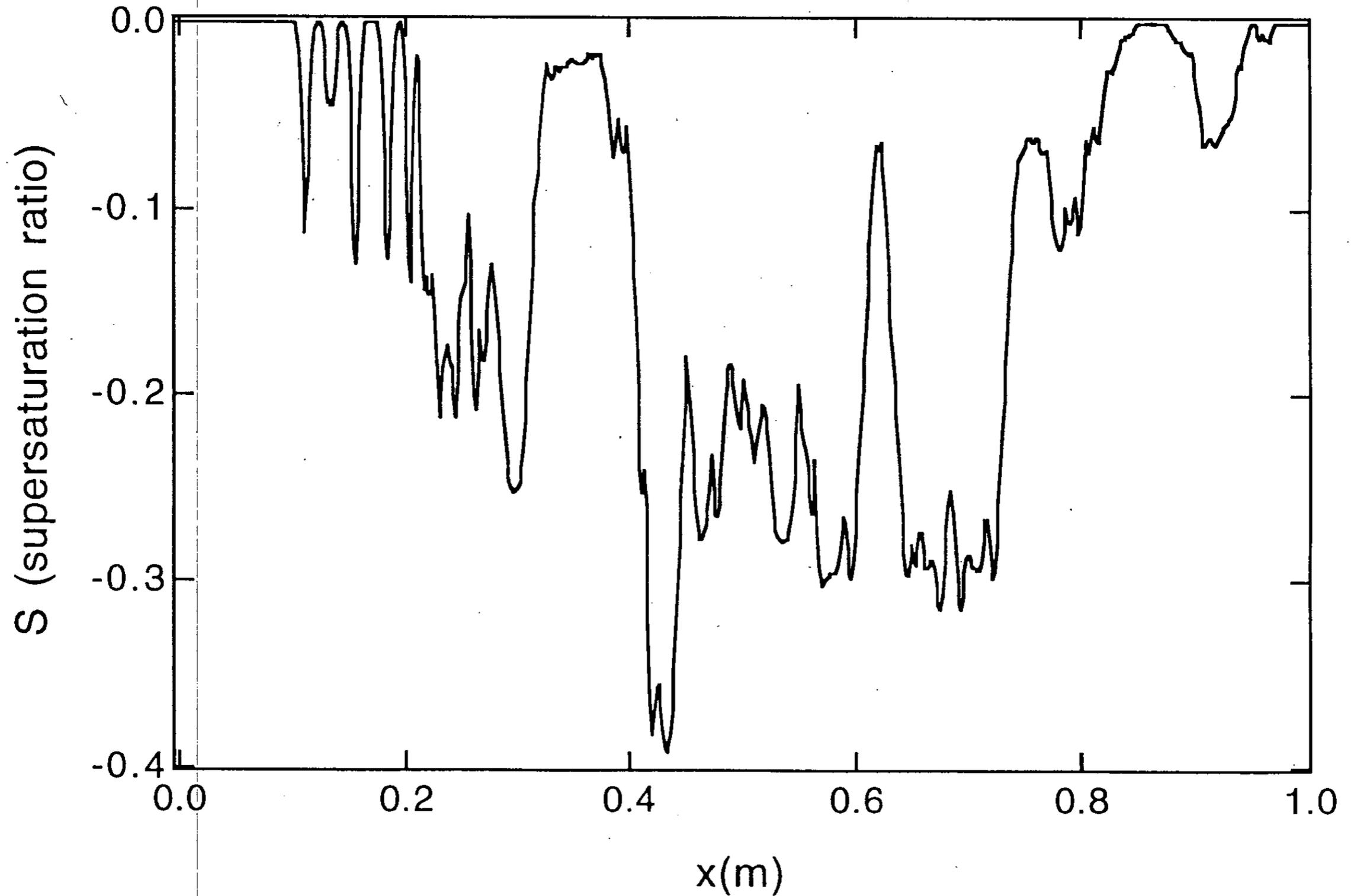
Figure 4.6: Standard deviation of the droplet radii just before entrainment until homogenization for entrainment fraction  $f = 0.2$  for the control case.

# Temperature Measured in Stratocumulus

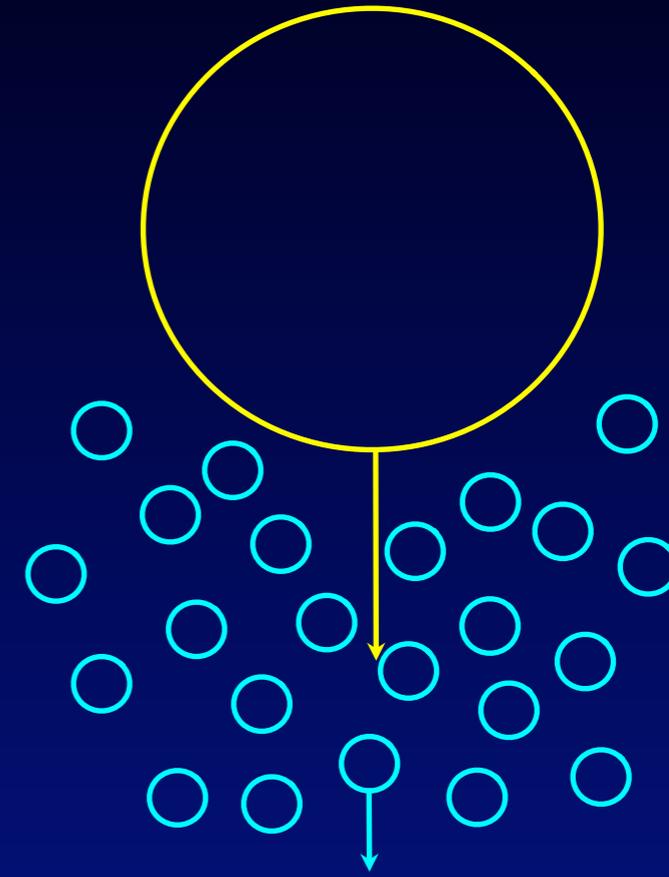
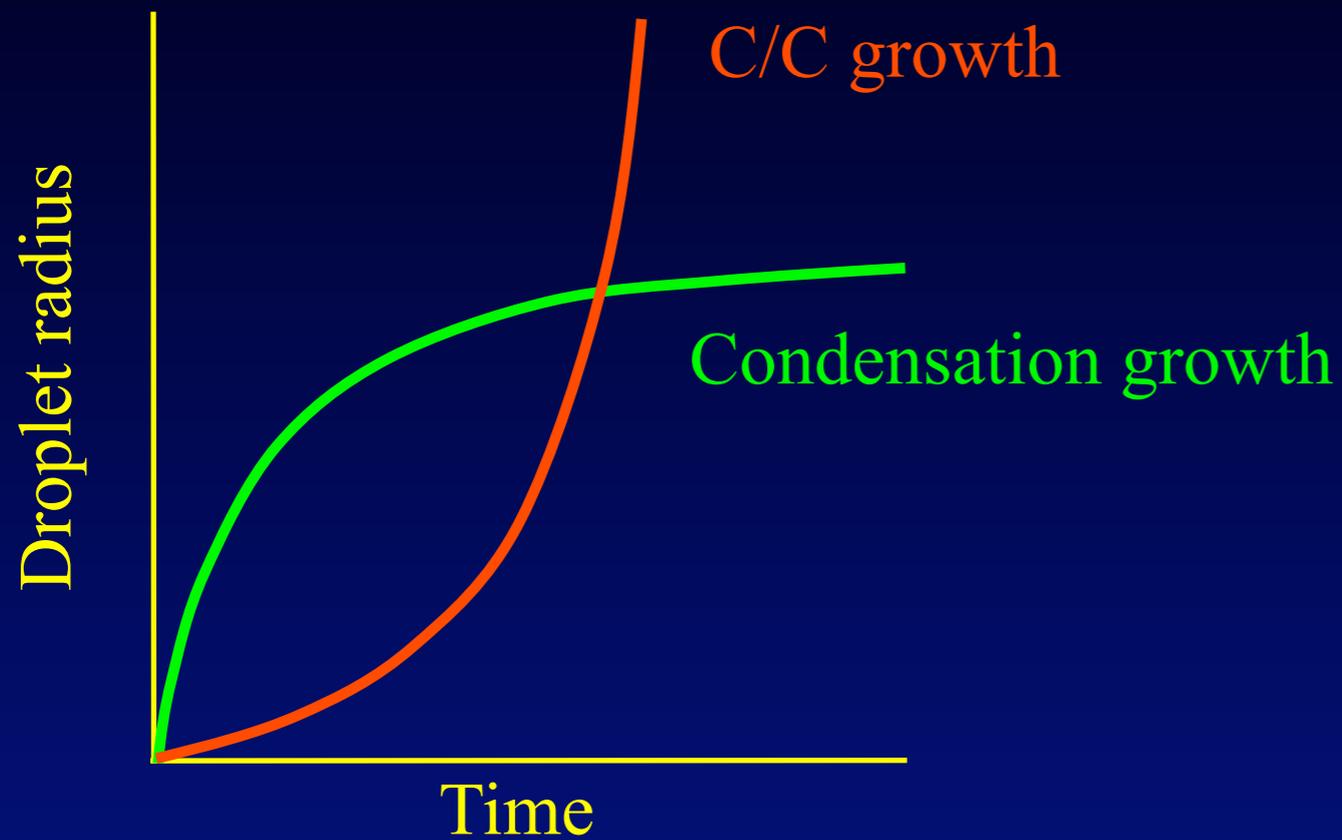


# Snapshot of supersaturation ratio during mixing

(from the *EMPM*)



# Collision-coalescence



- ◆ Growth of droplets into raindrops is achieved by **collision-coalescence**.
- ◆ Fall velocity of a droplet increases with size.
- ◆ Larger drops collect smaller cloud droplets and grow.

# Large droplets can initiate collision-coalescence growth

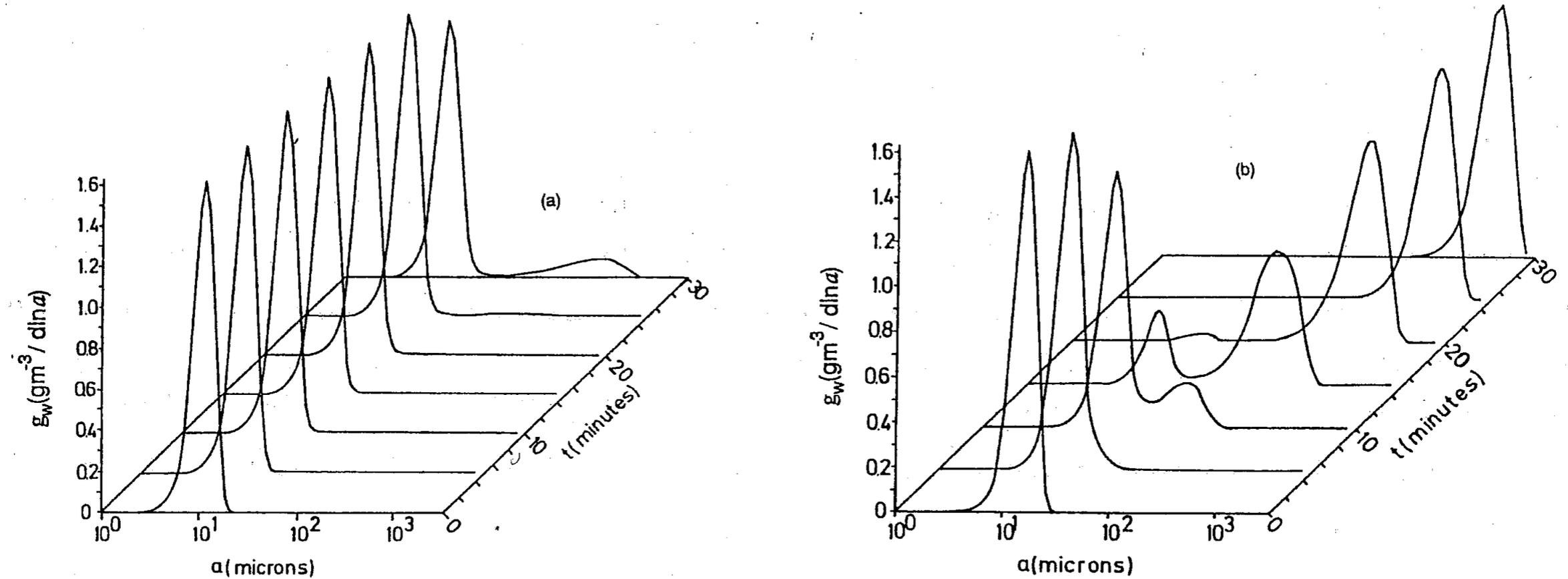
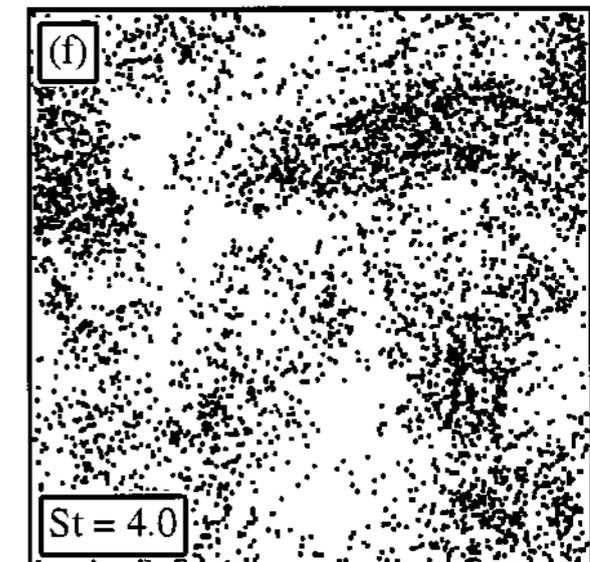
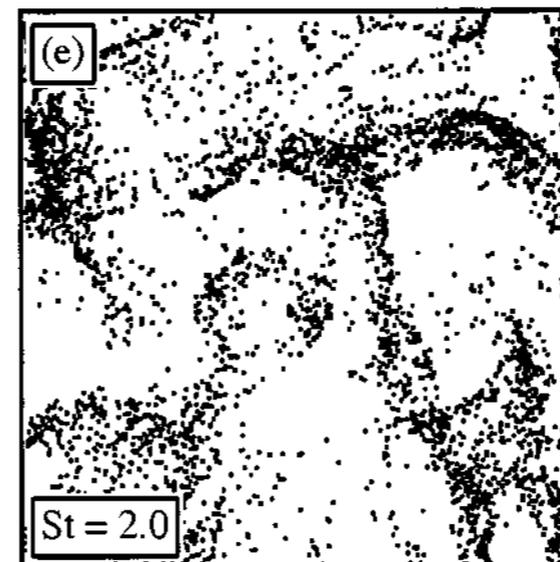
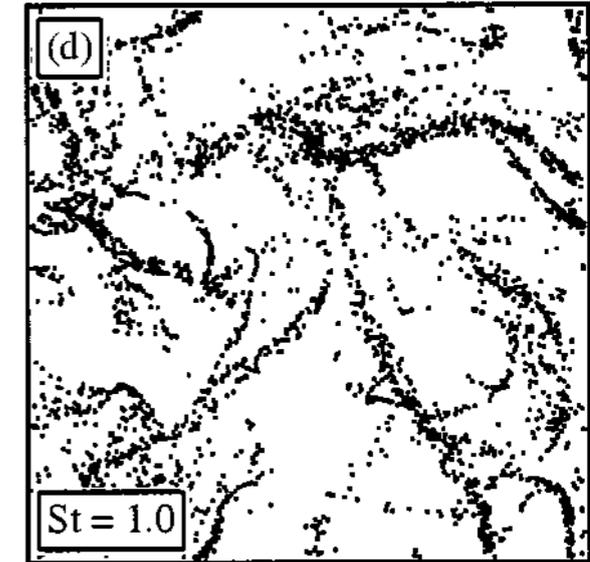
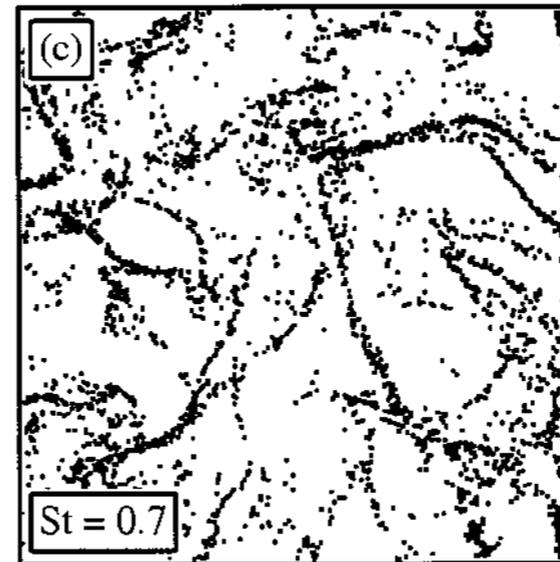
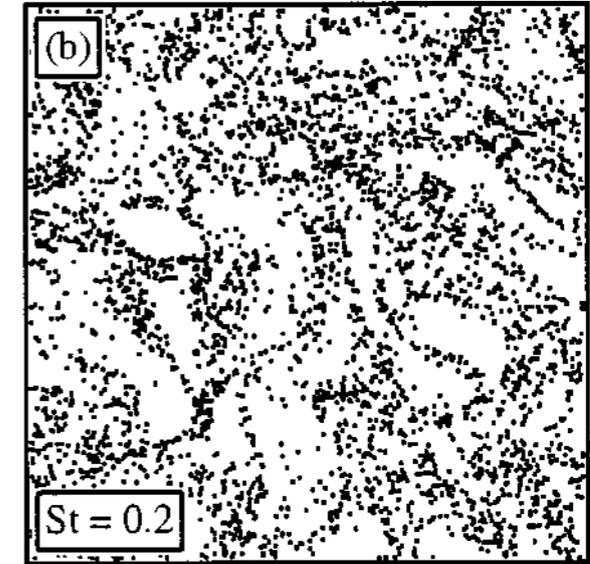
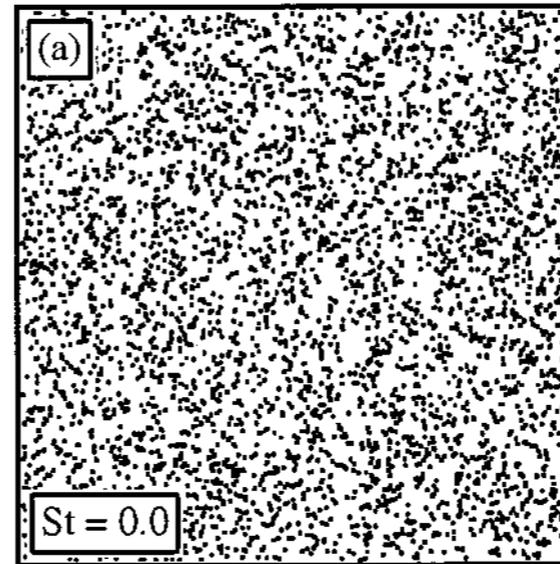


Fig. 15-8: Three-dimensional display of the time evolution of the drop mass distribution function as a function of drop radius, for an assumed initial spectrum of drops growing by collision and coalescence: (a)  $\bar{a} = 9 \mu\text{m}$ ,  $N_d = 237 \text{ cm}^{-3}$ ,  $w_L = 1 \text{ g m}^{-3}$ ; (b)  $\bar{a} = 13 \mu\text{m}$ ,  $N_d = 108 \text{ cm}^{-3}$ ,  $w_L = 1 \text{ g m}^{-3}$ . Based on the Berry Reinhardt method. (From Flossmann *et al.*, 1985, with changes.)

## Large droplets can initiate collision-coalescence growth

- *Processes that may contribute to large droplet production*
  - Entrainment and mixing of unsaturated air
  - Droplet clustering due to turbulence
  - Giant aerosols

# Clustering of inertial particles in turbulence increases collision rates



Direct numerical simulation results  
from Reade & Collins (2000)

# Parameterization of SGS Cloud Processes in LES

- SGS mixing is instantaneous in most LES.
- SGS variability does not affect DSD in any LES.
- SGS turbulence affects droplet collision rates in very few LES.

# Parameterization of SGS Cloud Processes in LES

(and how to improve, v. I)

- SGS mixing is instantaneous in most LES. (Decrease grid size or parameterize.)
- SGS variability does not affect DSD in any LES. (Decrease grid size.)
- SGS turbulence affects droplet collision rates in very few LES. (Modify collision kernel.)

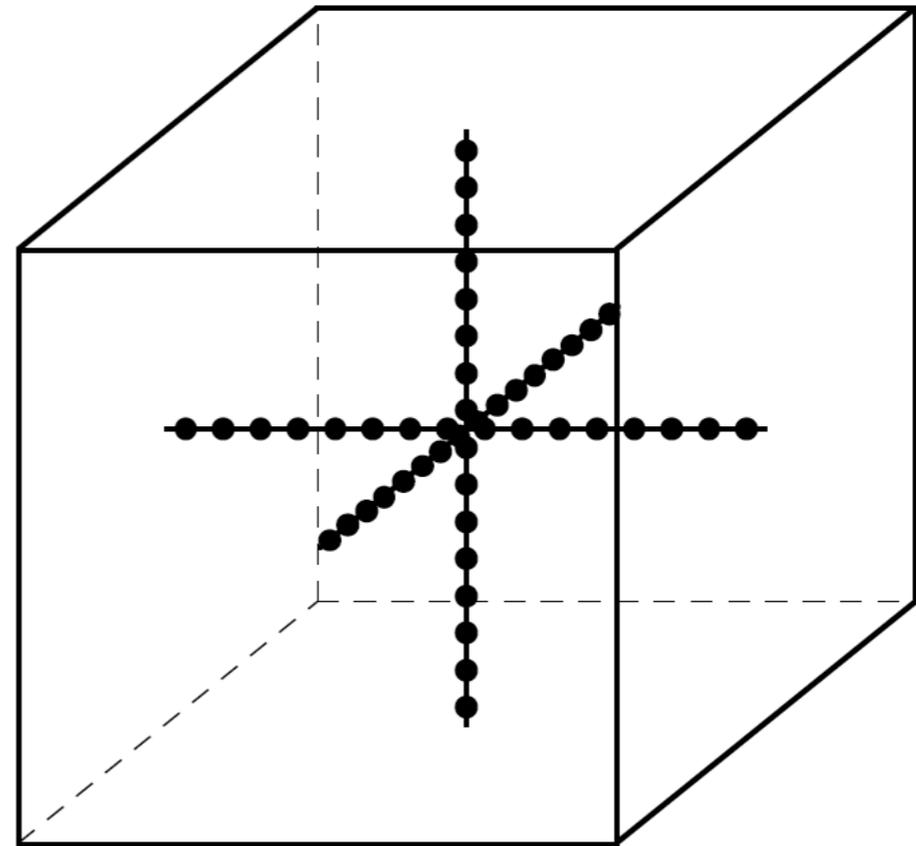
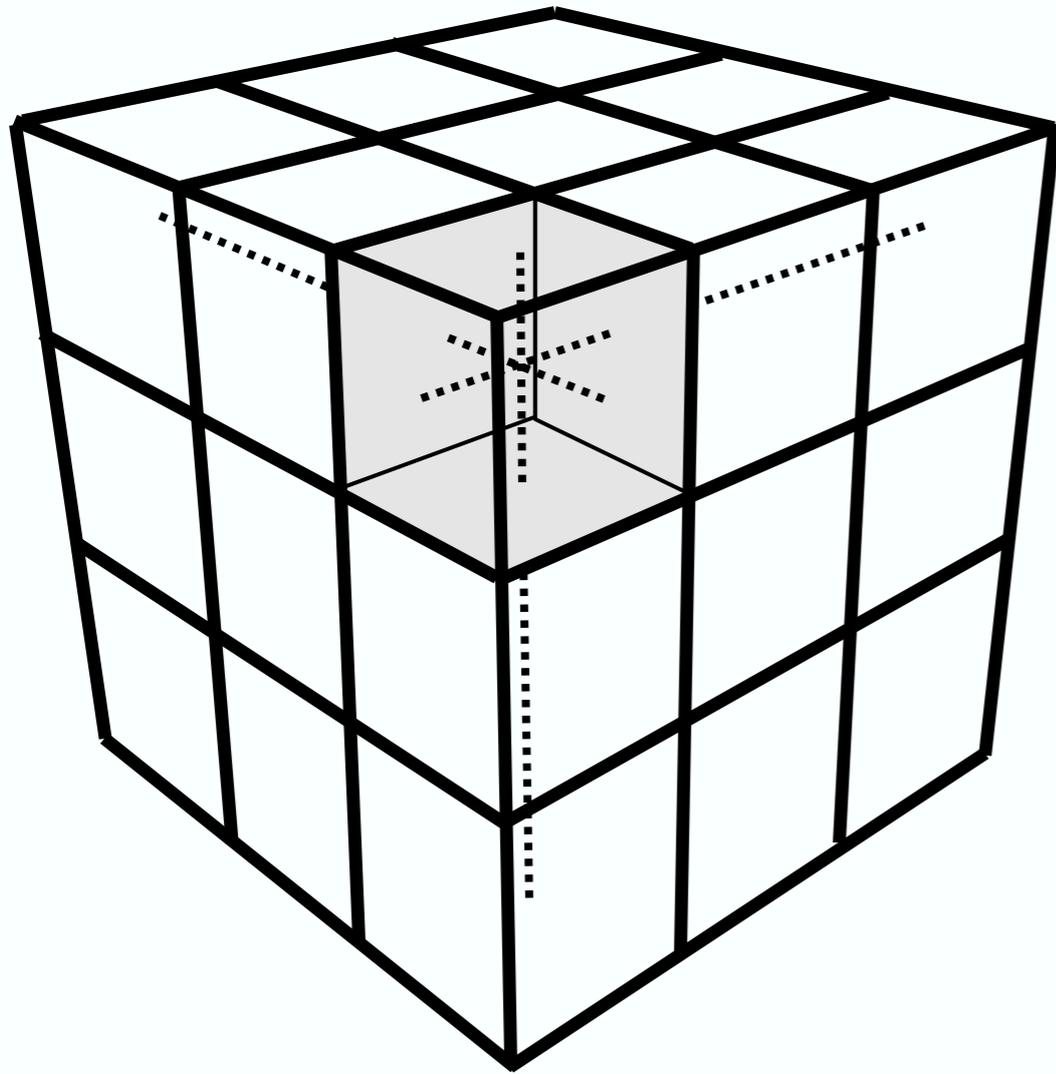
# How to resolve the small-scale variability?

- Decrease LES grid size?
  - To decrease LES grid size from 10 m to 1 cm would require  $10^9$  grid points per  $(10 \text{ m})^3$  and an increase in CPU time of  $10^{12}$ .
  - *This is not possible now or in the foreseeable future.*

# How to resolve the small-scale variability?

- Decrease dimensionality from 3D to 1D?
- To decrease grid size from 10 m to 1 cm would require only  $10^3$  grid points per  $(10 \text{ m})^3$ .
- *This is feasible now.*

# LES with 1D subgrid-scale model



# Summary

- Reducing the dimensionality is an established method.
- Removes or reduces the need for SGS parameterizations.
- It is very well suited for high-Reynolds number turbulent flows when small-scale mixing processes are important.

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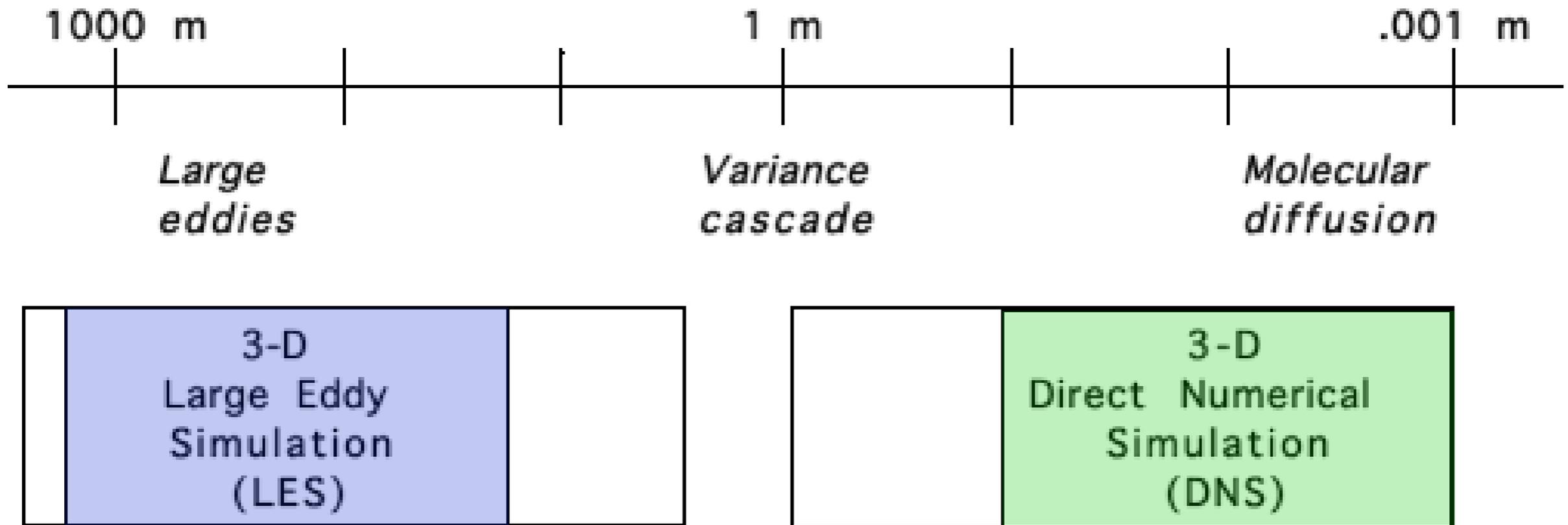
- **Large-eddy simulation (LES)**

- Description of LES

- Some applications of LES

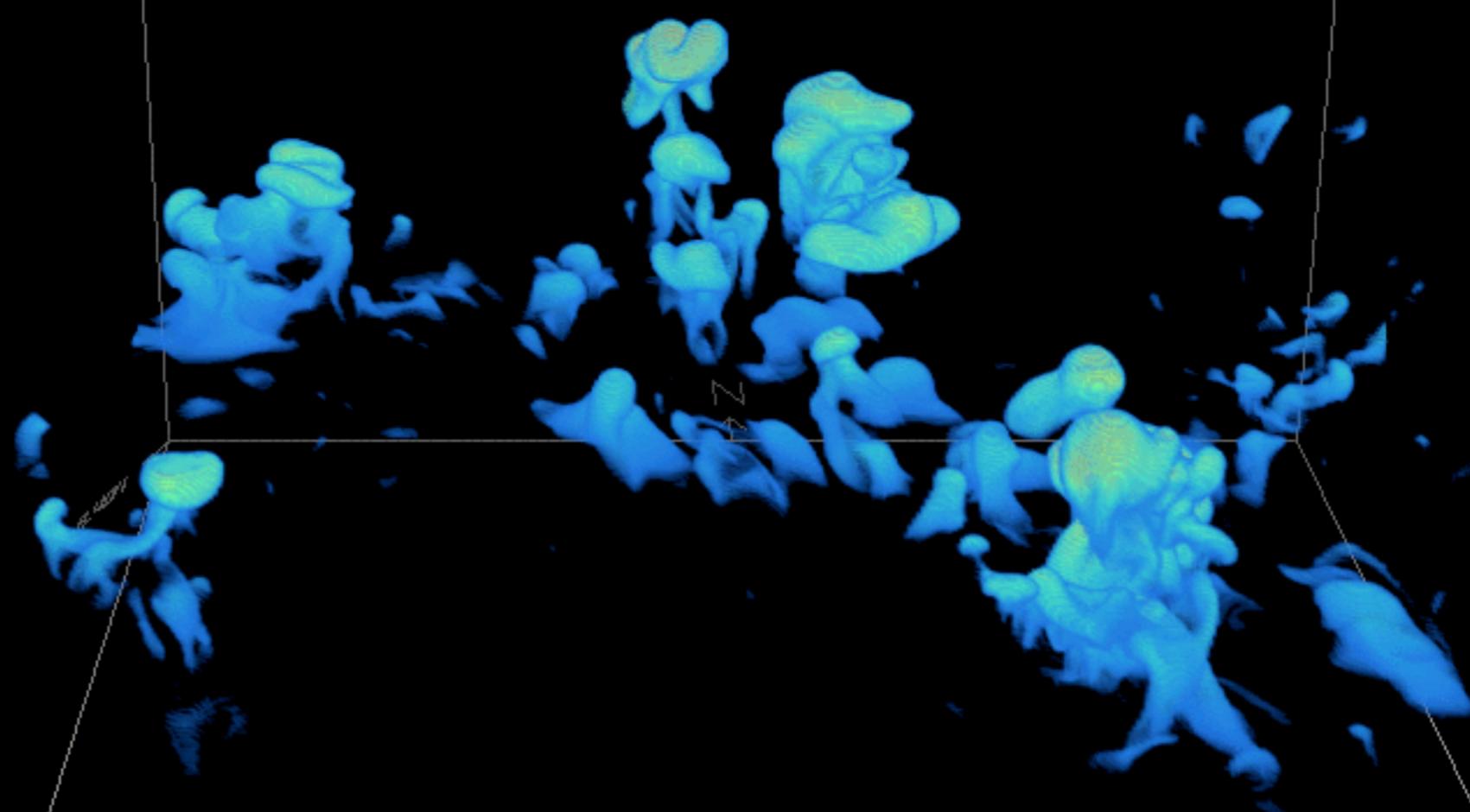
- Small-scale processes in LES

# *Scales of Atmospheric Turbulence*



**2010**

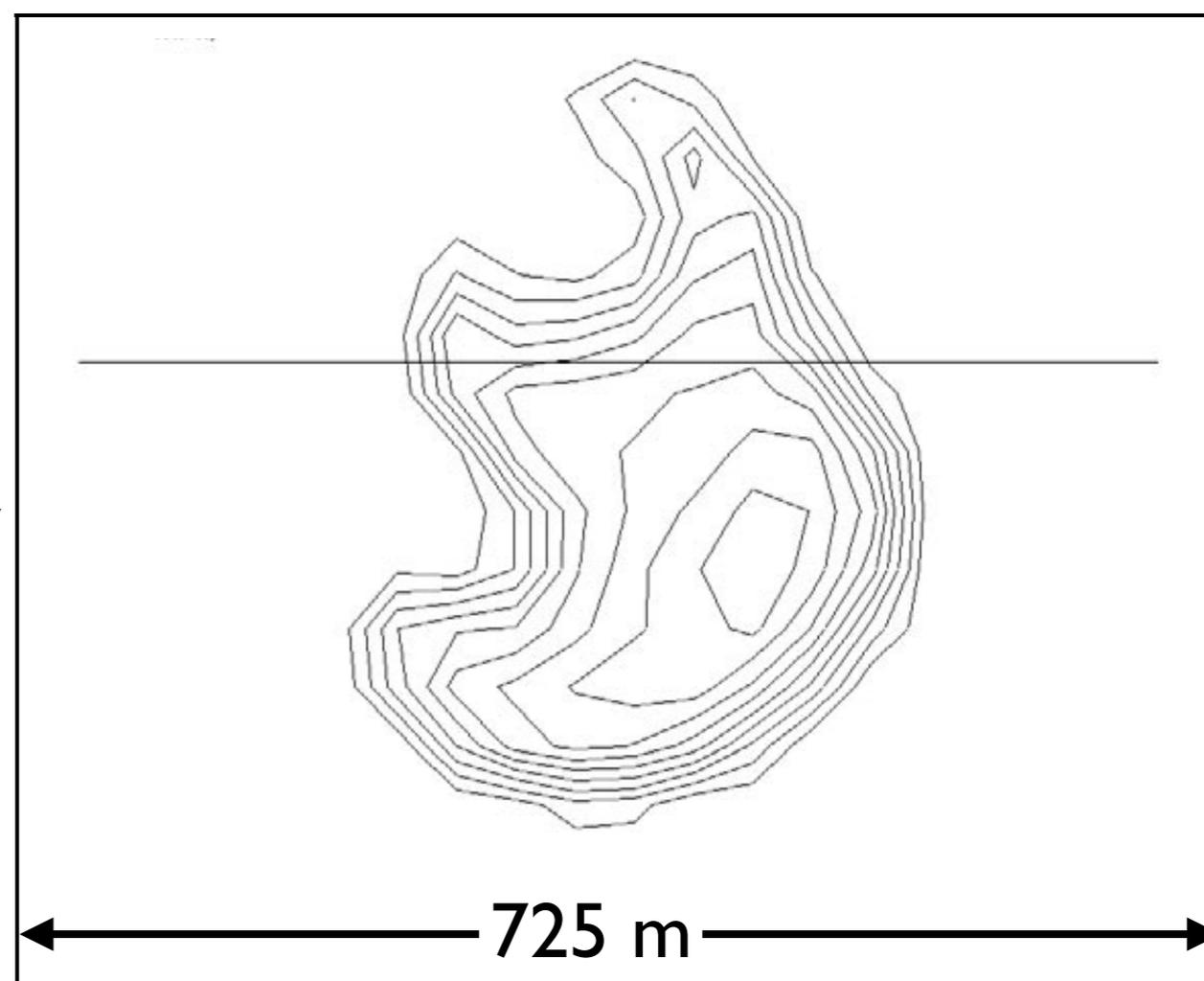
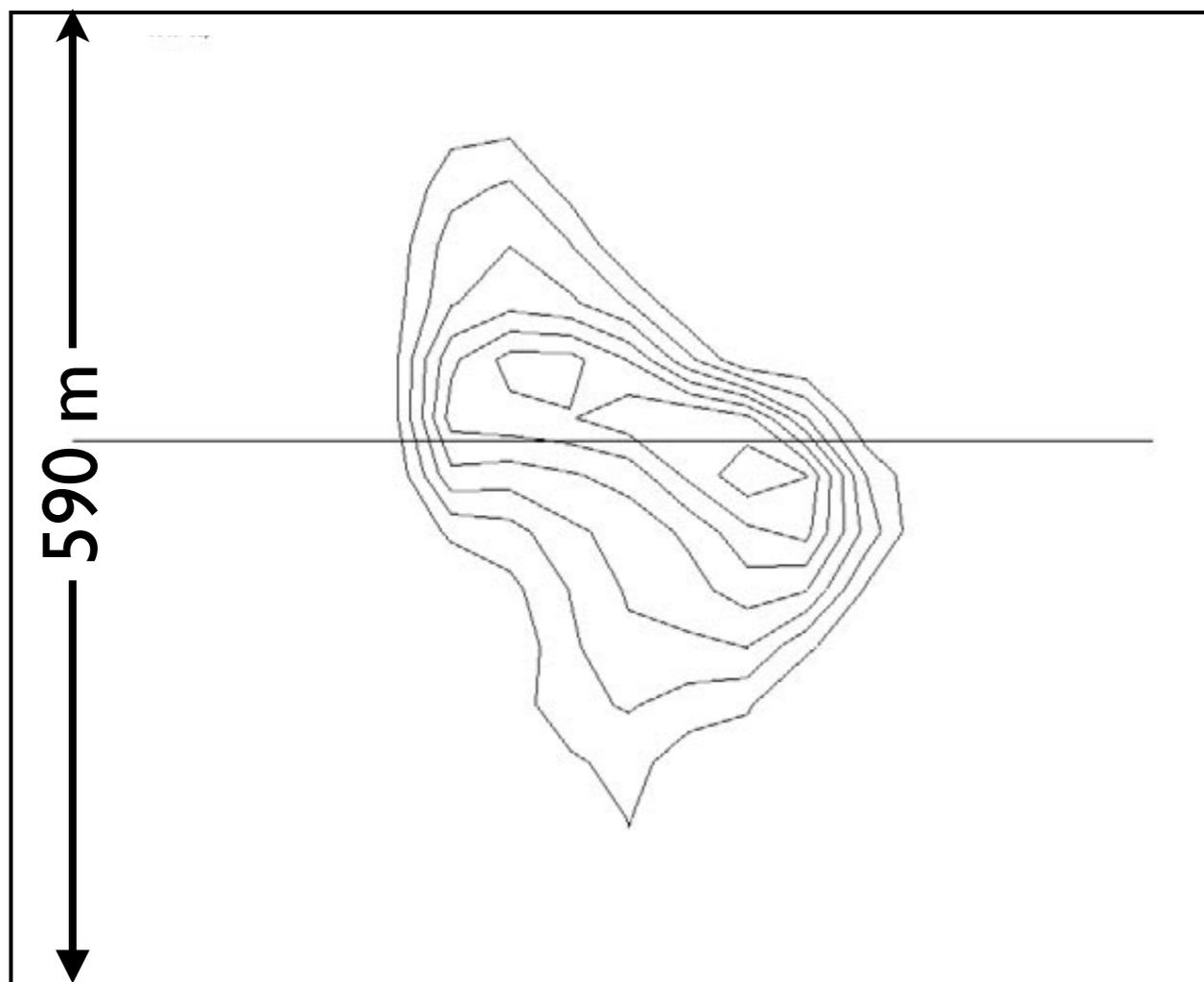
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Vis5D

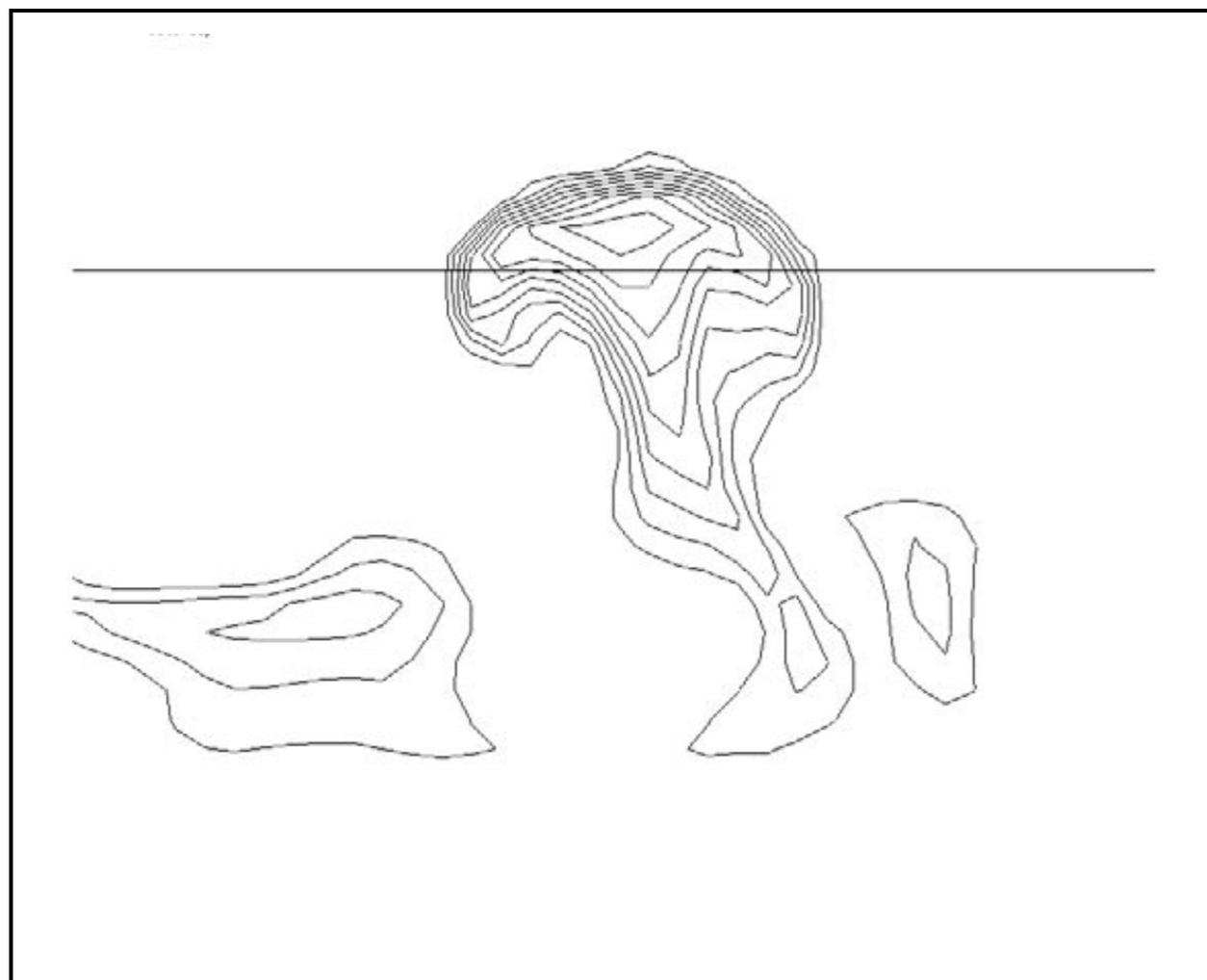
**Increase resolution in a LES of  
small cumulus clouds**

**grid size = 40 m**

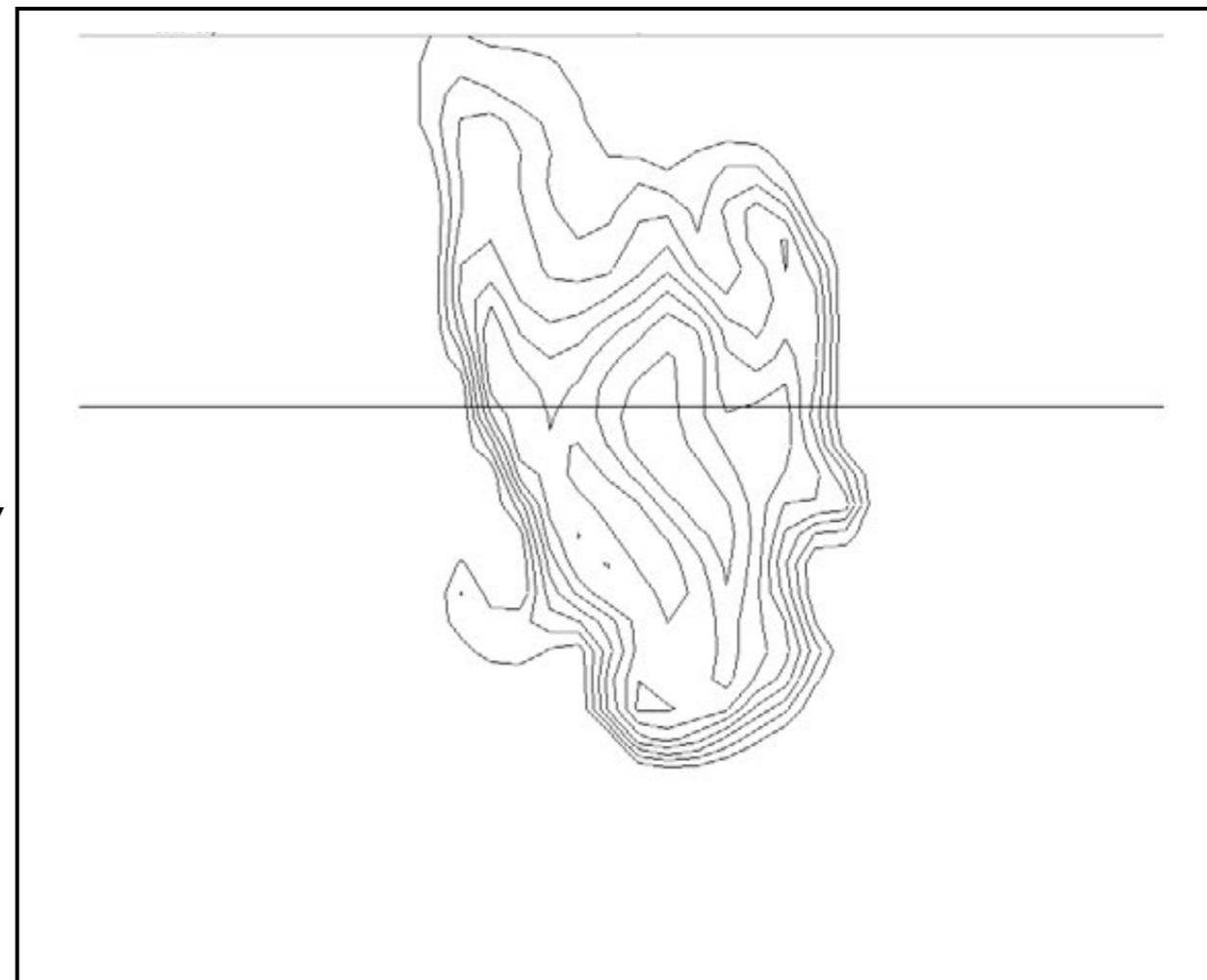


***liquid water mixing ratio***  
***(contour interval = 0.1 g/kg)***

**grid size = 20 m**

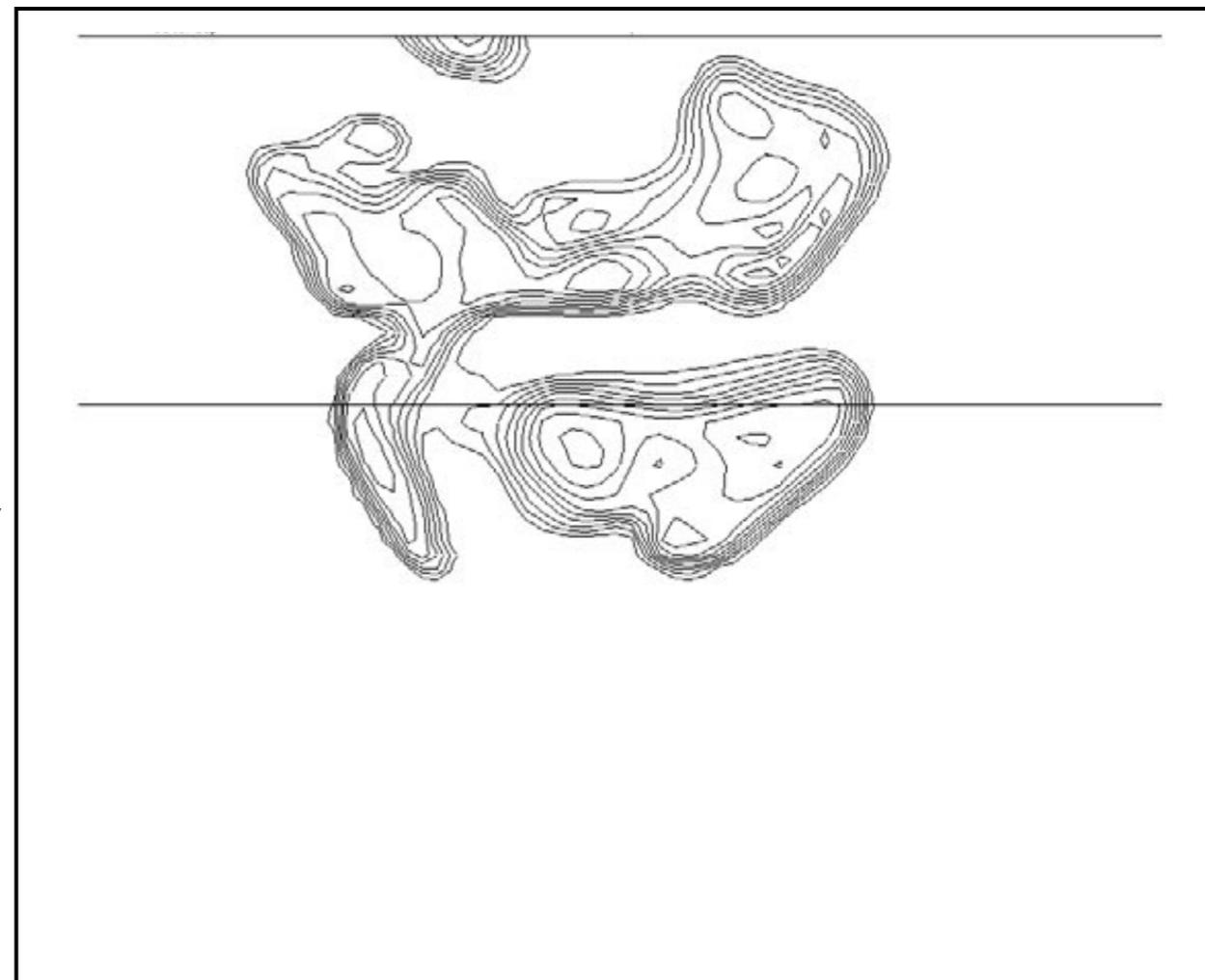
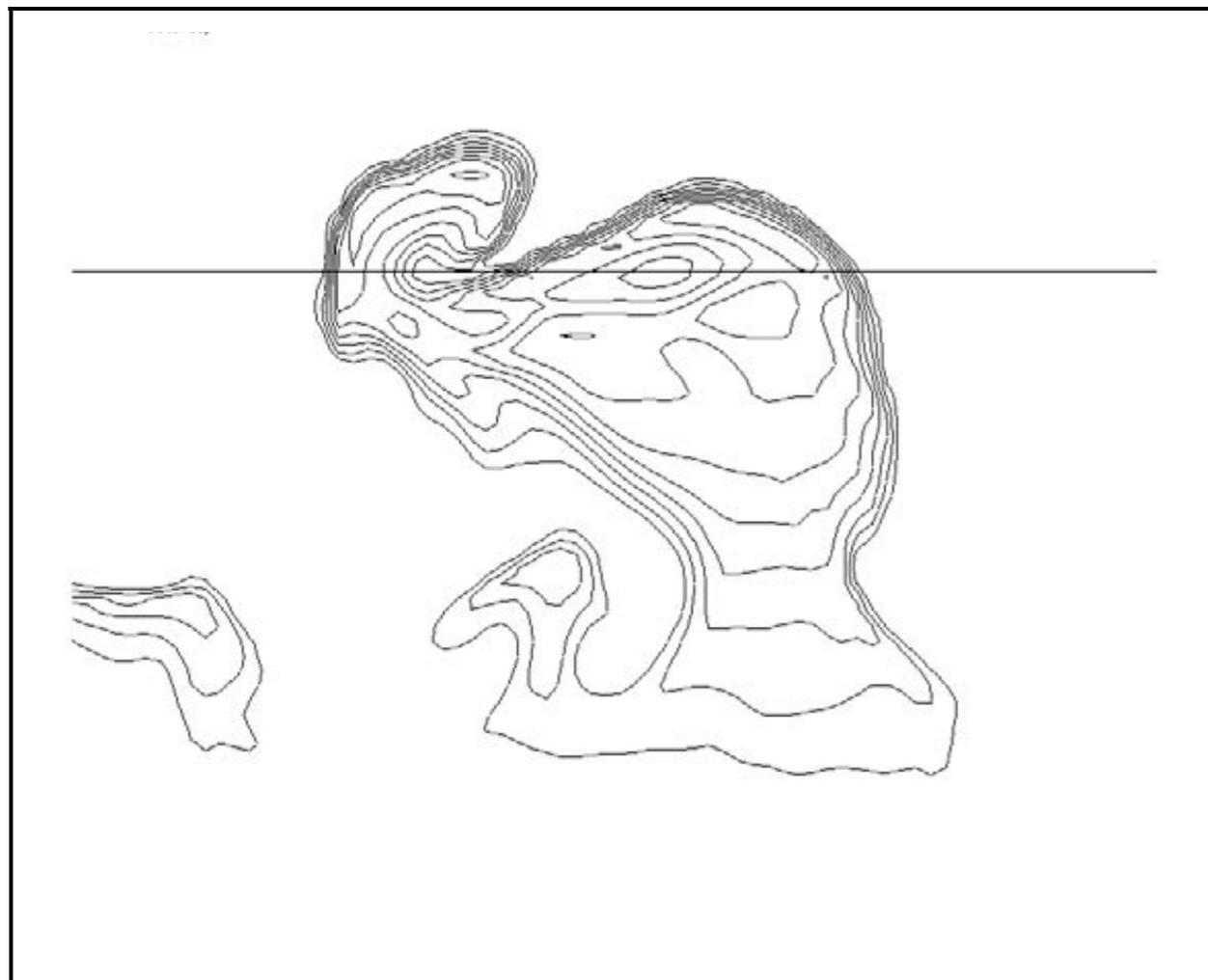


**x**



**x**

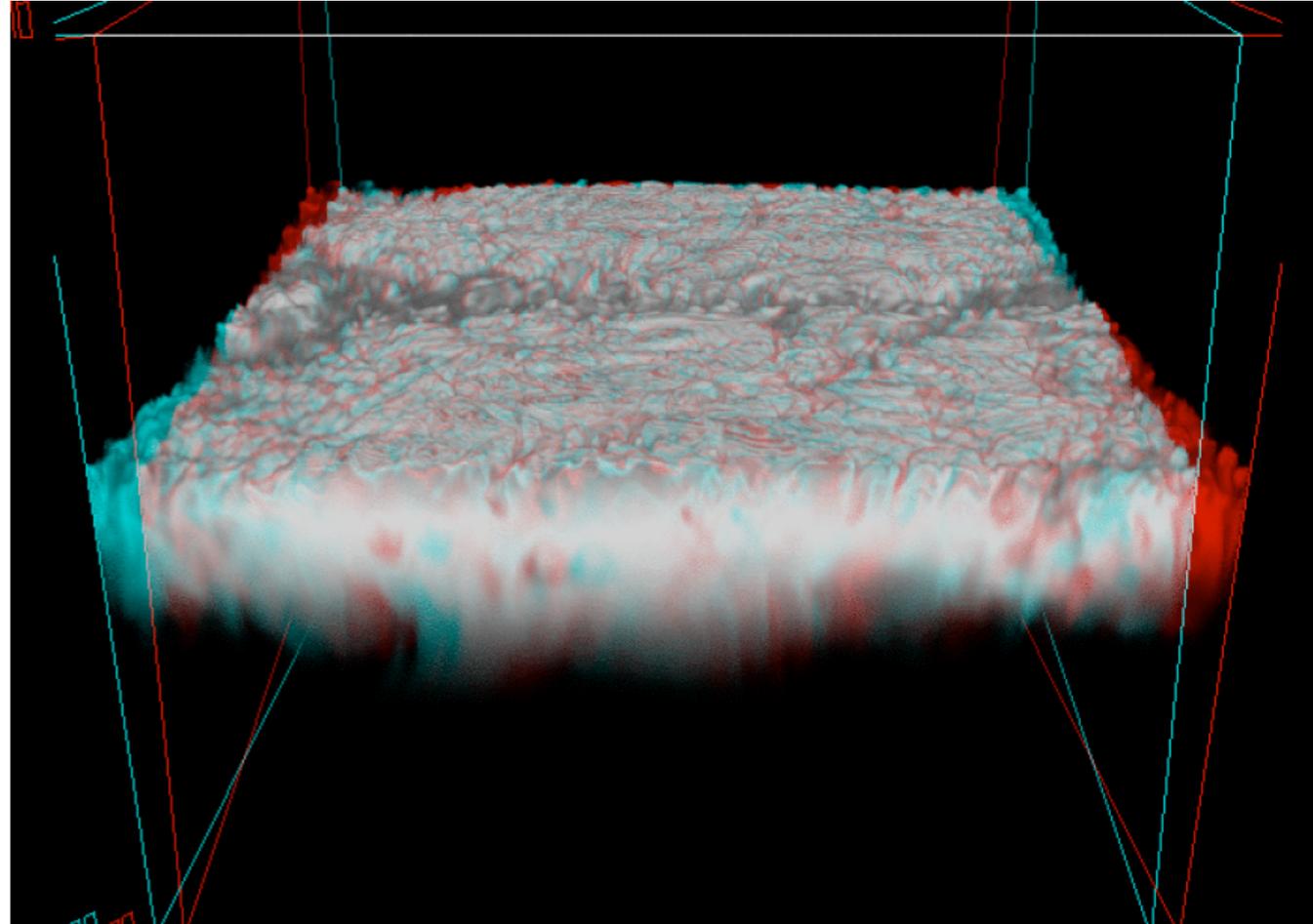
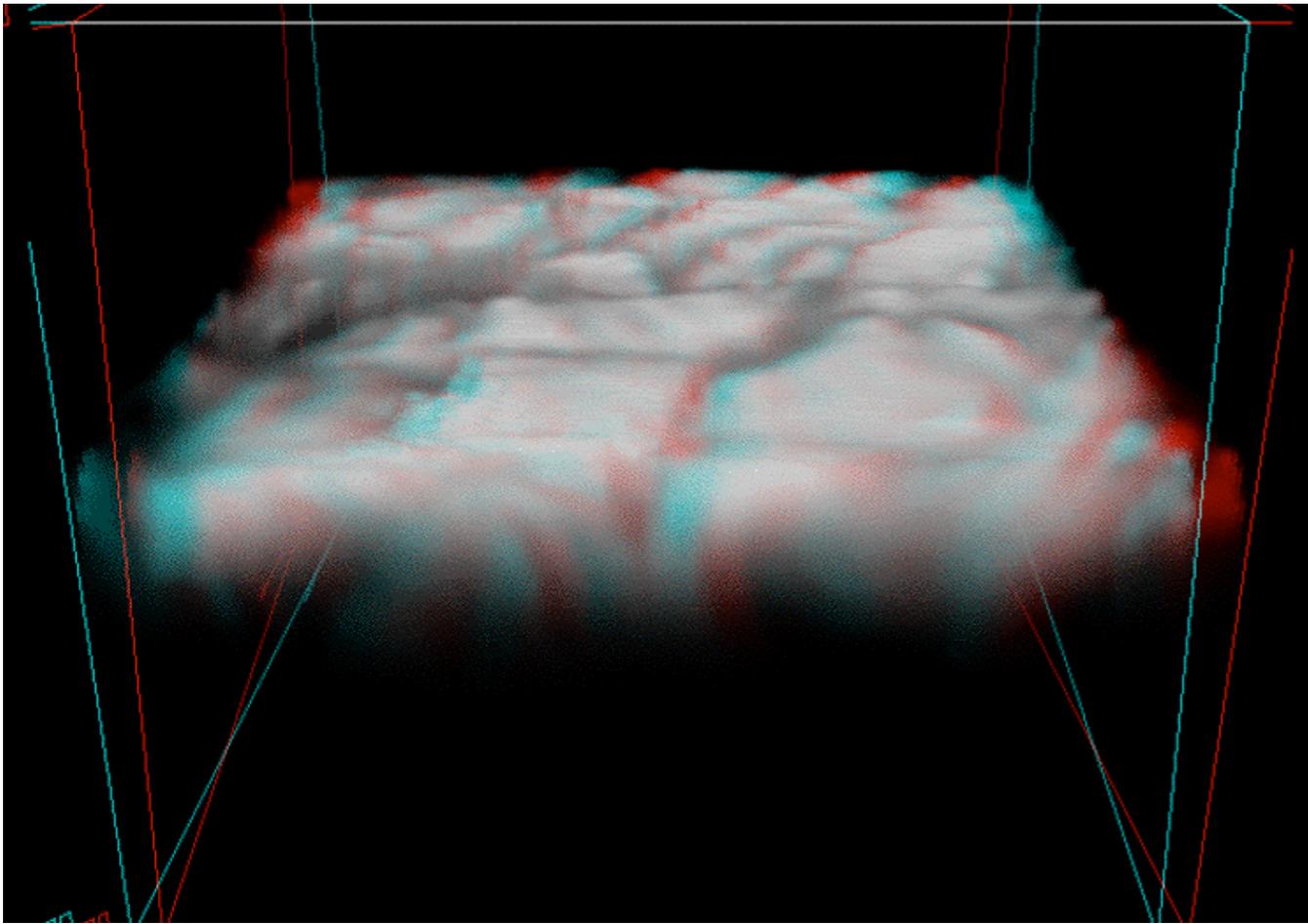
**grid size = 10 m**

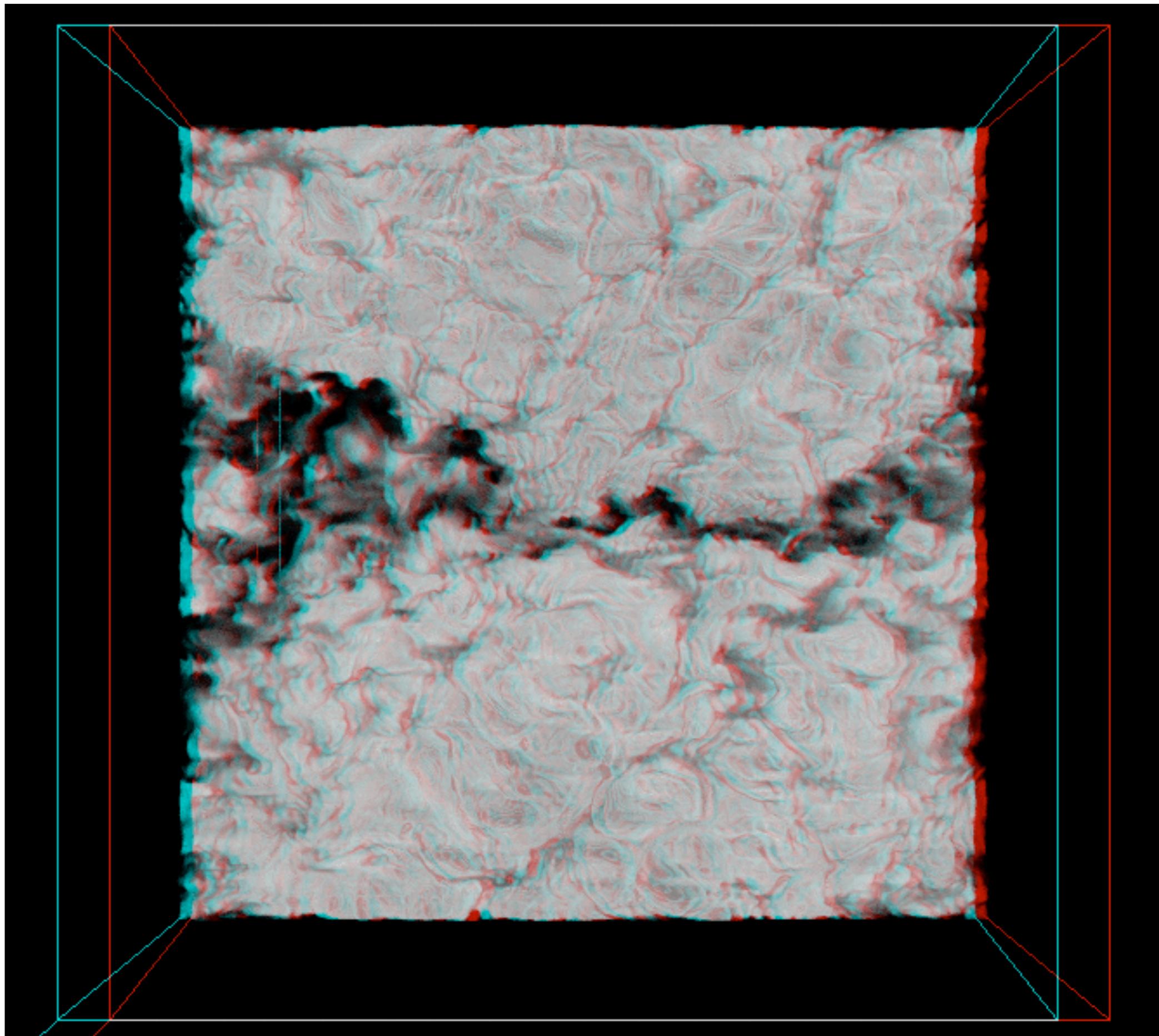


an example of high-resolution LES...

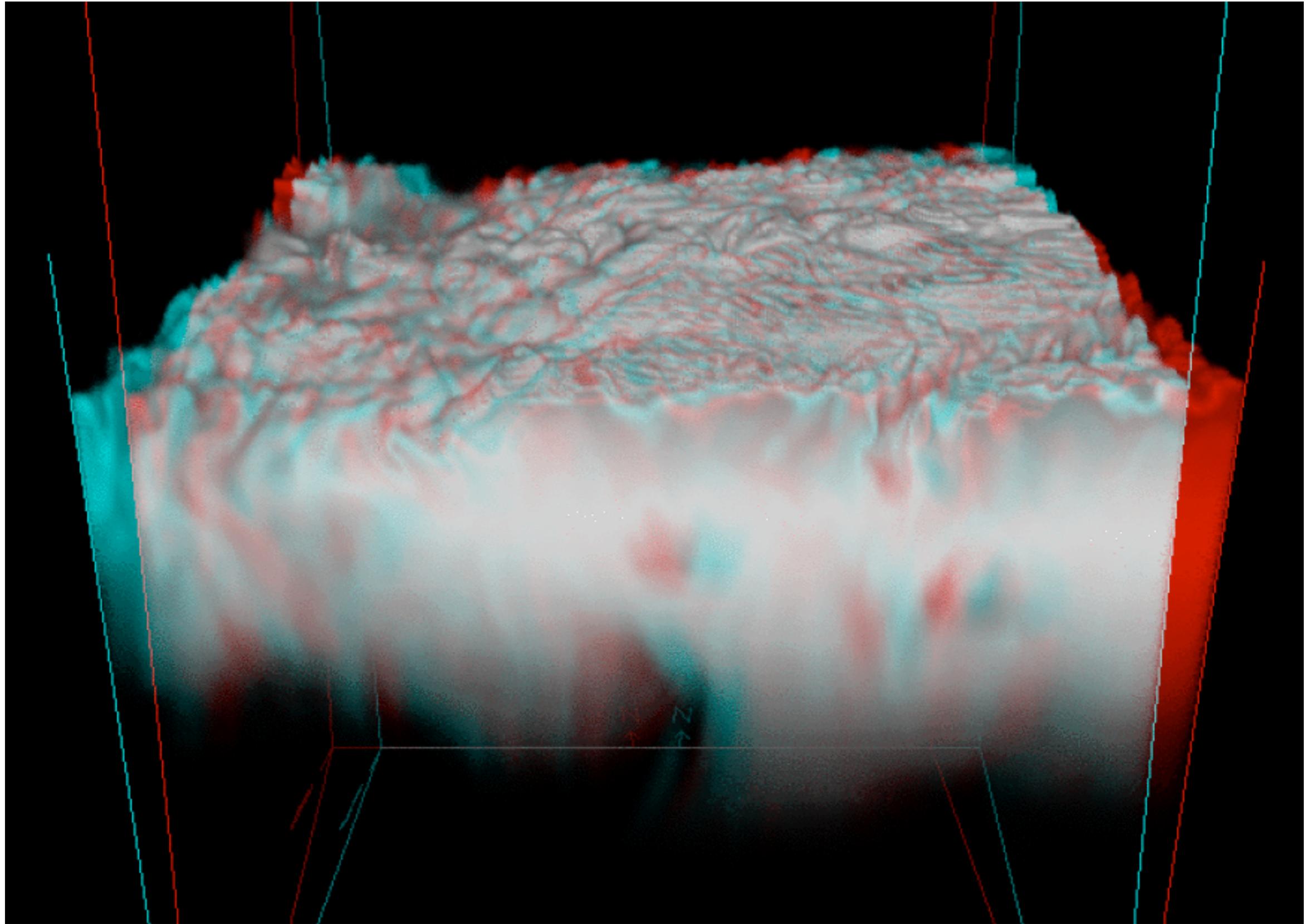
# Stratocumulus-topped boundary layer

- horizontal grid size = 6.25 m
- vertical grid size = 5 m near cloud top





a quarter of the domain



- Bridging the LES-DNS gap
- Large-eddy simulation (LES)
- **Parcel model**
- Linear Eddy Model (LEM)
- One-Dimensional Turbulence (ODT)
- Explicit Mixing Parcel Model (EMPM)
- ClusColl (Clustering and Collision Model)

## ● **Parcel model**

- No internal structure or variability.
- Simplest realistic framework for microphysics and turbulence interactions.
- Lagrangian framework avoids numerical artifacts due to advection.
- Can use to apply multiscale modeling to a growing cumulus turret.



Mt Lemmon

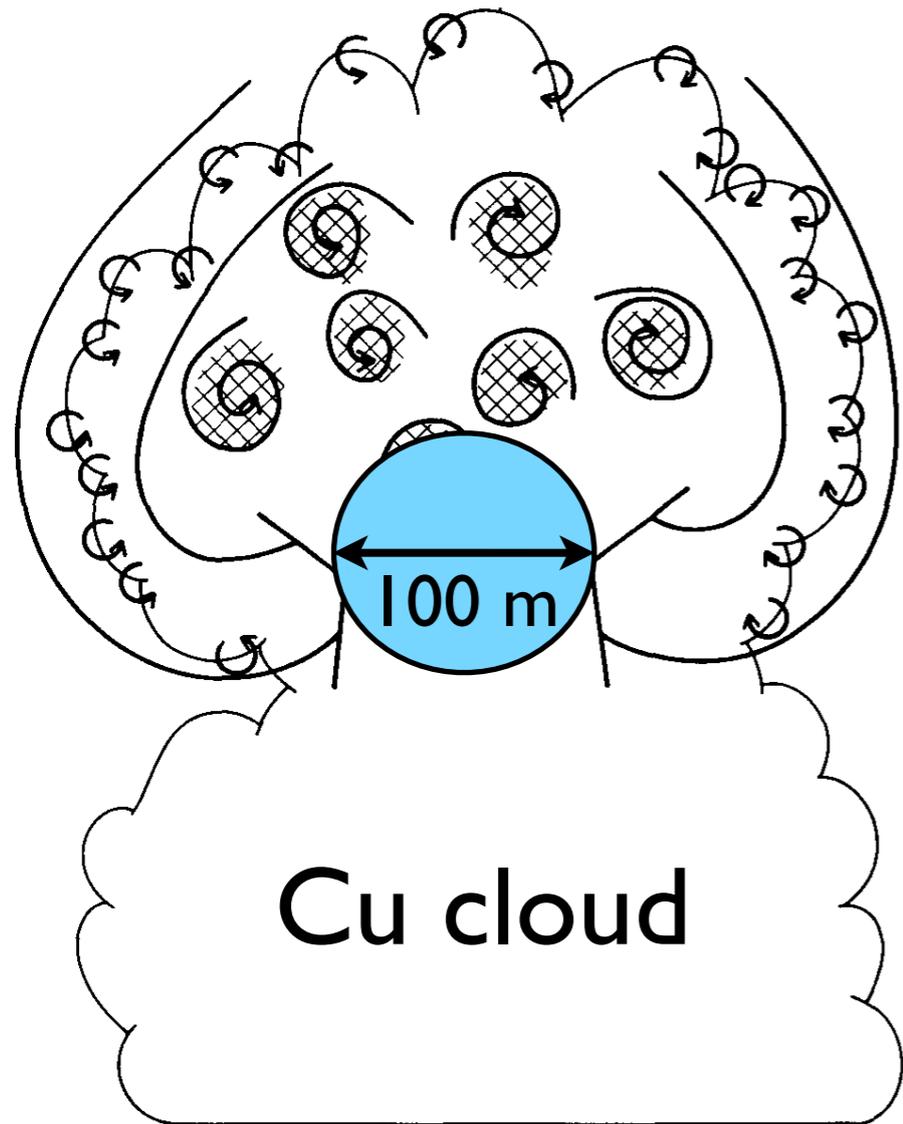
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Ogurus Cove, UT [www.esmjk.com](http://www.esmjk.com)

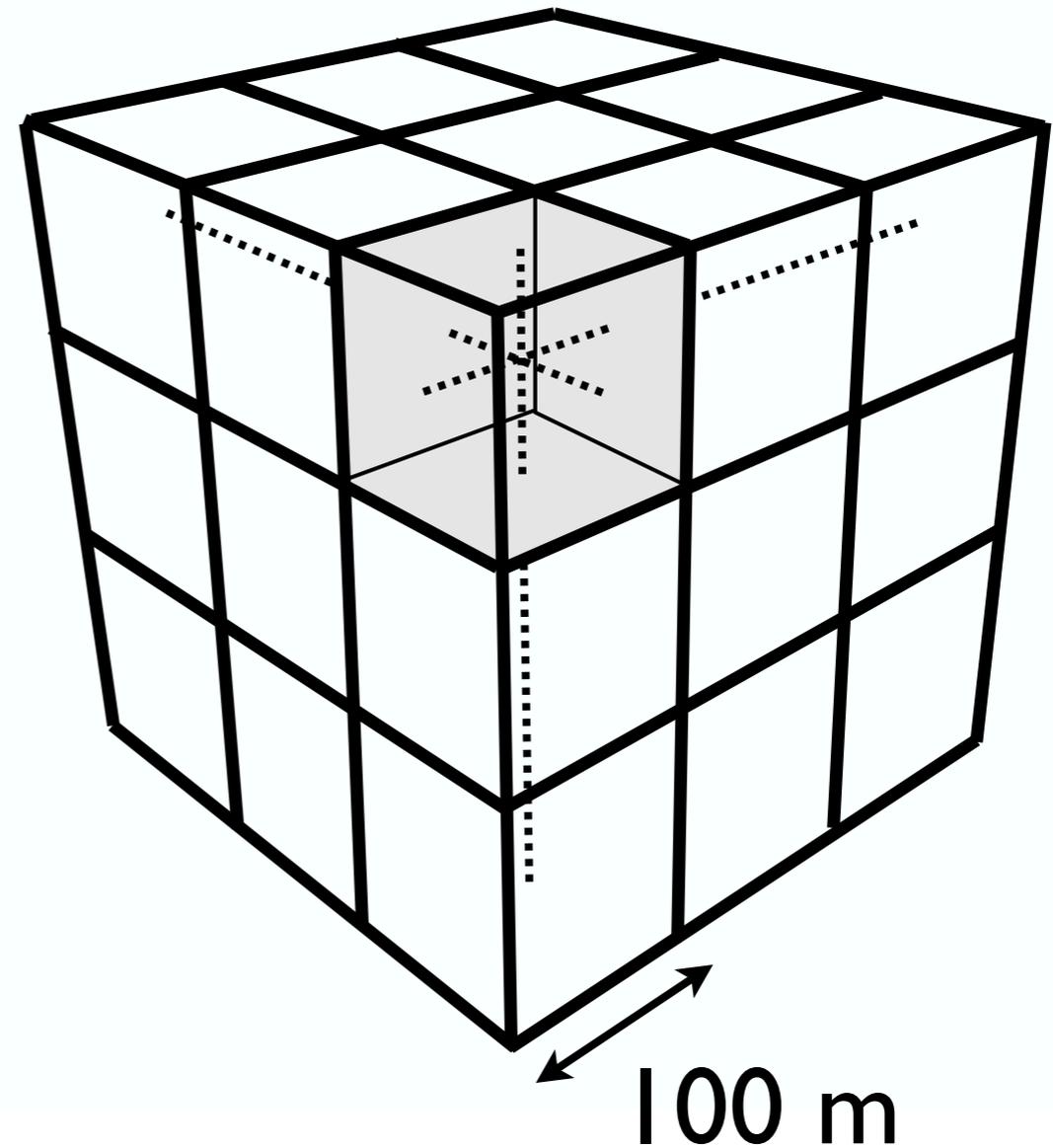


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## Classical Parcel model

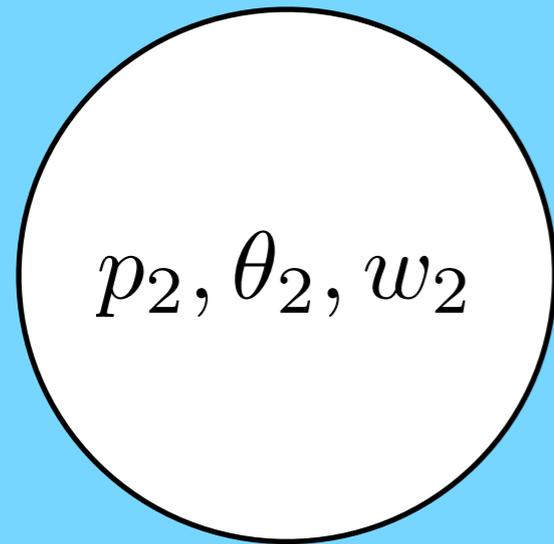


## Large-Eddy Simulation (LES) model



no sub-parcel or subgrid-scale variability

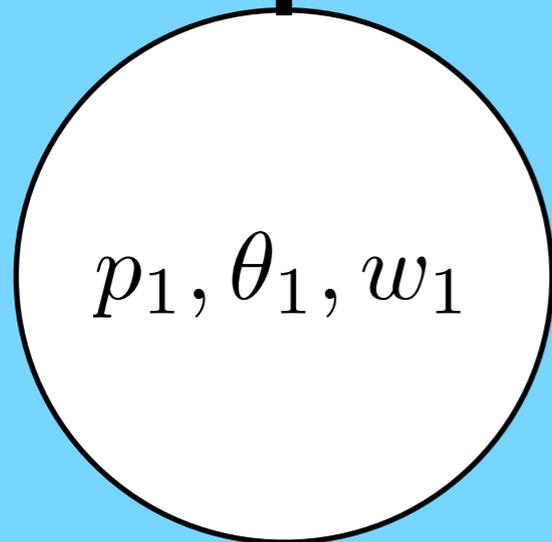
# Parcel Model



$p_2, \theta_2, w_2$

**State 2**

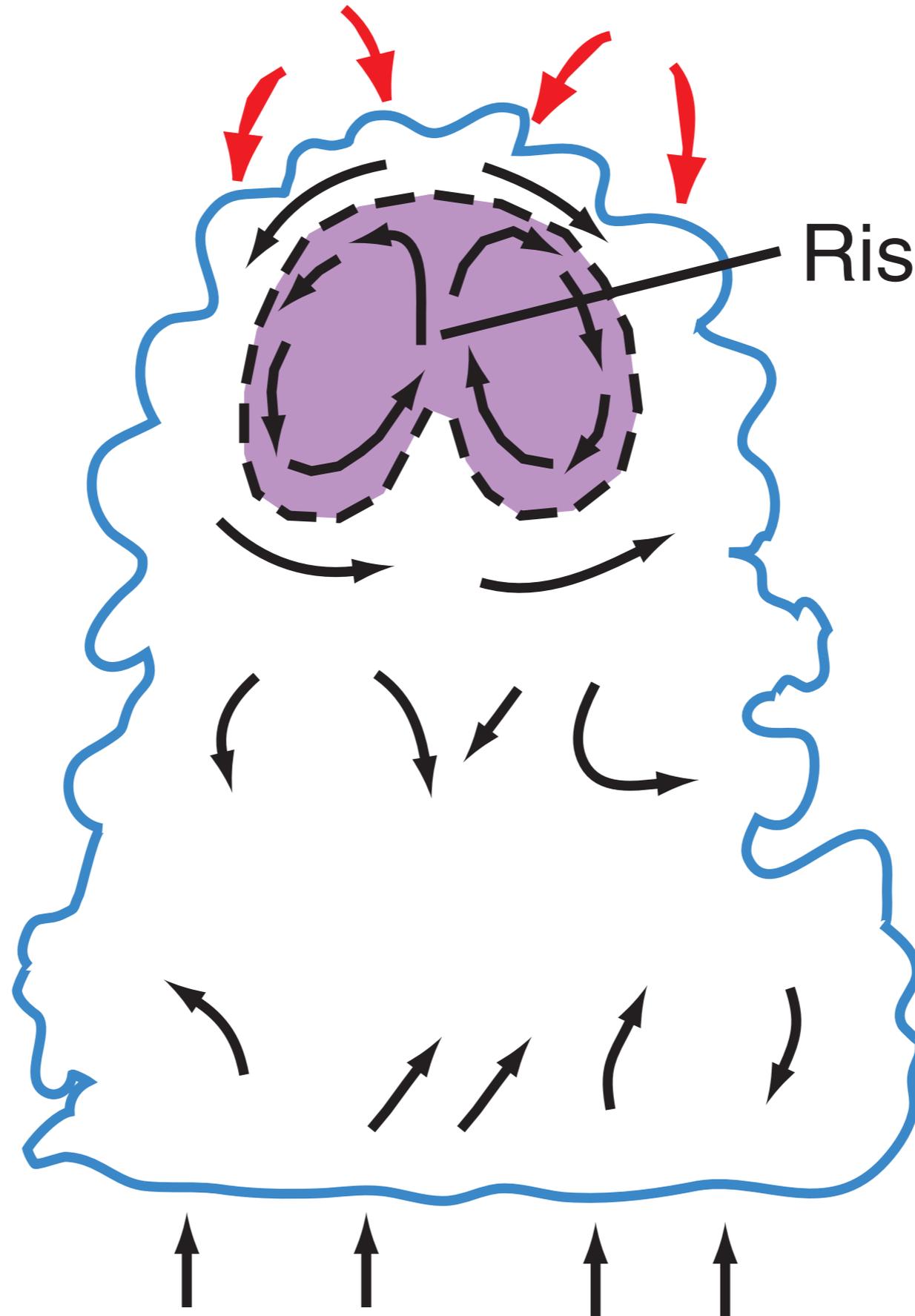
**thermodynamic process**



$p_1, \theta_1, w_1$

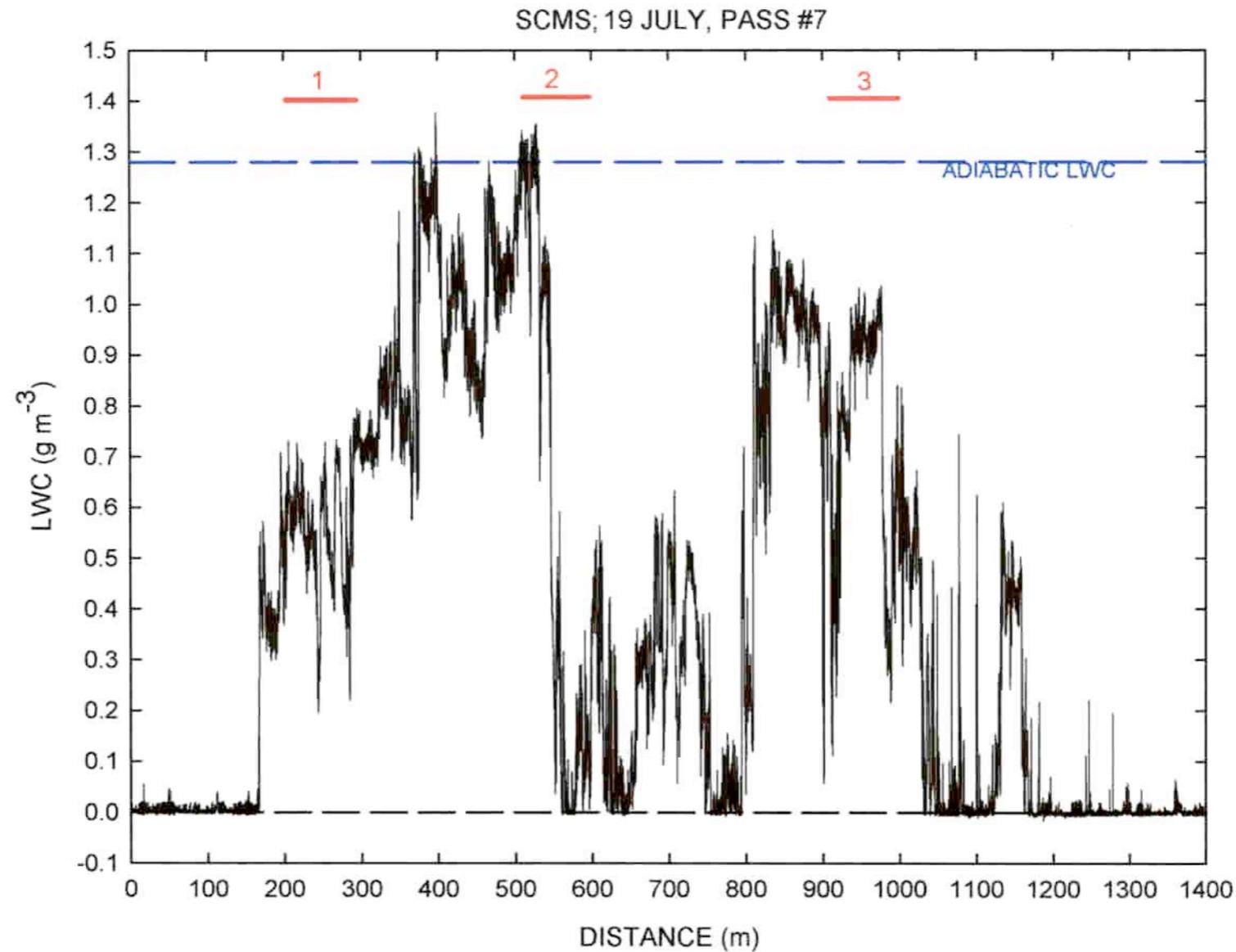
**State 1**

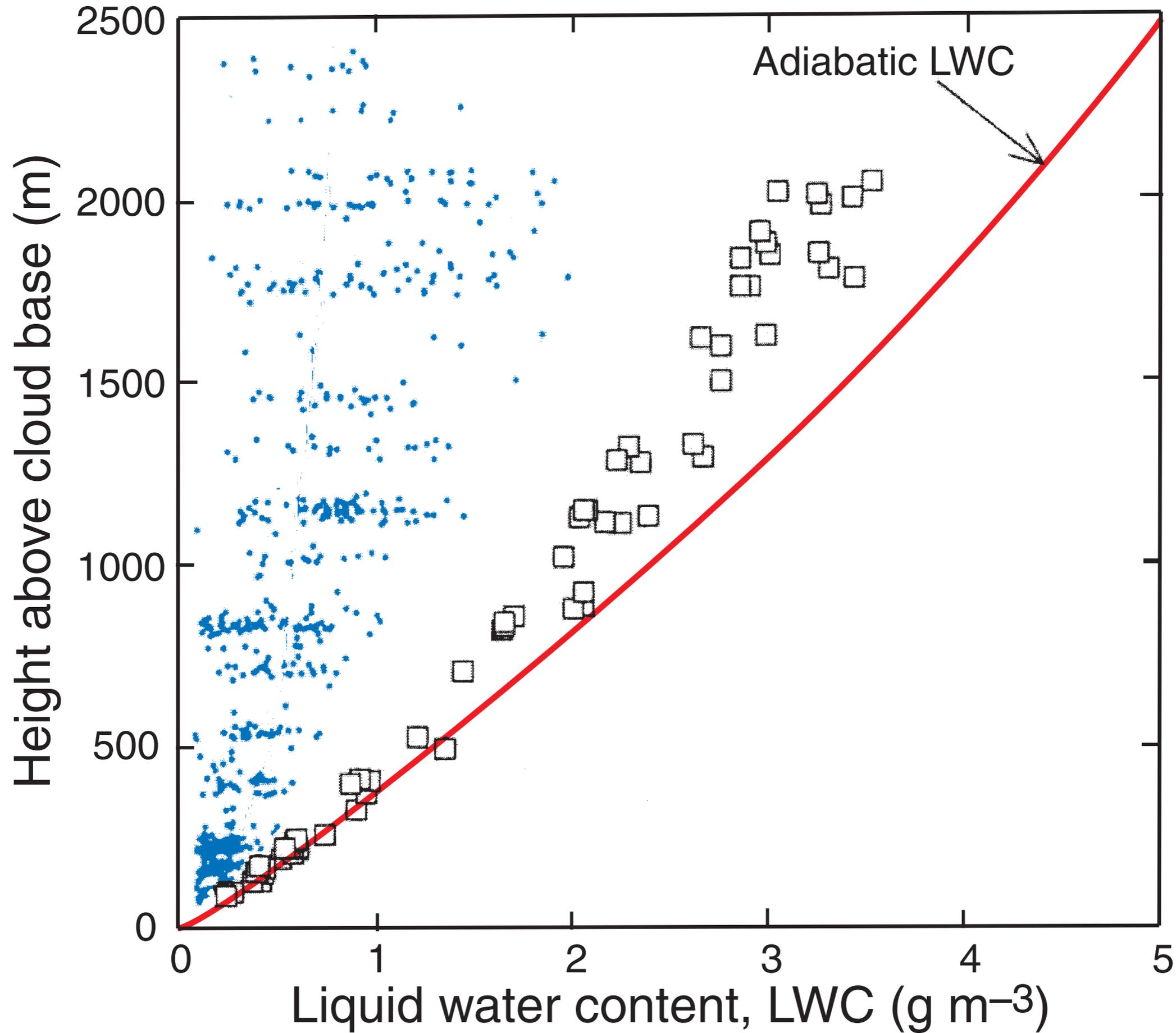
Entrainment



Rising thermal

# Small-scale Variability due to Entrainment and Mixing is Typical in Cumulus Clouds





- Bridging the LES-DNS gap
- Large-eddy simulation (LES)
- Parcel model
- **Linear Eddy Model (LEM)**
- One-Dimensional Turbulence (ODT)
- Explicit Mixing Parcel Model (EMPM)
- ClusColl (Clustering and Collision Model)

- **Linear Eddy Model (LEM)**

- Evolves **scalar** spatial variability on all relevant turbulence scales using one dimension.
- Distinguishes turbulent deformation and molecular diffusion.
- Turbulence properties are **specified**.

## Modeling Mixing

- Small-scale turbulence is important for mixing, unlike for dynamics where it provides dissipation for large-scale structures.
- Most parameterizations of mixing can't accurately predict molecular mixing.

Most make no distinction between turbulent advection and molecular diffusion.

This distinction is crucial for an accurate representation of mixing.

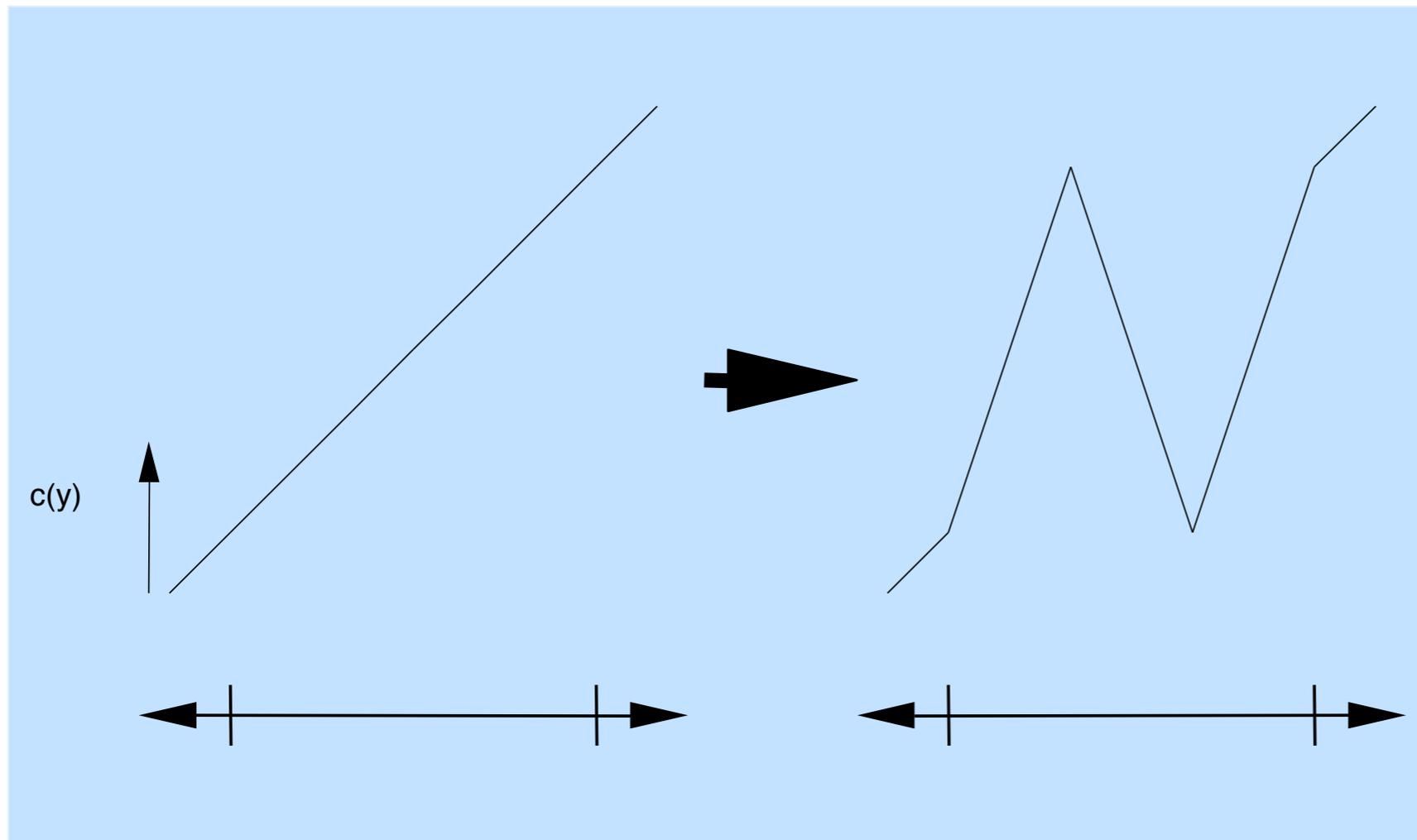
- In the atmosphere, the extreme range of length scales present magnifies the difficulties.

# Linear Eddy Model

(Kerstein 1988)

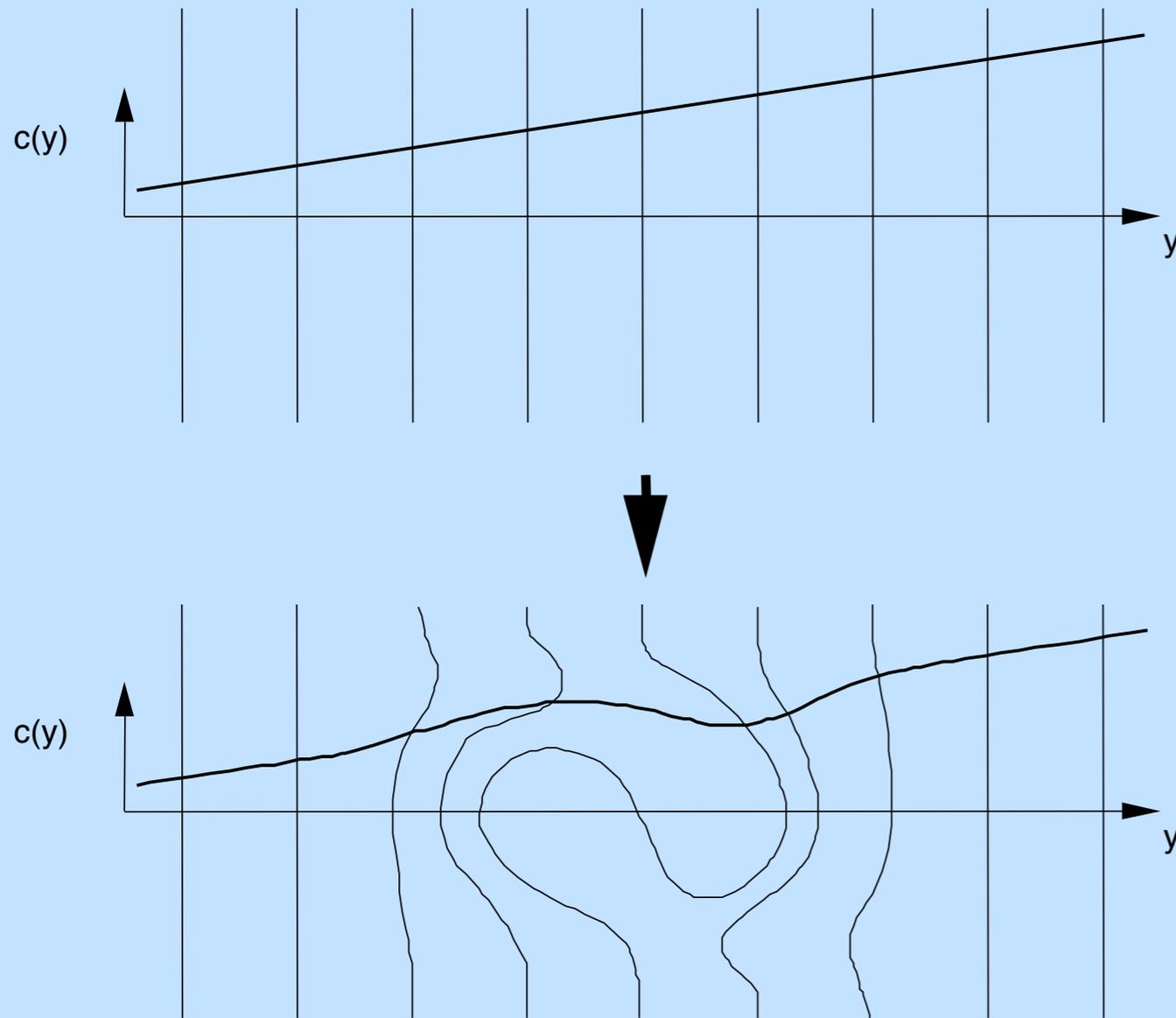
- Distinguishes turbulent deformation and molecular diffusion.
- The mixing process:
  1. turbulent deformation (increases gradients)
  2. molecular diffusion (reduces gradients)
- The linear eddy model:
  - 1-D so all relevant scales can be represented.
  - Molecular diffusion is explicit.
  - Turbulent deformation is represented by rearrangement events:
    - Size of event represents the eddy size.
    - Distribution of eddy sizes is obtained from Kolmogorov scaling laws for high  $Re$  turbulence.
    - Process is consistent with turbulent kinetic energy cascade.

Advection is modeled as a sequence of *triplet maps* that preserve desired advection properties, even in 1D



The triplet map captures compressive strain and rotational folding effects, and causes no property discontinuities

Advection is modeled as a sequence of *triplet maps* that preserve desired advection properties, even in 1D

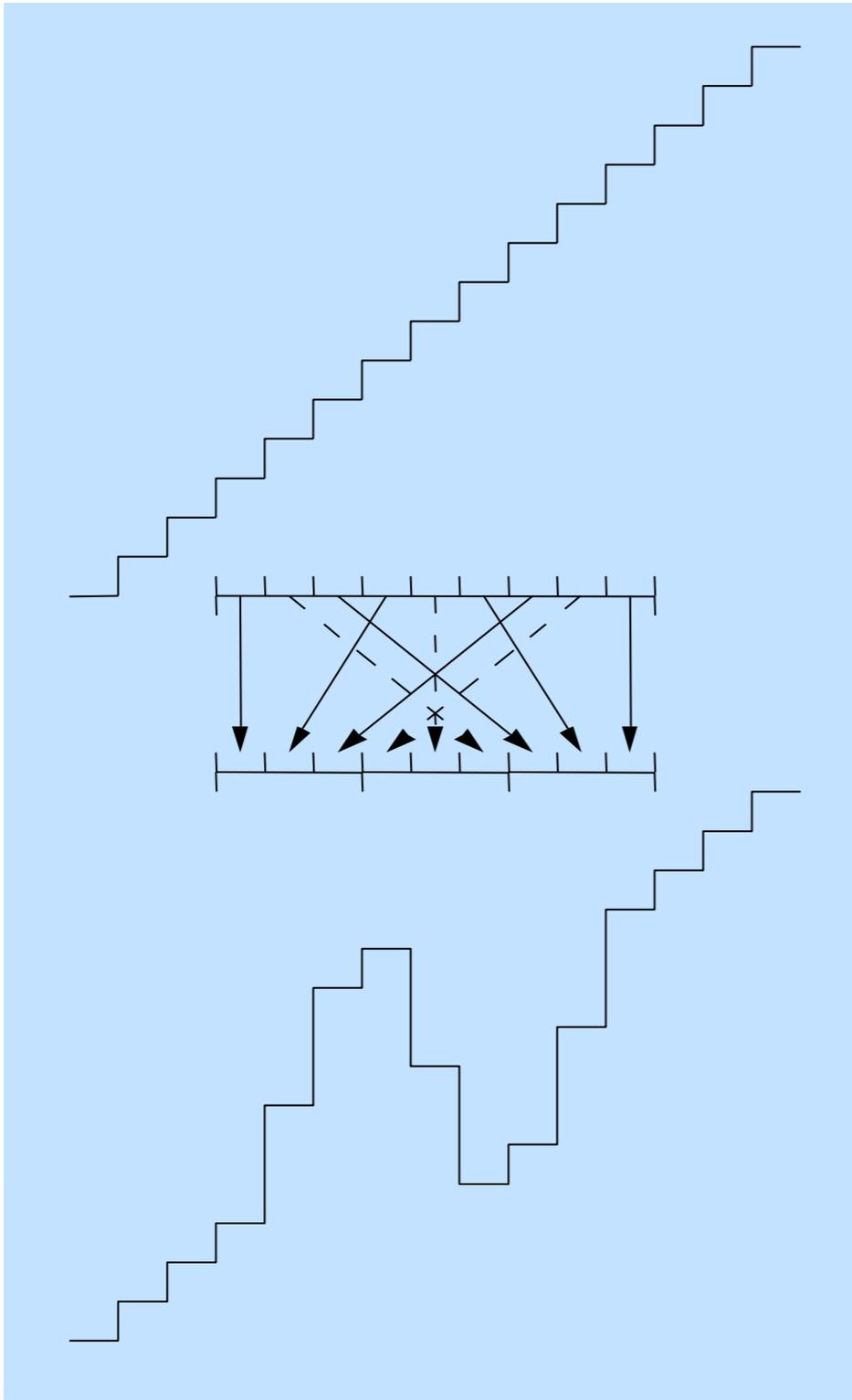


This procedure imitates the effect of a 3D eddy on property profiles along a line of sight

The triplet map (1D eddy)

- moves fluid parcels without intermixing their contents
- conserves fluid properties
- does not cause property discontinuities
- reduces fluid separations by at most a factor of 3

Advection is modeled as a sequence of *triplet maps* that preserve desired advection properties, even in 1D



The triplet map is implemented numerically as a permutation of fluid cells (or on an adaptive mesh)

# Triplet Map for Fluid Elements

Each triplet map has a location, size, and time.

- *Location* is randomly chosen.
- *Size*  $l$  is randomly chosen from a distribution that matches inertial range scalings.
  - Smallest map (eddy) is Kolmogorov scale,  $\eta$ .
  - Largest eddy is  $L$ , usually domain size.
- Eddies occur at a *rate* determined by the large eddy time scale and eddy size range.

# Triplet Map

## Eddy Size Distribution

$$f(l) = \begin{cases} \frac{5}{3} \frac{l^{-8/3}}{\eta^{-5/3} - L^{-5/3}}, & \text{if } \eta \leq l \leq L \\ 0, & \text{otherwise} \end{cases}$$

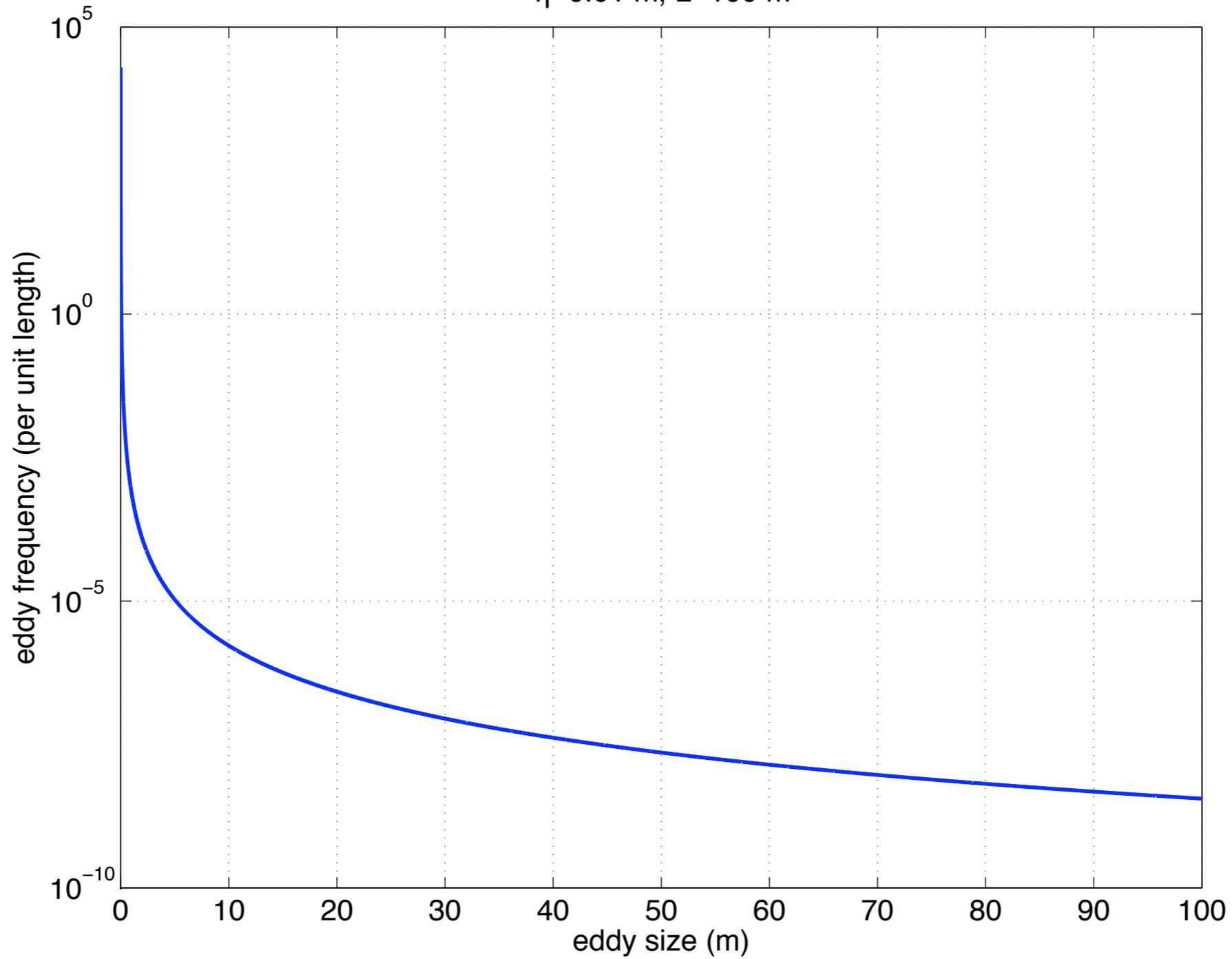
## Eddy Rate

$$\Lambda = \frac{54}{5} \frac{D_T}{L^3} \left( \frac{L}{\eta} \right)^{5/3}.$$

$D_T \approx 0.1 L^{4/3} \epsilon^{1/3}$  is the turbulent diffusivity due to all eddies.

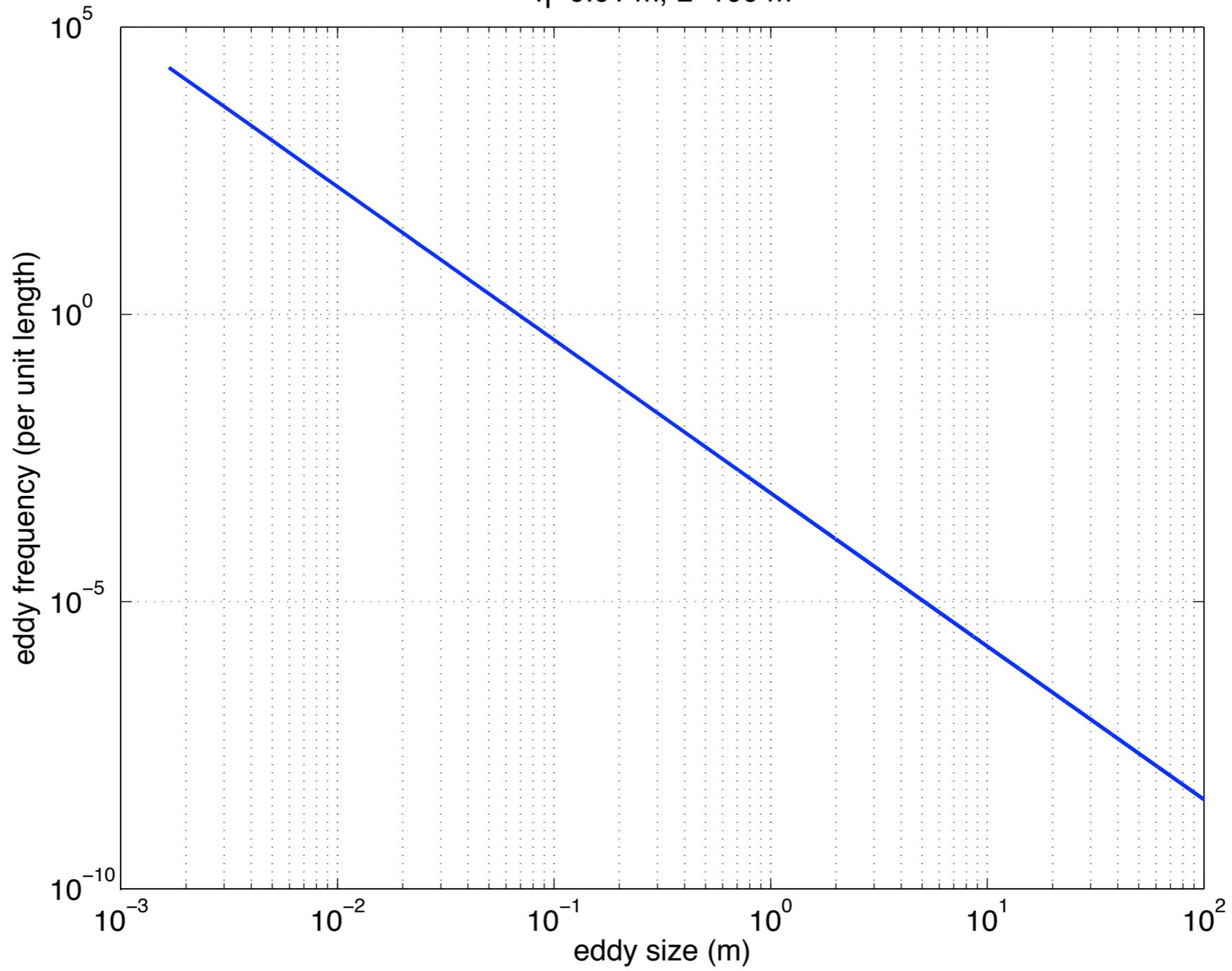
# Eddy Size Distribution

$\eta=0.01$  m,  $L=100$  m



# Eddy Size Distribution

$\eta=0.01$  m,  $L=100$  m



# Molecular Diffusion

$$\frac{\partial \phi_i}{\partial t} = D_M \frac{\partial^2 \phi_i}{\partial x^2}$$

$D_M$  is the *molecular* diffusivity of the scalar  $\phi_i$ .

# Scale reduction in the LEM

Let  $D(t)$  be the length of a fluid segment and the corresponding scale reduction rate be  $dD/dt$ .

Inertial range scaling suggests that

$$\frac{dD}{dt} \approx \frac{-D}{\tau_D} \equiv -(D\epsilon)^{1/3}$$

where  $\tau_D \equiv (D^2/\epsilon)^{1/3}$  is the time scale for turbulent breakdown of an eddy of size  $D$  and  $\epsilon$  is the dissipation rate of turbulence kinetic energy.

We can show that the LEM obeys this equation as well, if we interpret  $dD/dt$  as the LEM's time- and space-averaged rate of scale reduction.

# Triplet map vs. 3D turbulence

## Strengths

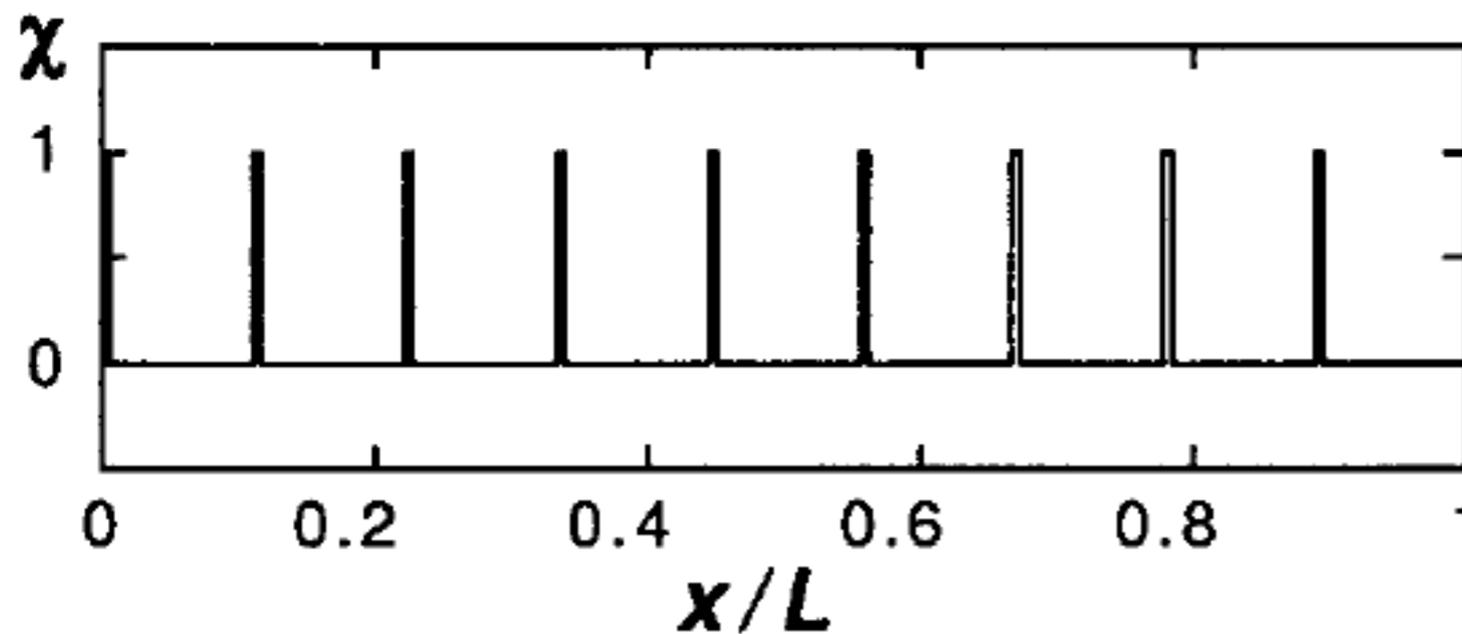
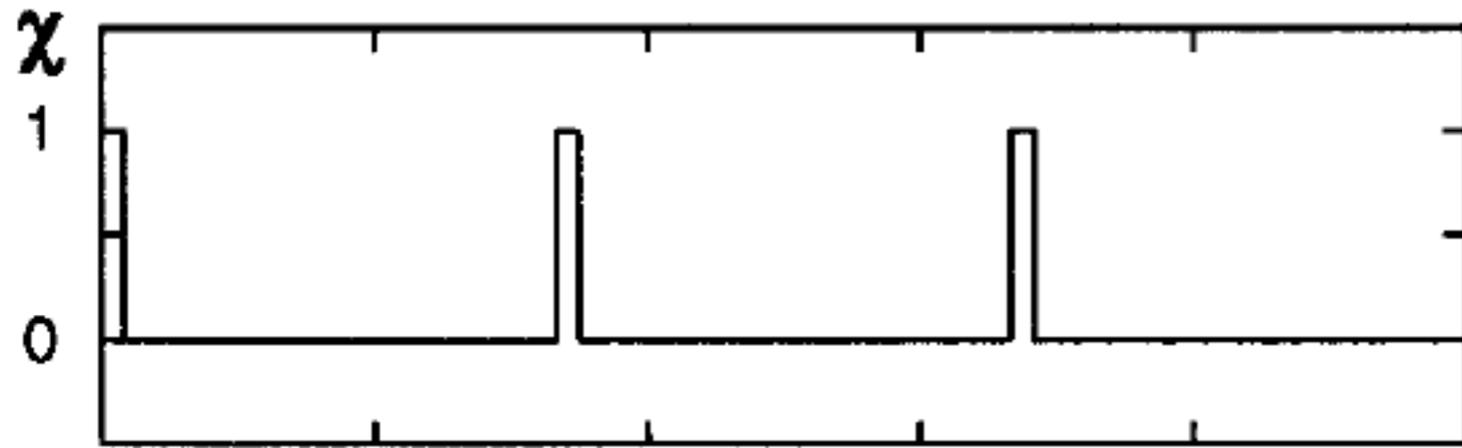
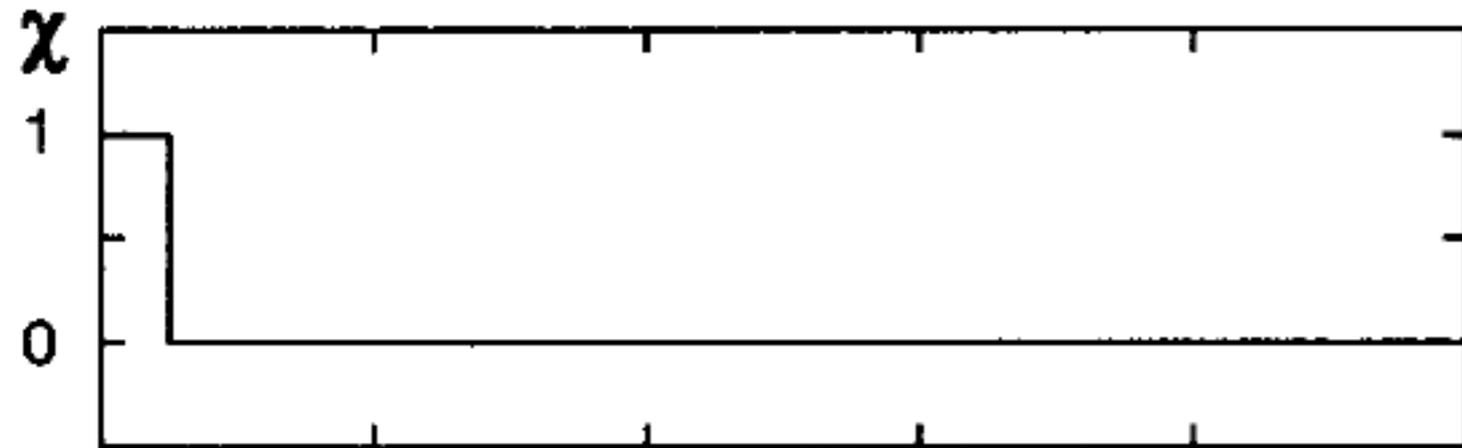
- **Transport:** map frequency is set so that fluid transport matches turbulent eddy diffusivity.
- **Length scale reduction:** by matching the inertial-range size-vs.-frequency distribution of eddy motions, the rate of length scale reduction as a function of fluid parcel size is consistent with 3D turbulence.
- **Intermittency:** Random sampling of triplet map occurrences and sizes reproduces, qualitatively and to some degree quantitatively, intermittency properties of 3D turbulence.
- **Mixing:** In conjunction with molecular diffusion, the map sequence reproduces mixing features.

# Triplet map vs. 3D turbulence

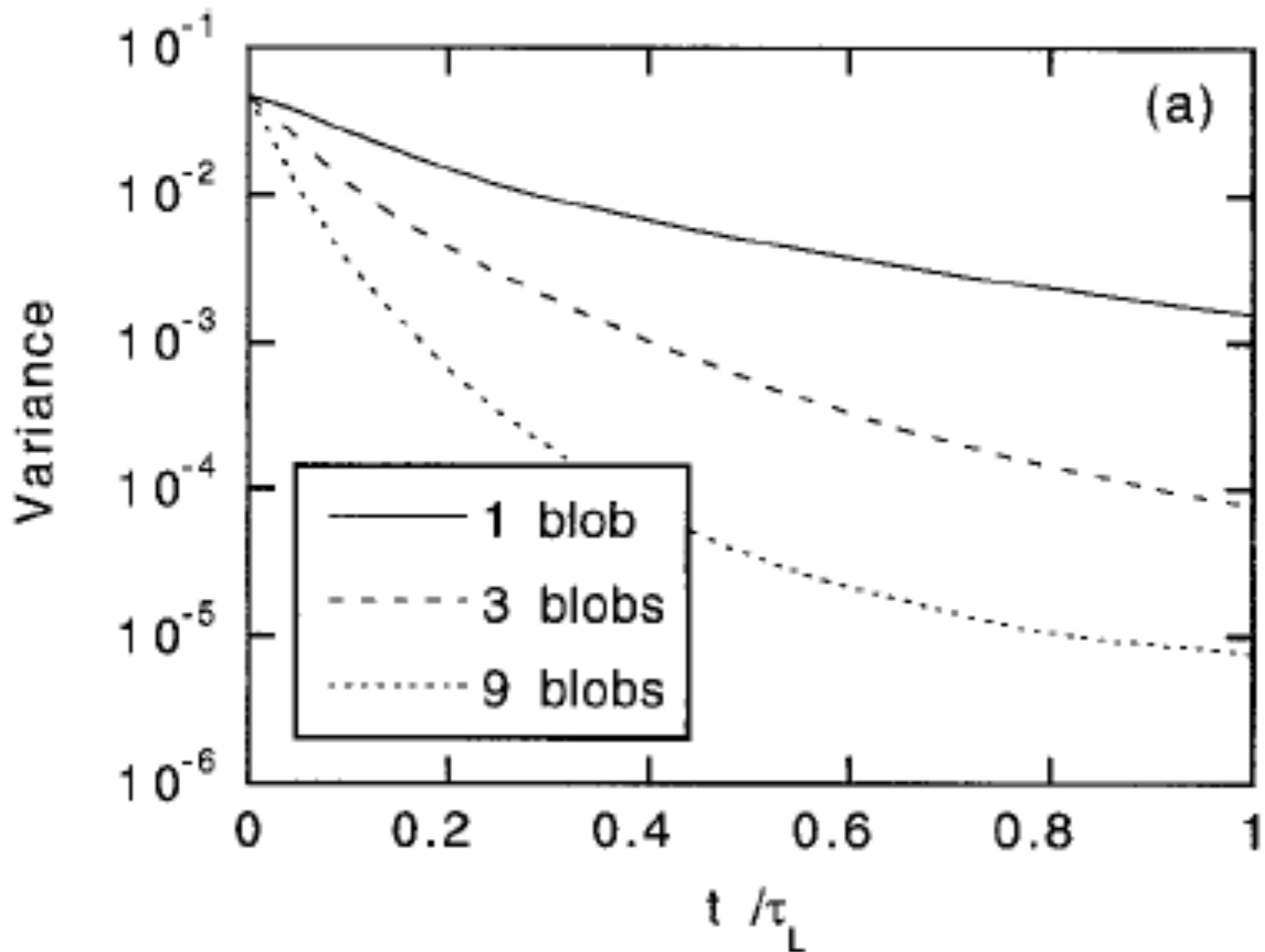
## Weaknesses

- Omits effects of time persistence of turbulent motions.
- When diffusive time scales are shorter than turbulent time scales, diffusion can suppress scalar fluctuations faster than they are generated in 3D turbulence.
- In some cases, turbulence spreads a slow-diffusing scalar faster than a fast-diffusing scalar. This is a multi-dimensional effect that 1D advection cannot capture.

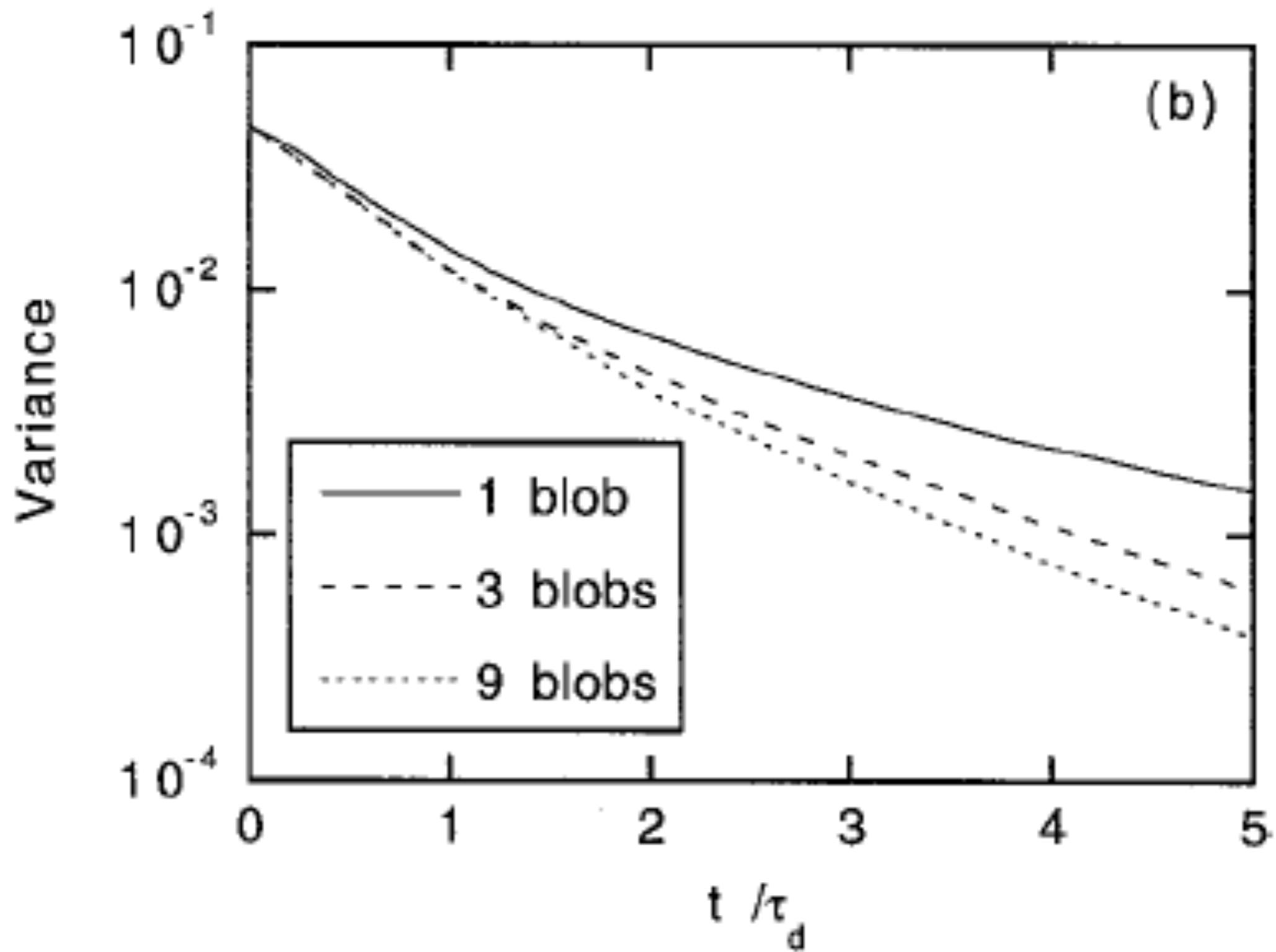
# Use the LEM to simulate mixing



**Perform 100  
realizations  
starting from  
these initial  
conditions.**



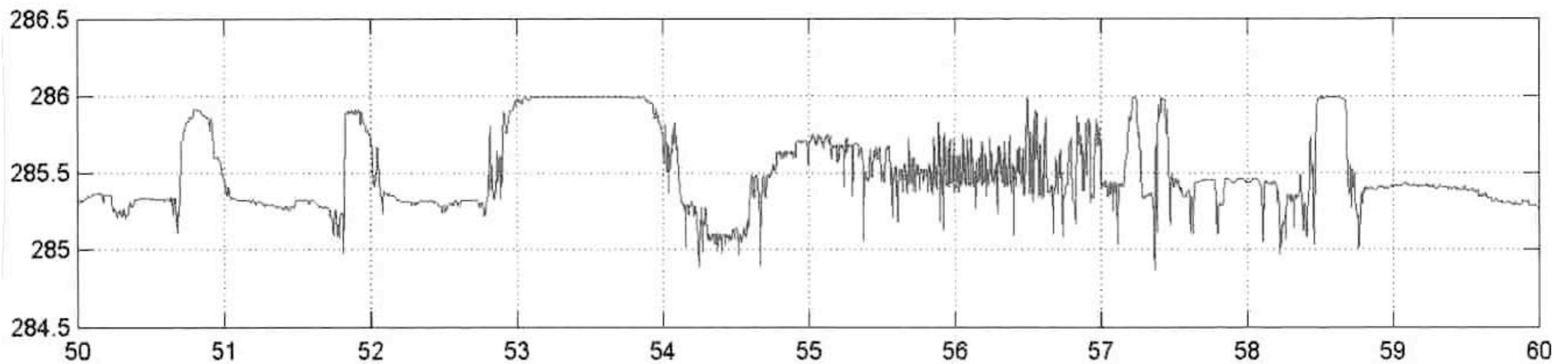
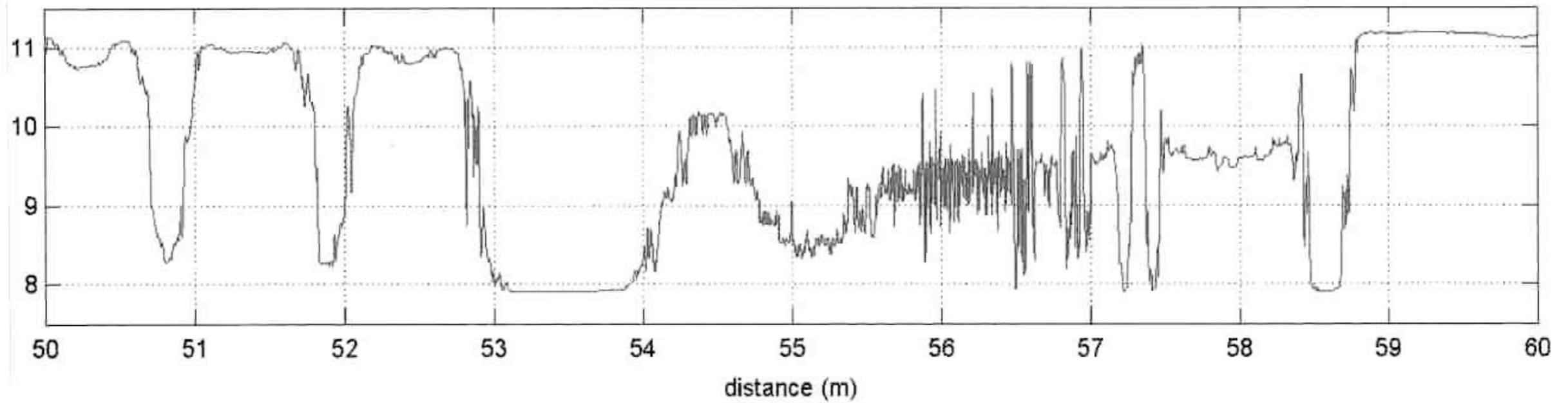
$\tau_L$  is a large-eddy time scale  $\sim (L^2/\epsilon)^{1/3}$



$\tau_d$  is the mixing time scale  $\sim (d^2 / \epsilon)^{1/3}$

**Example of using the LEM  
to study isobaric mixing of  
clear and cloudy air**

# LEM water vapor and temperature fields



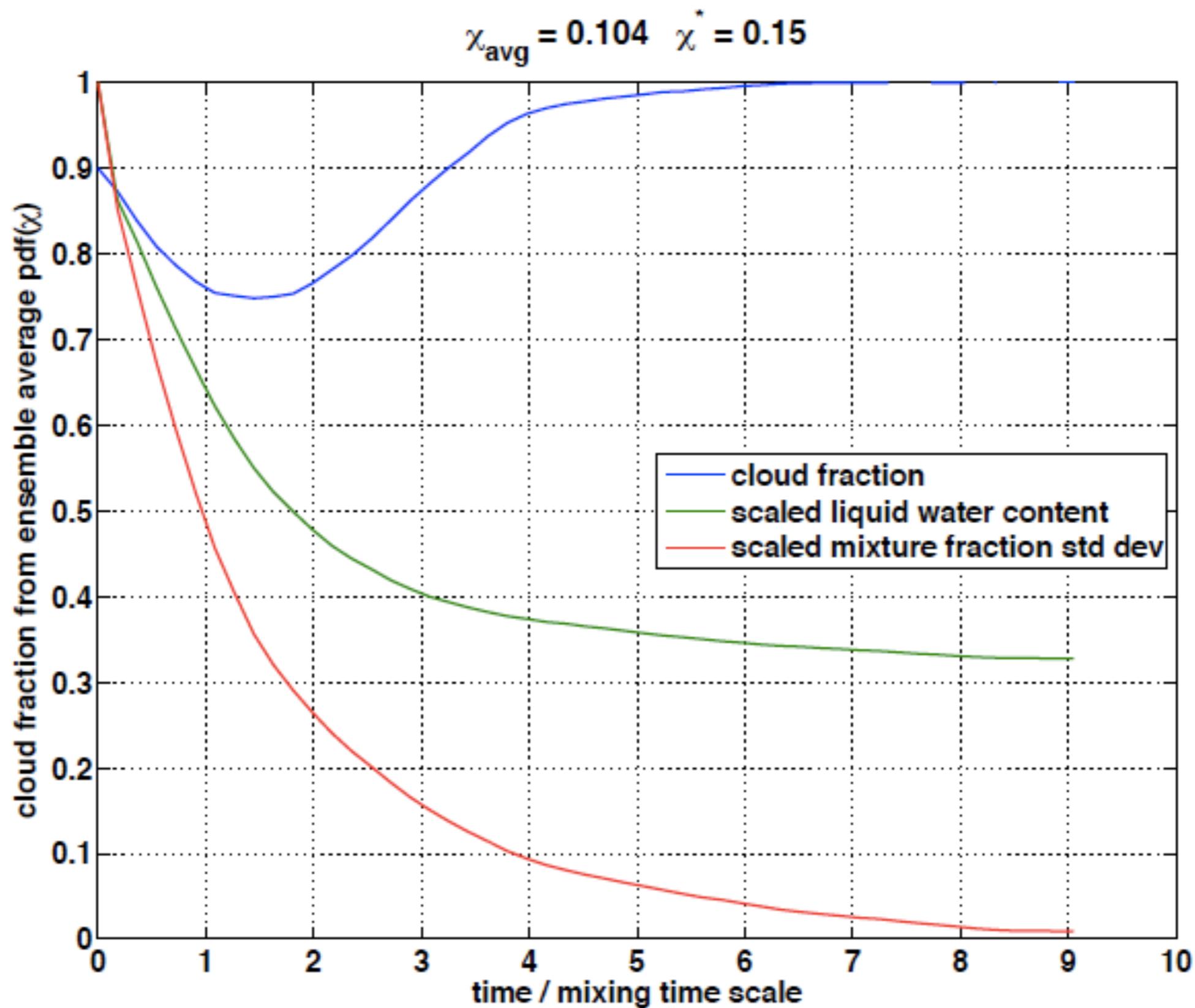


Figure 4: Evolution of cloud fraction, liquid water content, and mixture fraction std dev during mixing after entrainment of one blob of size  $d/L = \bar{\chi} = 0.1$ , with  $\chi^* = 0.15$ , based on 100 realizations of the linear eddy model.

- Bridging the LES-DNS gap
- Large-eddy simulation (LES)
- Parcel model
- Linear Eddy Model (LEM)
- **One-Dimensional Turbulence (ODT)**
- Explicit Mixing Parcel Model (EMPM)
- ClusColl (Clustering and Collision Model)

- **One-Dimensional Turbulence (ODT)**

- Evolves **scalar and velocity** spatial variability on all relevant turbulence scales using one dimension.
- Distinguishes turbulent deformation and molecular diffusion.
- Turbulence properties are **predicted.**



- Bridging the LES-DNS gap
- Large-eddy simulation (LES)
- Parcel model
- Linear Eddy Model (LEM)
- One-Dimensional Turbulence (ODT)

- Explicit Mixing Parcel Model (EMPM)
- ClusColl (Clustering and Collision Model)

# Turbulent Motion of *Fluid Elements* can be Represented by Applying I-D Maps

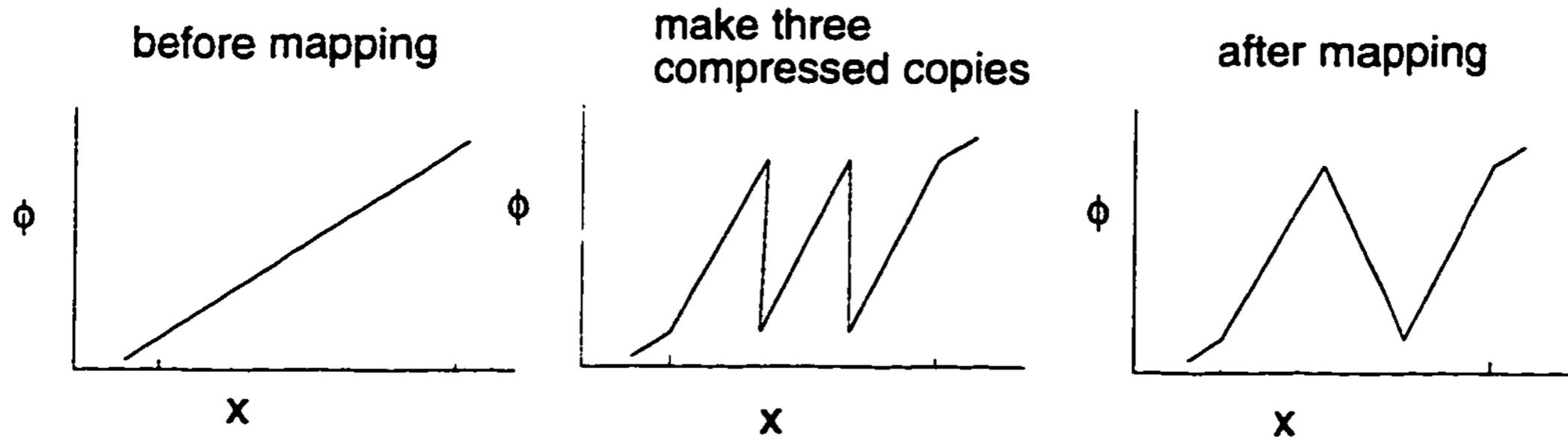


Figure 2.1. Schematic diagram of a triplet mapping event.

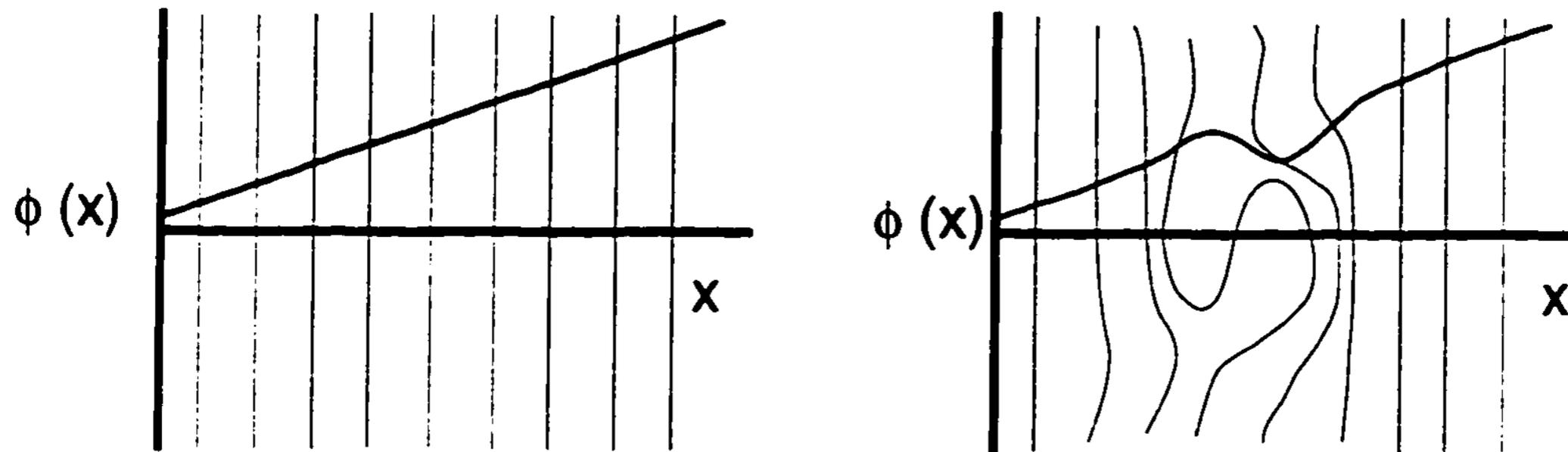
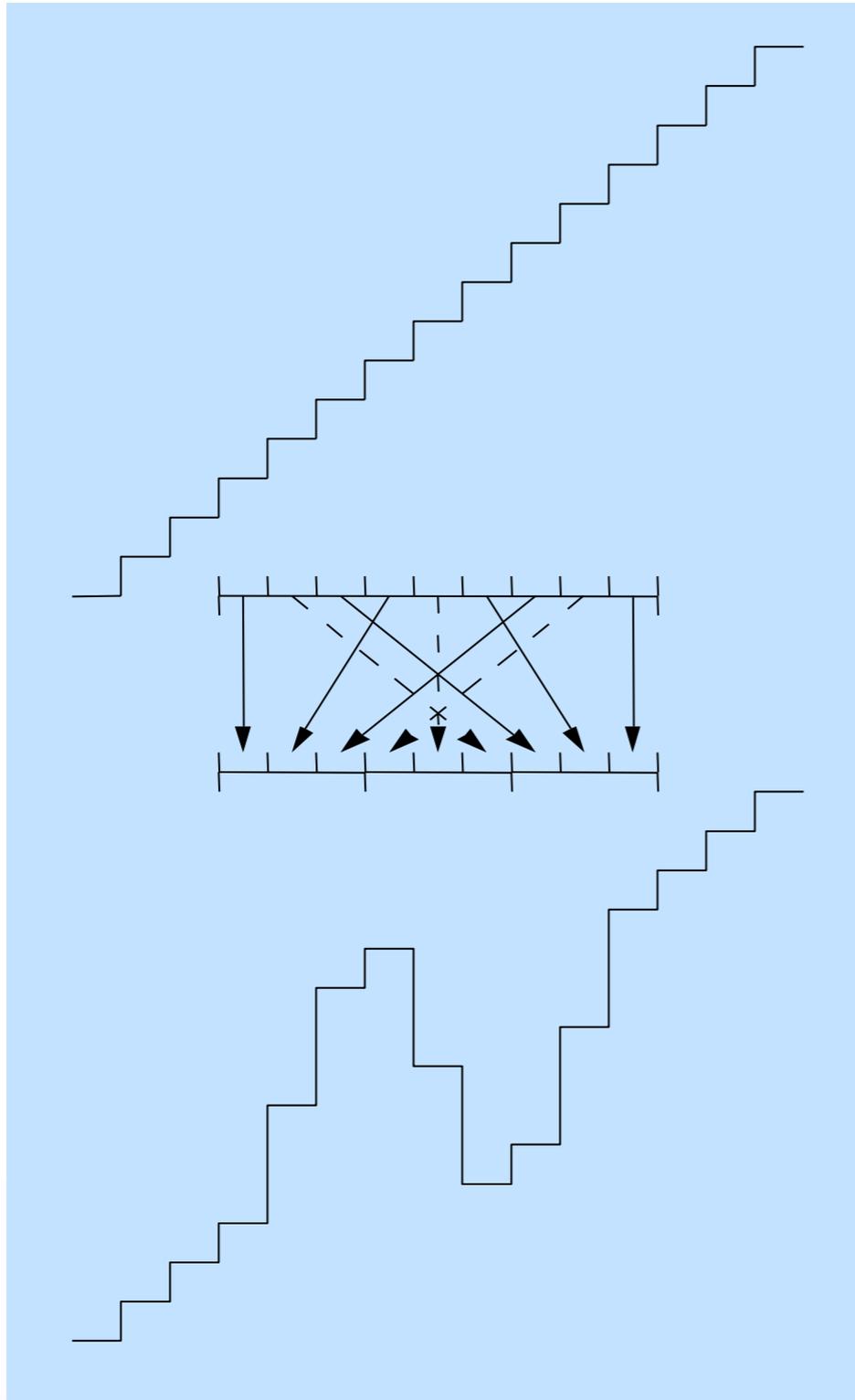


Figure 2.2. Effect of a single counterclockwise eddy on a scalar field with linear gradient.

Advection is modeled as a sequence of *triplet maps* that preserve desired advection properties, even in 1D



The triplet map is implemented numerically as a permutation of fluid cells (or on an adaptive mesh)

- Bridging the LES-DNS gap
- Large-eddy simulation (LES)
- Parcel model
- Linear Eddy Model (LEM)
- One-Dimensional Turbulence (ODT)
- **Explicit Mixing Parcel Model (EMPM)**
- ClusColl (Clustering and Collision Model)

- **Explicit Mixing Parcel Model (EMPM)**

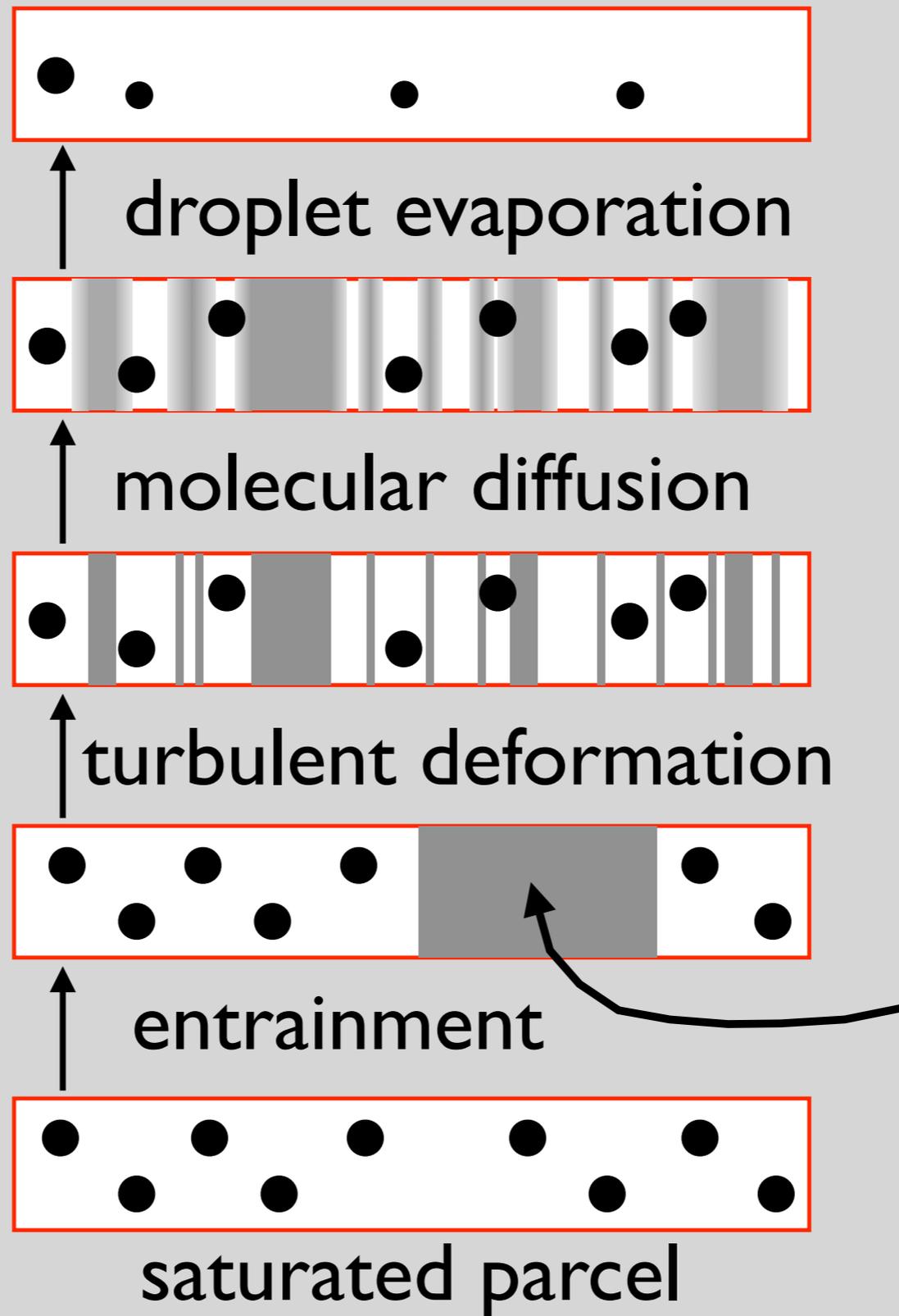
- Combines the Linear Eddy Model with:

- A parcel model.
- Stochastic entrainment events.
- Bulk or droplet microphysics.
- Specified ascent speed.

- Cloud droplets can grow or evaporate according to their local environments.



# EMPM with droplets and entrainment



# Explicit Mixing Parcel Model (EMPM)

- The EMPM predicts the evolving in-cloud variability due to entrainment and finite-rate turbulent mixing using a 1D representation of a rising cloudy parcel.
- The 1D formulation allows the model to resolve fine-scale variability down to the smallest turbulent scales ( $\sim 1$  mm).
- The EMPM can calculate the growth of several thousand individual cloud droplets based on each droplet's local environment.

Krueger, S. K., C.-W. Su, and P. A. McMurry, 1997: Modeling entrainment and fine-scale mixing in cumulus clouds. *J. Atmos. Sci.*, 54, 2697–2712.

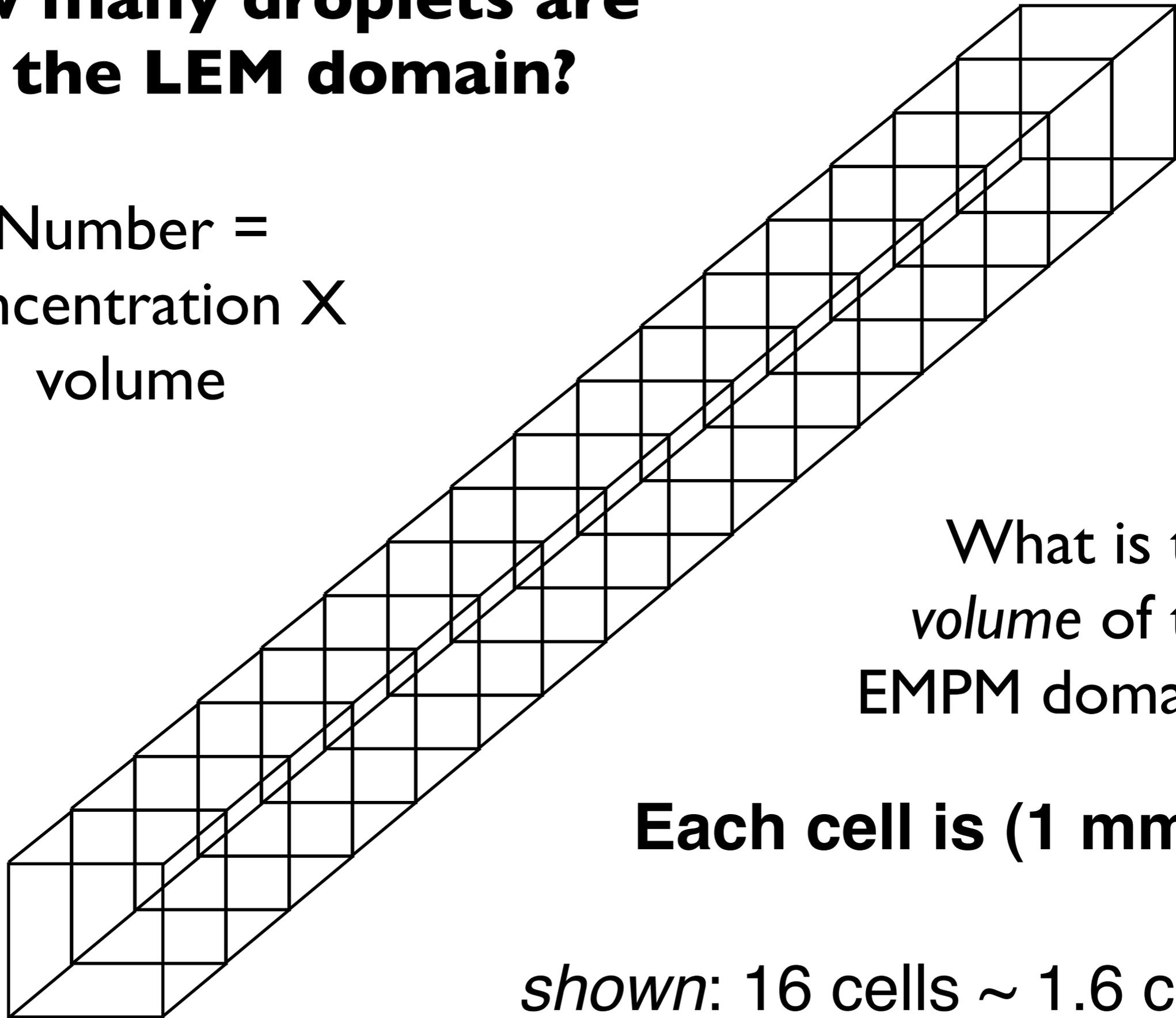
Su, C.-W., S. K. Krueger, P. A. McMurry, and P. H. Austin, 1998: Linear eddy modeling of droplet spectral evolution during entrainment and mixing in cumulus clouds. *Atmos. Res.*, 47–48, 41–58.

# EMPM Variables

- Bulk microphysics:
  - Liquid water static energy
  - Total water mixing ratio
- Droplet microphysics:
  - Temperature
  - Water vapor mixing ratio

# How many droplets are in the LEM domain?

Number =  
concentration  $\times$   
volume

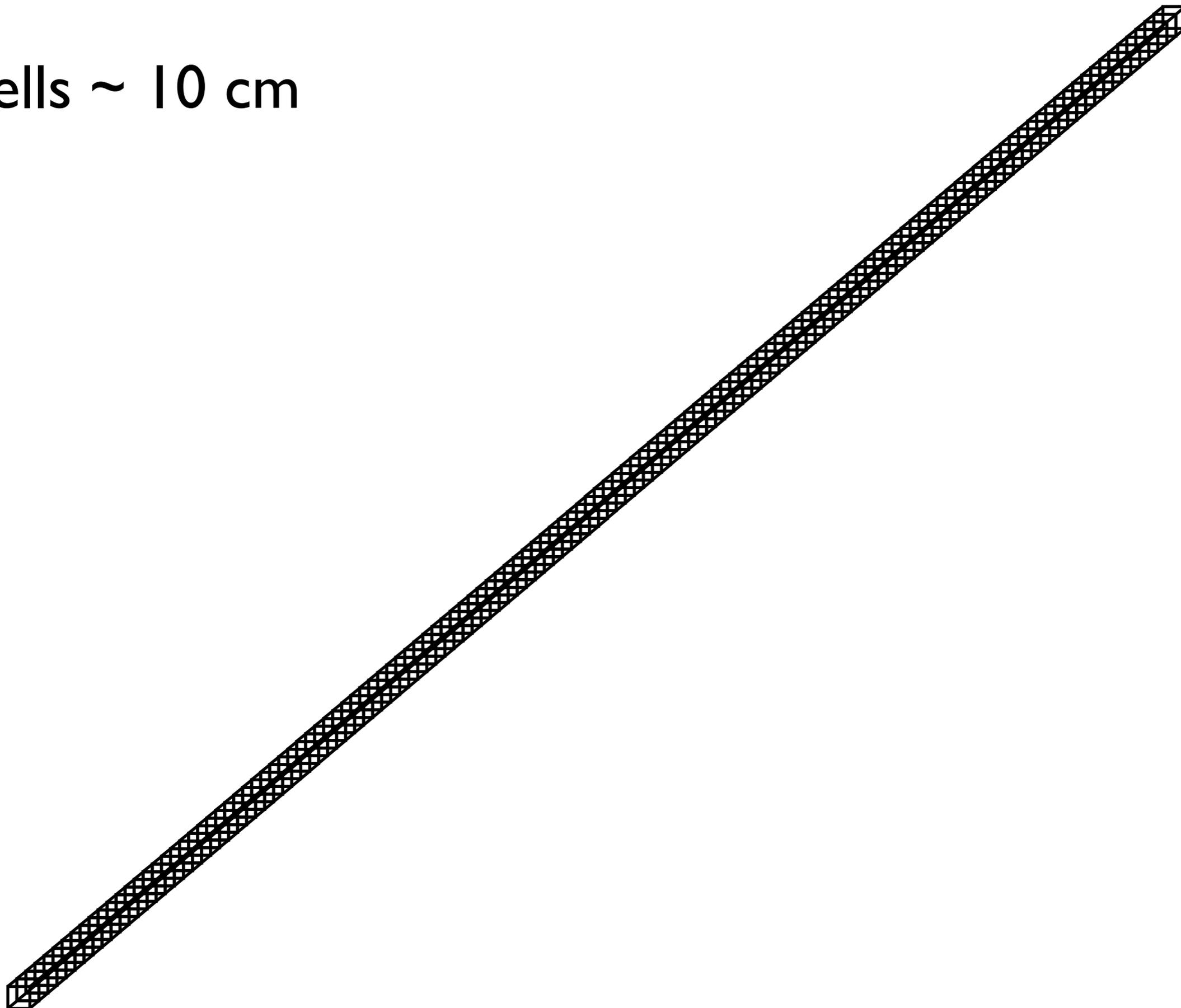


What is the  
*volume* of the  
EMPM domain?

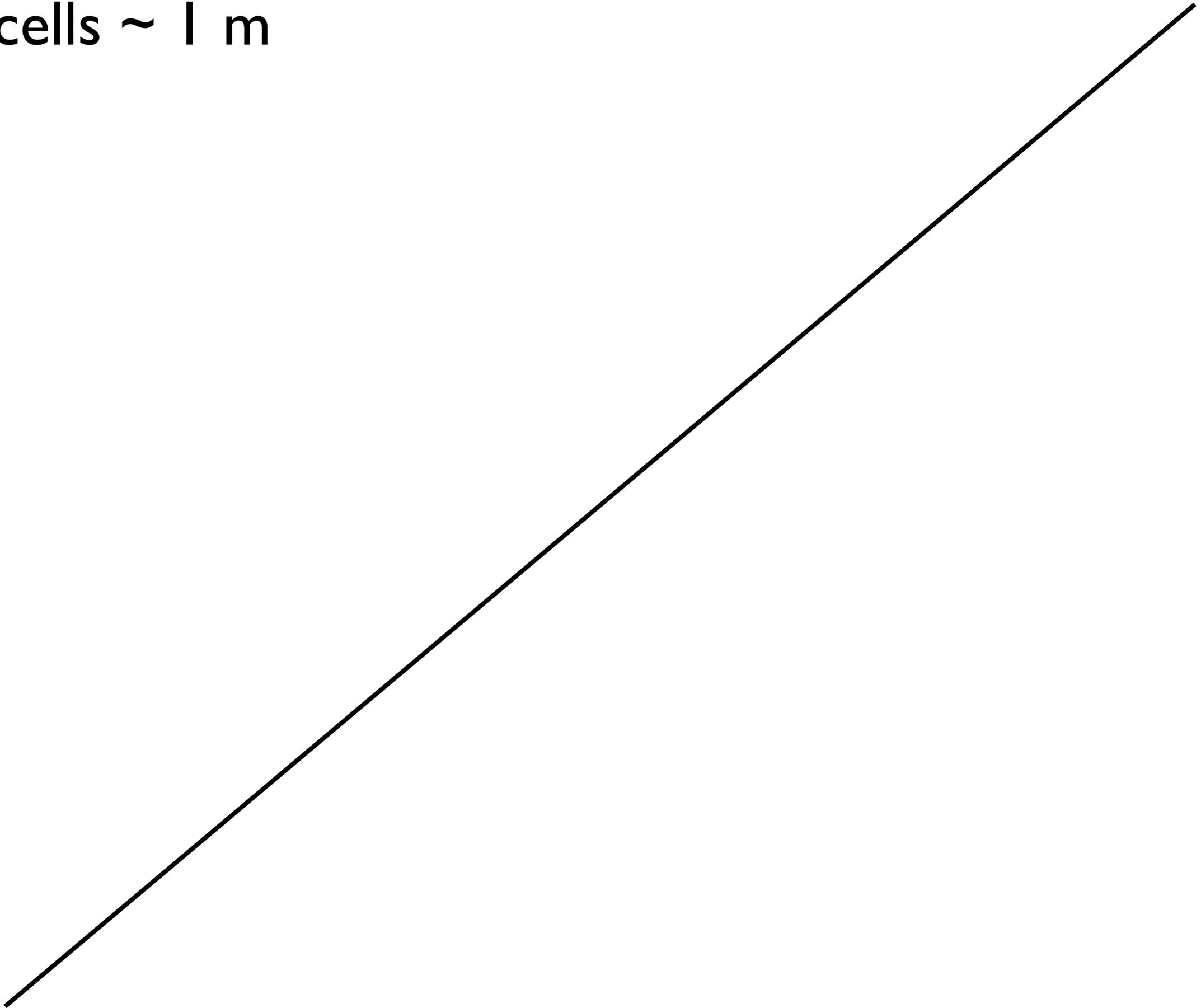
**Each cell is  $(1 \text{ mm})^3$**

*shown: 16 cells  $\sim$  1.6 cm*

128 cells ~ 10 cm



1024 cells ~ 1 m



$$\begin{aligned} \text{Volume of a 20 m-long domain} &= \\ 20 \text{ m} \times 1 \text{ mm} \times 1 \text{ mm} &= \\ 20 \times 100 \text{ cm} \times 0.1 \text{ cm} \times 0.1 \text{ cm} &= \\ 20 \text{ cm}^3 & \end{aligned}$$

$$\begin{aligned} \text{Number of droplets in domain} &= \\ \text{concentration} \times \text{domain volume} &= \\ 100 \text{ cm}^{-3} \times 20 \text{ cm}^3 &= \\ 2000 & \end{aligned}$$

$$\begin{aligned} \text{Number of droplets per cell} &= \\ \text{concentration} \times \text{cell volume} &= \\ 100 \text{ cm}^{-3} \times 0.001 \text{ cm}^3 &= \\ 0.1 \text{ (1 droplet per cm)} & \end{aligned}$$

What is the average droplet separation in 3D?

Each droplet occupies a volume of

$$V/N = 0.01 \text{ cm}^3$$

This is a cube with sides of length

$$(0.01 \text{ cm}^3)^{1/3} = 0.2 \text{ cm} = 2 \text{ mm}$$

# EMPM Required Inputs

- Required for a classical (instant mixing) parcel model calculation:

Thermodynamic properties of cloud-base air

Updraft speed

Entrainment rate

Thermodynamic properties of entrained air

Aerosol properties

- In addition, the EMPM requires:

Parcel size

Entrained blob size,  $d$

Turbulence intensity (e.g., dissipation rate,  $\epsilon$ )

# Droplet growth by diffusion of water vapor

$$r_j \frac{dr_j}{dt} = \frac{S - A_1 + A_2}{A_3 + A_4}$$

$r_j$  is the radius of the  $j$ th droplet,  $A_1$  and  $A_2$  are the correction factors for droplet curvature and solute effects,  $A_3$  and  $A_4$  are the heat conduction and vapor diffusion terms, and  $S$  is the supersaturation.

**In the EMPM, droplets move relative to the fluid at their terminal velocities.**

# Droplet Microphysics

droplet radius:  $r_j \frac{dr_j}{dt} = \frac{S - A_1 + A_2}{A_3 + A_4},$

supersaturation  $S = \frac{q_v}{q_{vs}} - 1,$

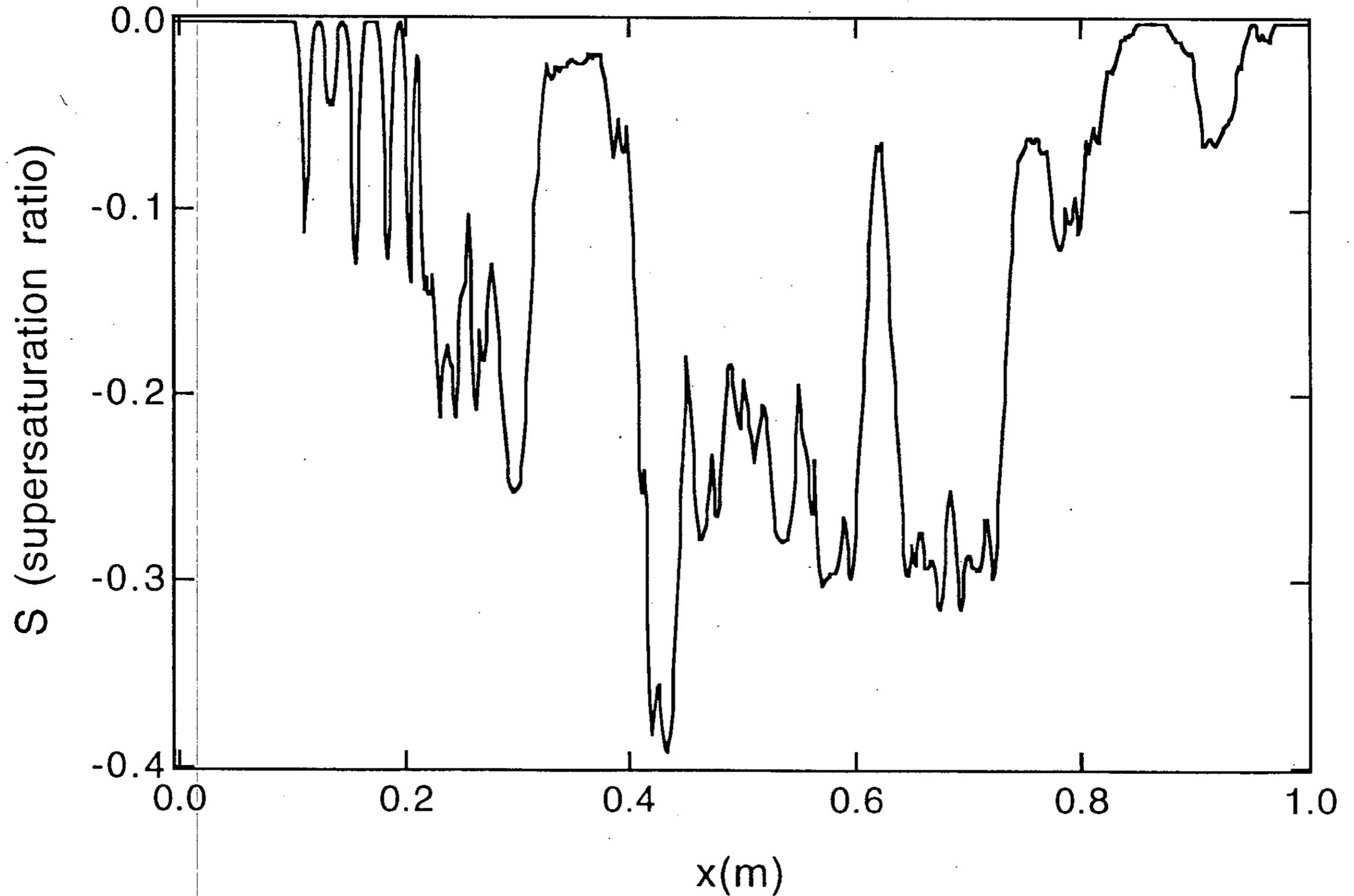
water vapor:  $\left. \frac{dq_v}{dt} \right|_{\text{phase change}} = - \sum_j 4\pi N_j \rho_w r_j^2 \frac{dr_j}{dt},$

temperature:

$$\left. \frac{dT}{dt} \right|_{\text{phase change}} = - \frac{L_v}{c} \left. \frac{dq_v}{dt} \right|_{\text{phase change}} - w \frac{g}{c},$$

# Snapshot of supersaturation ratio during mixing

(from the *EMPM*)



# Droplet histories during mixing

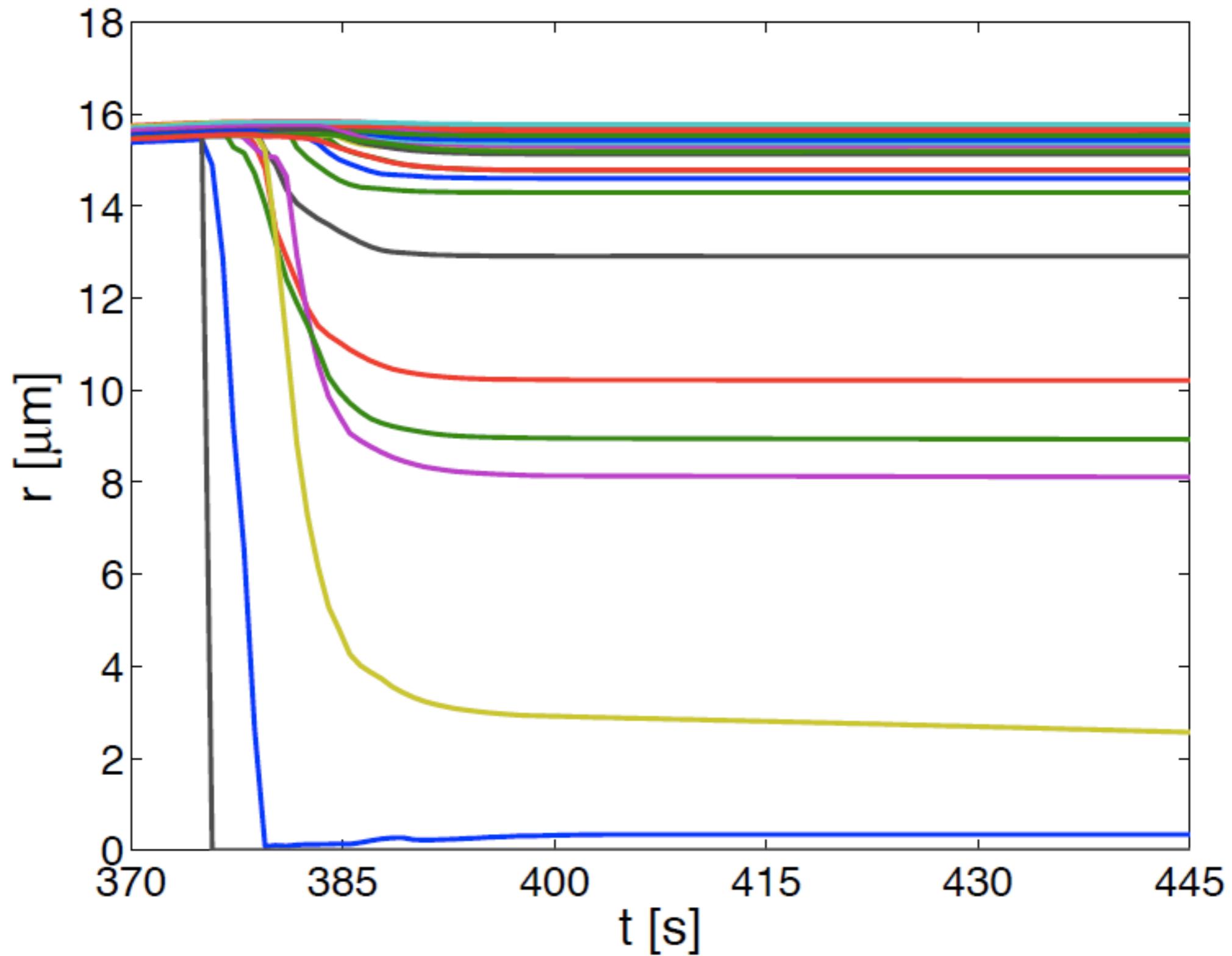
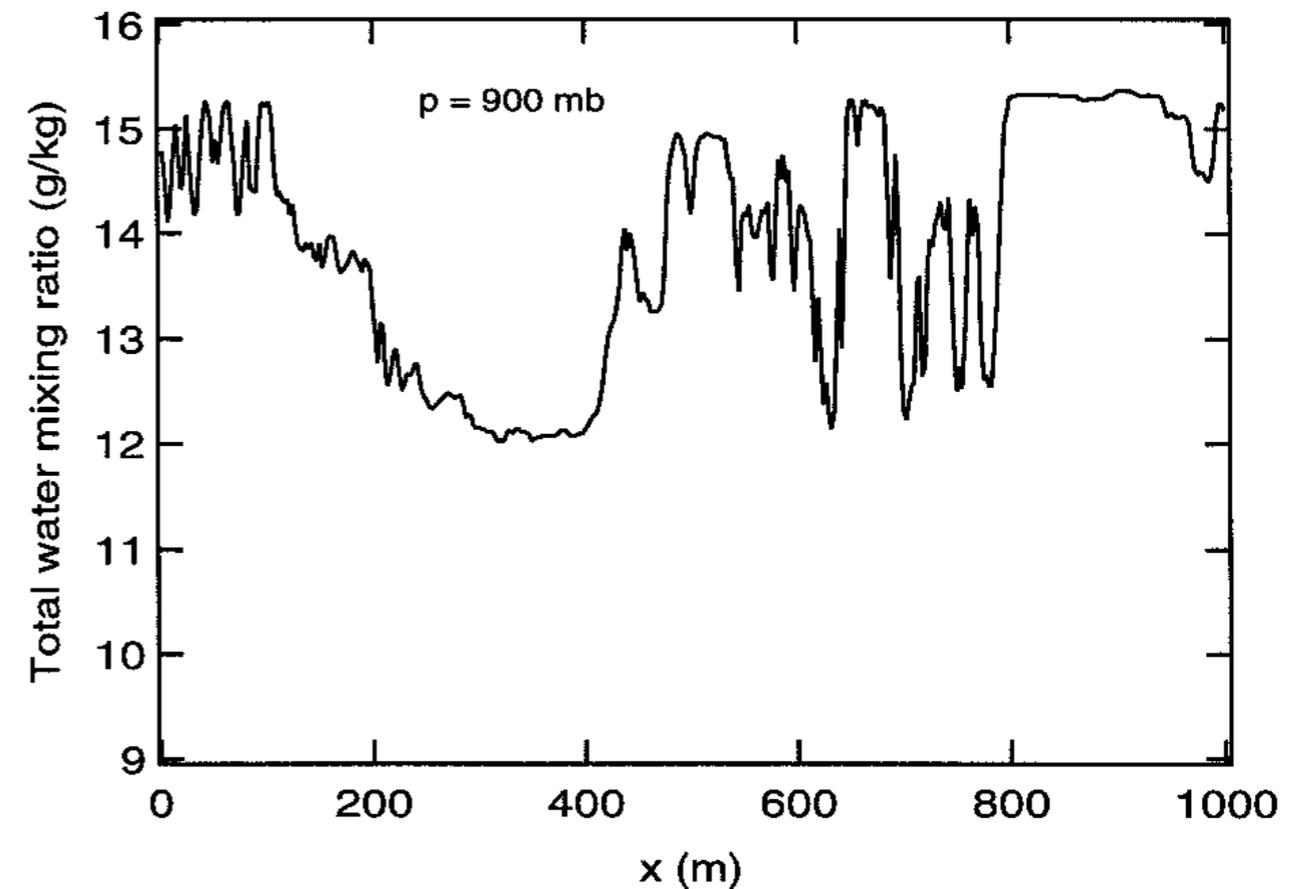
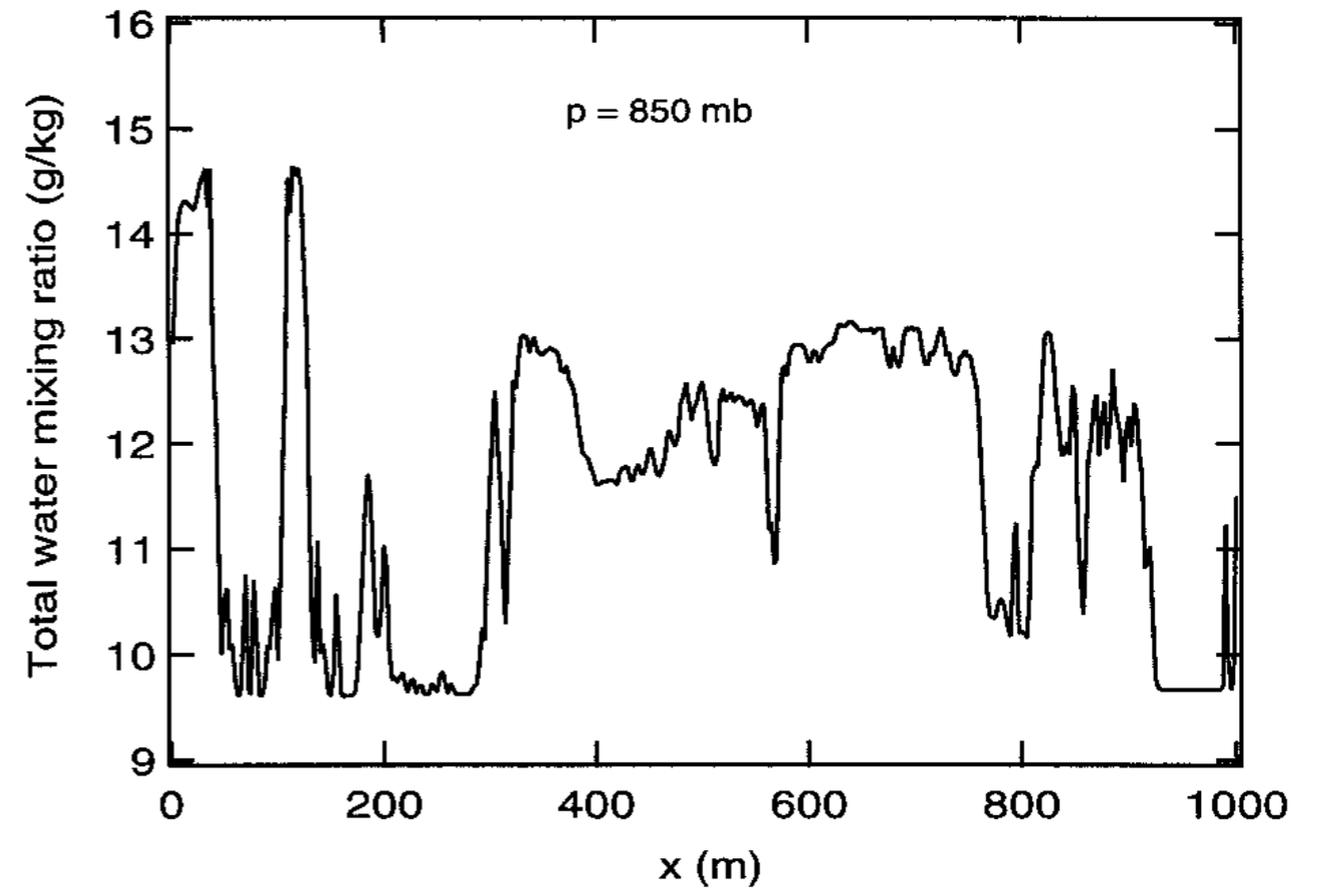


Figure 4.10: Radius histories of 30 droplets for  $f = 0.1$  and  $RH_e = 0.219$ .

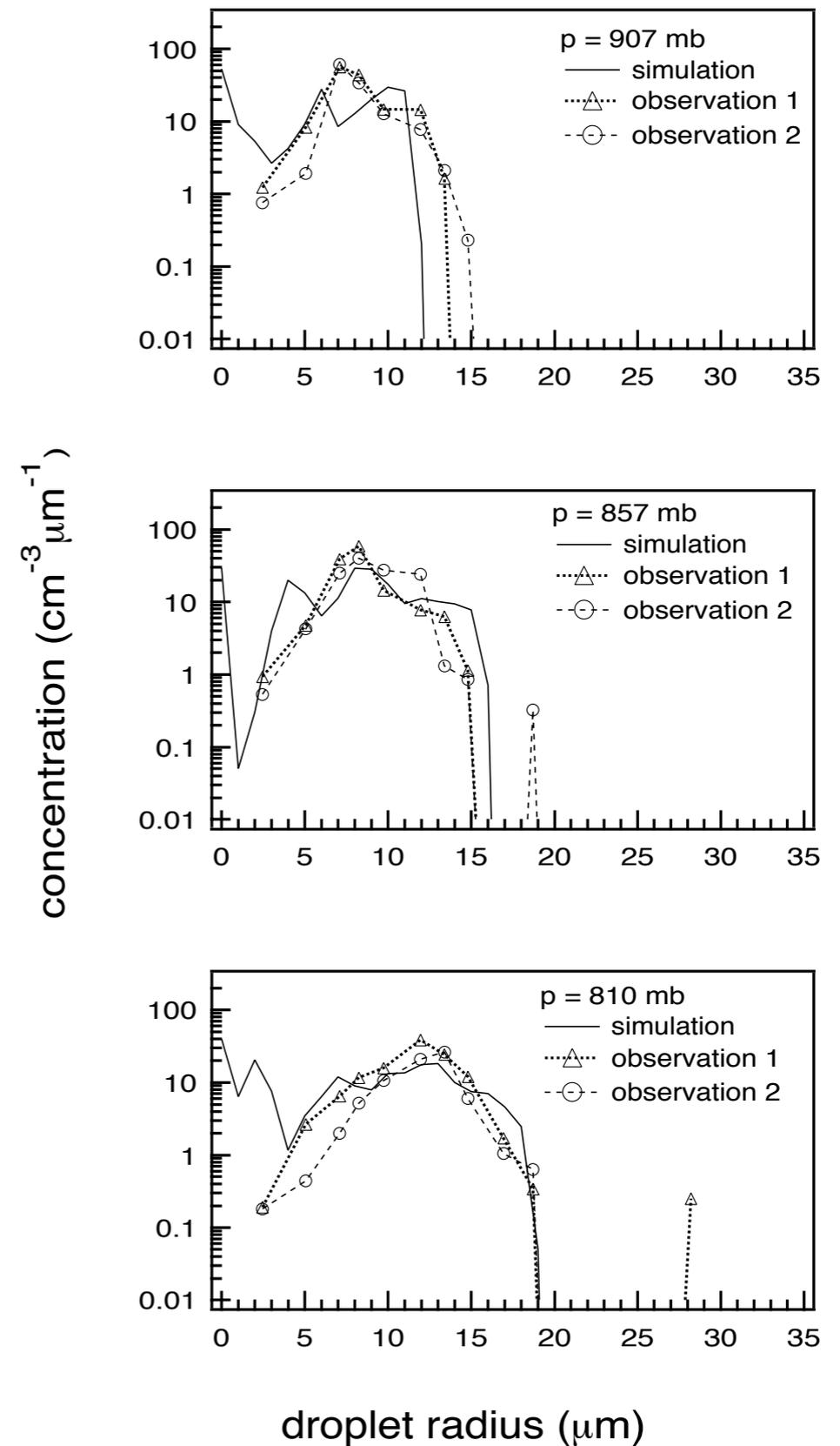
## Comparison to Measurements

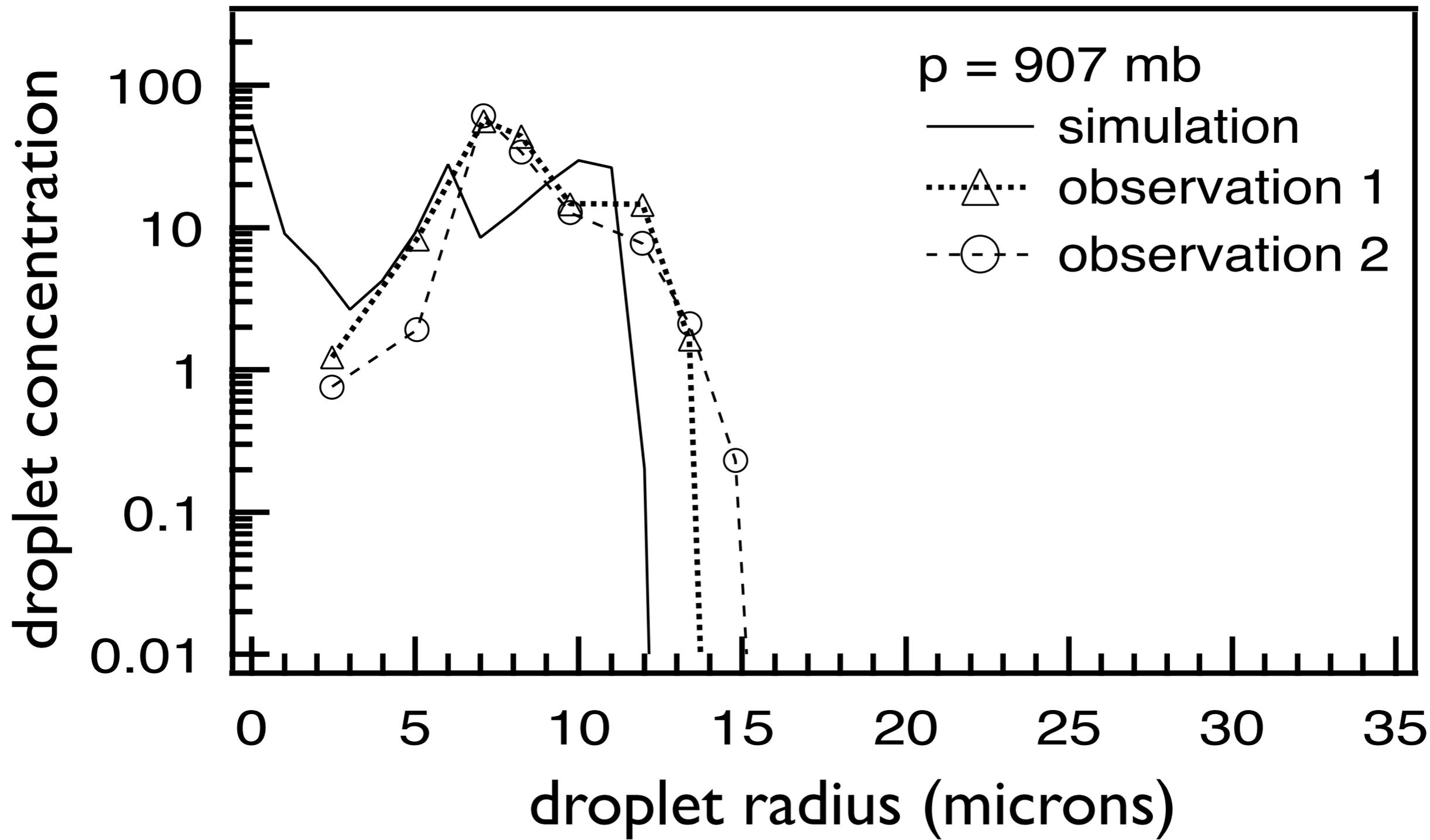
EMPM results can be directly compared to high-rate aircraft measurements of temperature, water vapor, liquid water content, and droplet size spectra.

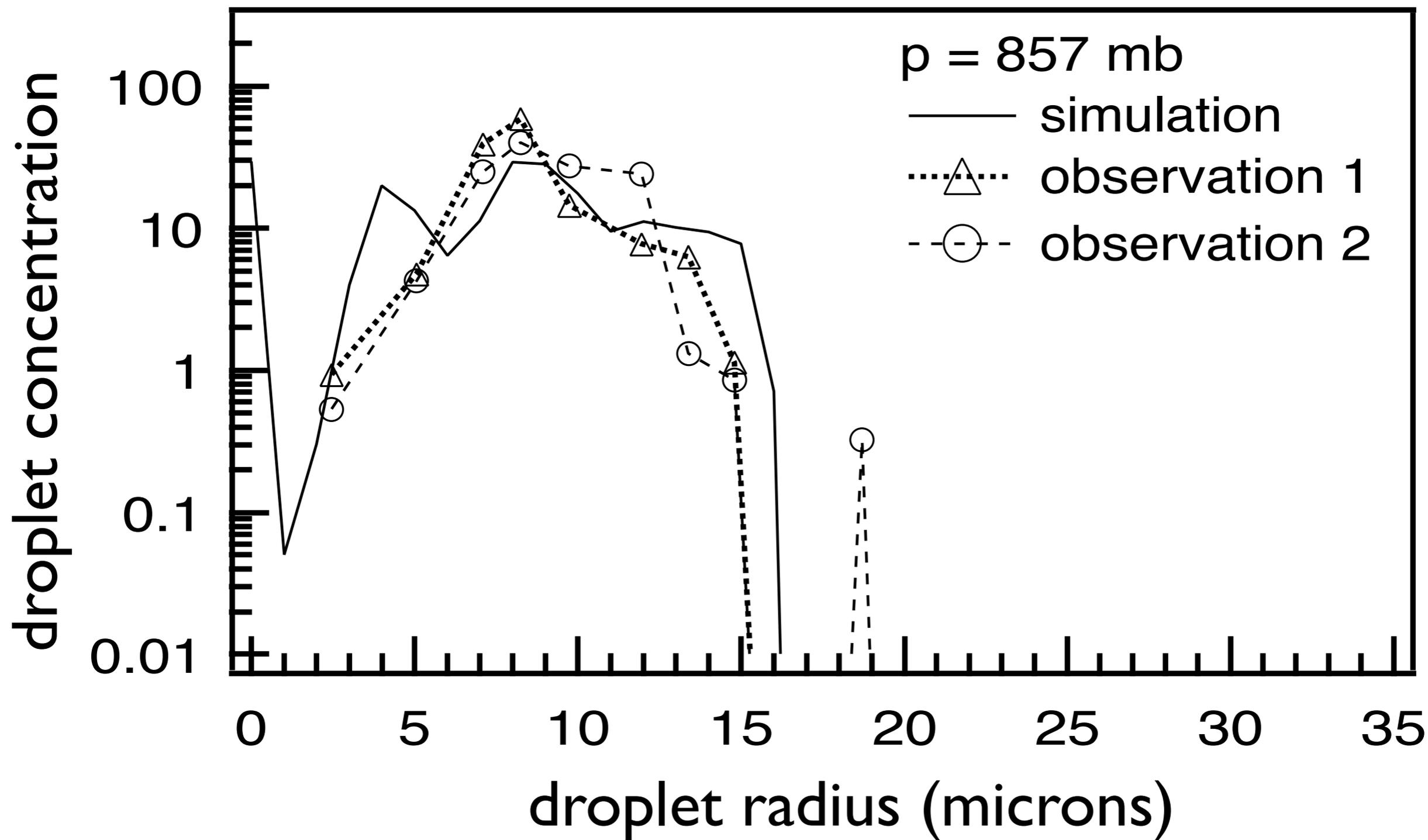


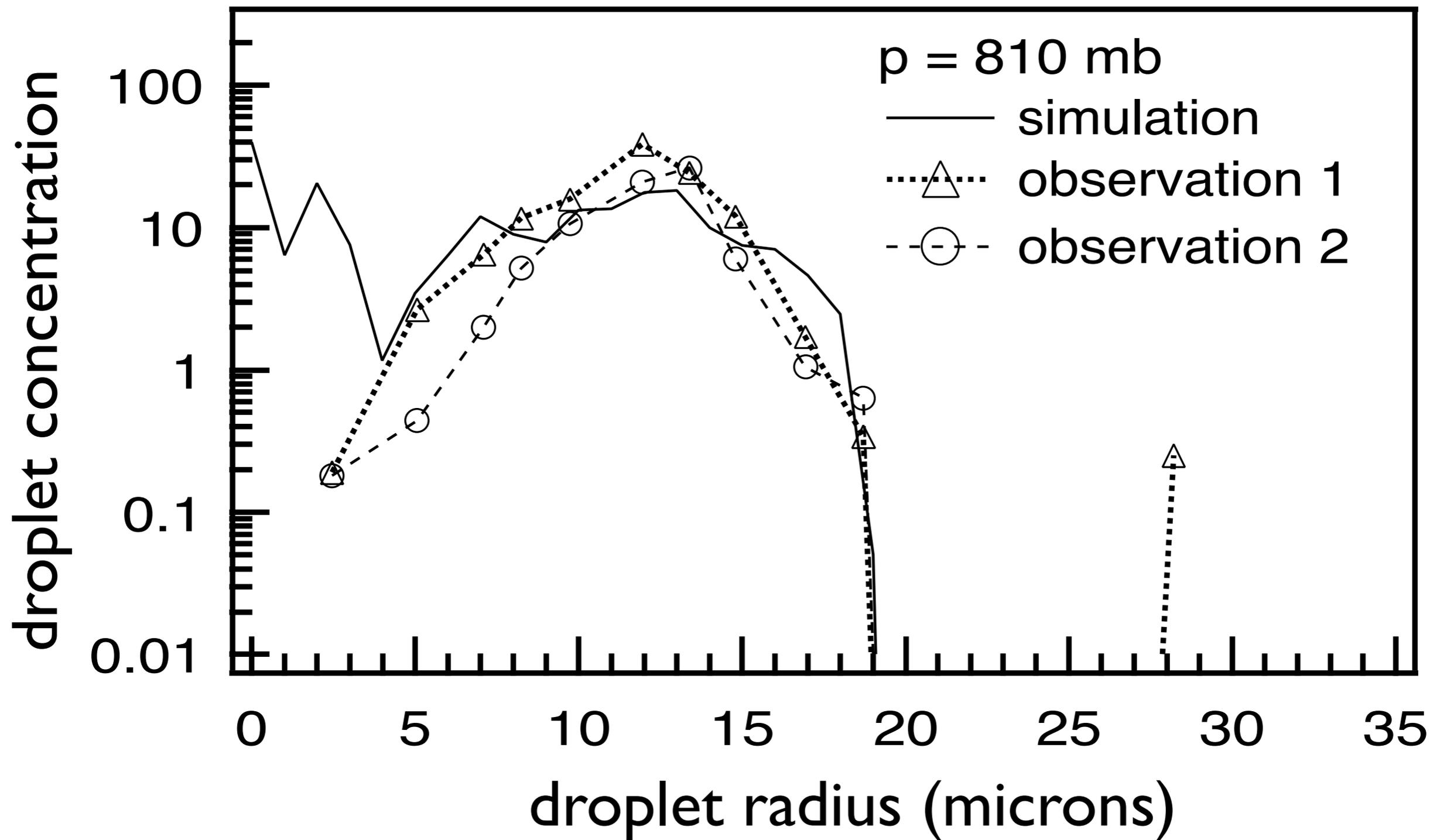
# Applying the EMPM to Hawaiian Cumuli

The EMPM produced realistic, broad droplet size spectra that included super-adiabatic-sized droplets. The computed spectra agreed with those measured by aircraft.







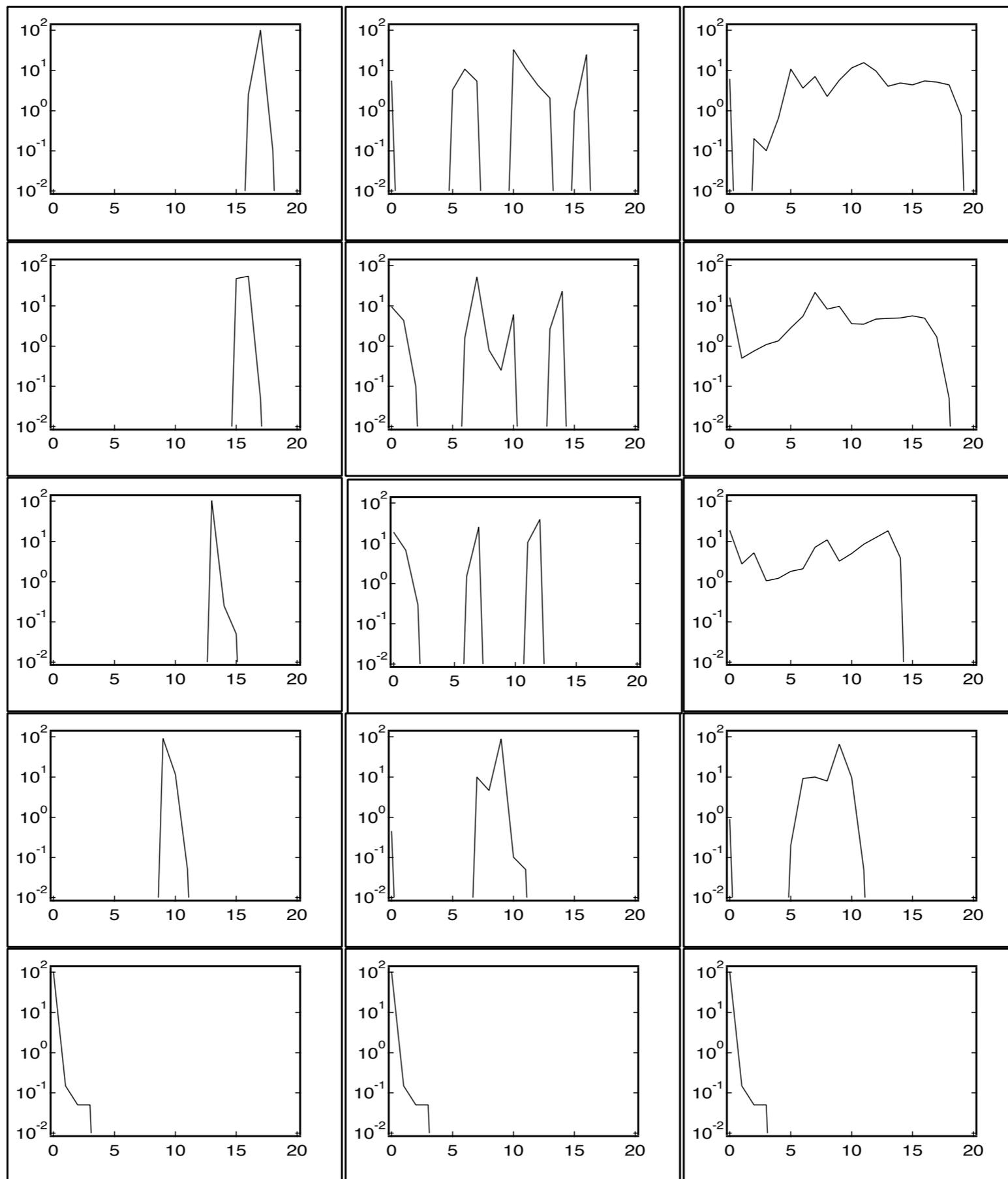


no entrainment +  
finite-rate mixing

entrainment +  
instant mixing

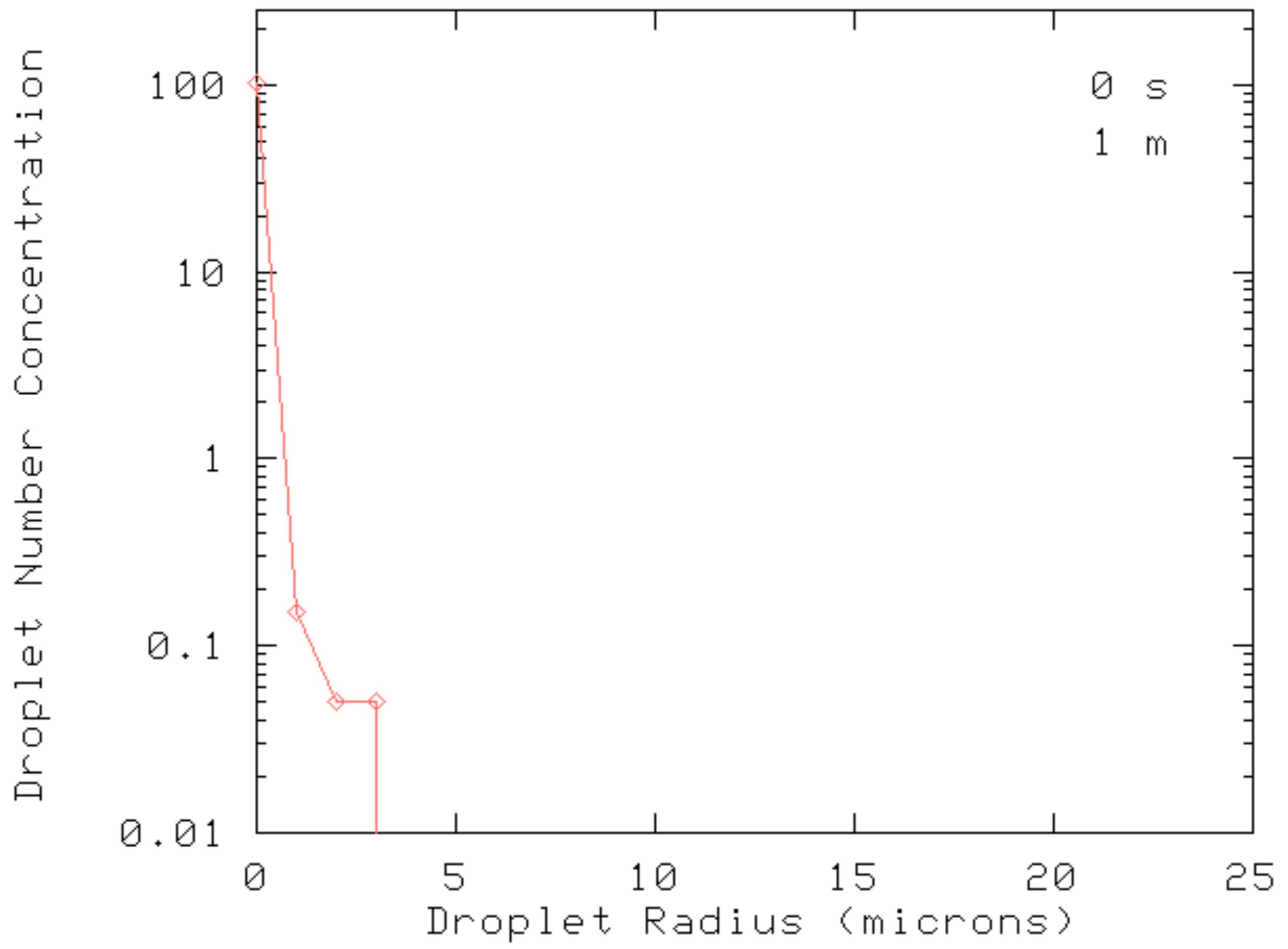
entrainment +  
finite rate mixing

droplet concentration ( $\text{cm}^{-3} \mu\text{m}^{-1}$ )



droplet radius ( $\mu\text{m}$ )

# Finite Rate Mixing with Entrainment



# **Mixing and Evaporation Time Scales**

# Mixing Time Scales

## Eddy mixing time scale

The eddy mixing time scale for a blob of entrained air of size  $d$  is

$$\tau_{\text{eddy}} \equiv \left( \frac{d^2}{\varepsilon} \right)^{1/3}, \quad (6)$$

where  $\varepsilon$  is the dissipation rate of turbulent kinetic energy.

## Sedimentation mixing time scale

Droplet sedimentation transports droplets into the entrained air. The time scale for sedimentation of droplets of radius  $r$  into a blob of entrained air of size  $d$  is

$$\tau_{\text{sed}} \equiv \frac{d}{V_t}, \quad (7)$$

where  $V_t = cr^2$  is the Stokes terminal velocity, and  $c = 1.19 \times 10^8 \text{ m}^{-1} \text{ s}^{-1}$ .

# A generalized eddy mixing time scale

When there is droplet sedimentation and turbulent mixing, the two processes acting together allow the entrained air to evaporate droplets more quickly than if either process was acting alone. While droplets are falling into the entrained air, the average width of the entrained air filaments is simultaneously decreasing due to scale reduction by turbulent eddies. A generalized eddy mixing time scale that accounts for both droplet sedimentation and turbulent mixing is

$$\tau_{\text{eddy}}^* \equiv \frac{1}{\frac{1}{\tau_{\text{eddy}}} + \frac{1}{\tau_{\text{sed}}}}, \quad (8)$$

where  $\tau_{\text{eddy}}$  and  $\tau_{\text{sed}}$  are defined by (6) and (7), respectively.

# Mixing Time Scale

$$\tau = \left( \frac{d^2}{\epsilon} \right)^{1/3},$$

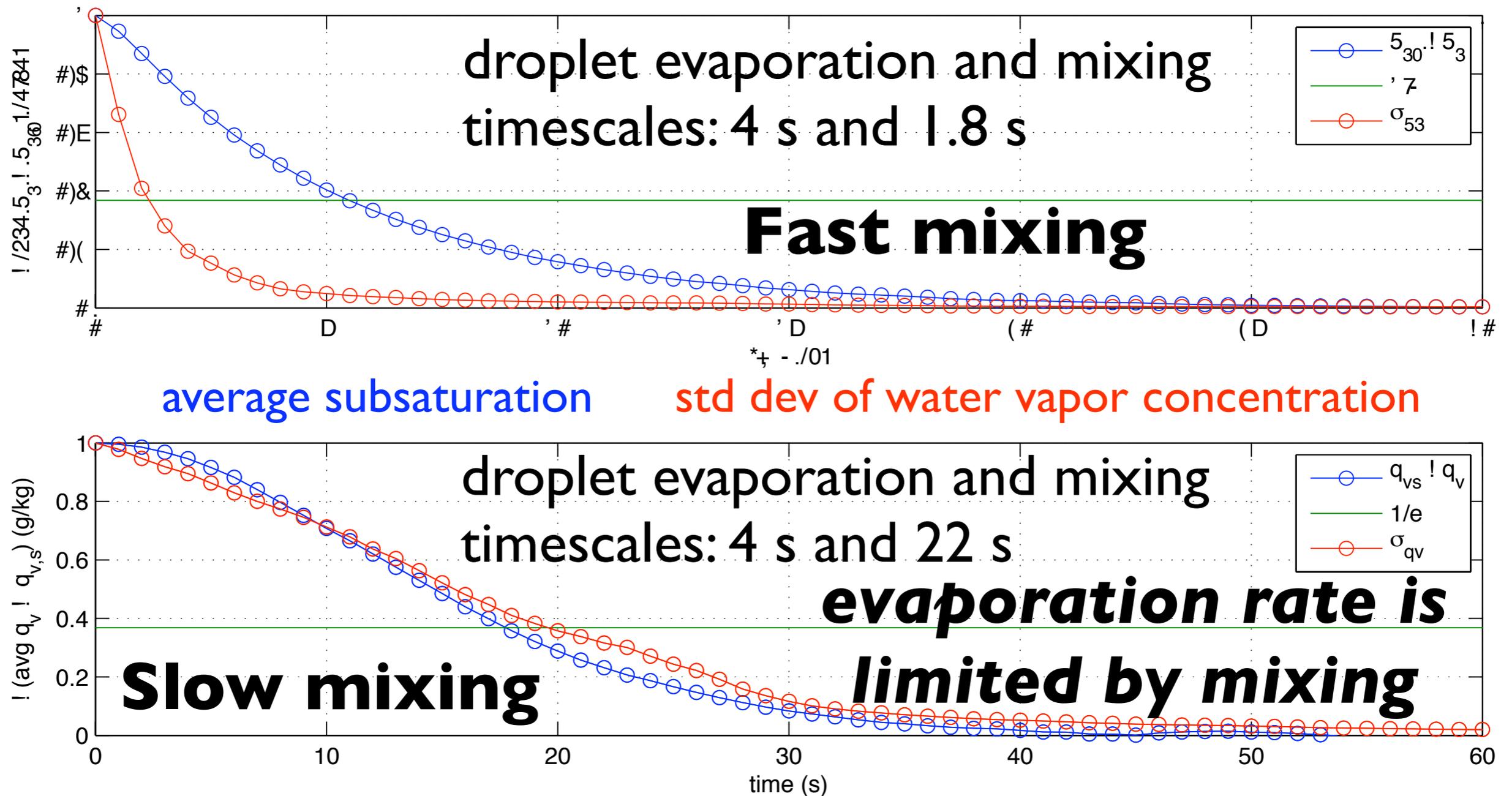
$d$  is entrained blob size,  $\epsilon$  is dissipation rate of turbulence kinetic energy.

For a **cumulus cloud**,  $U \sim 2$  m/s,  $L \sim 1000$  m, so  $\epsilon \sim U^3/L = 10^{-2}$  m<sup>2</sup>/s<sup>3</sup>. For  $d = 100$  m,  $\tau \sim 100$  s.

**Classic (instant mixing) parcel model** is recovered when

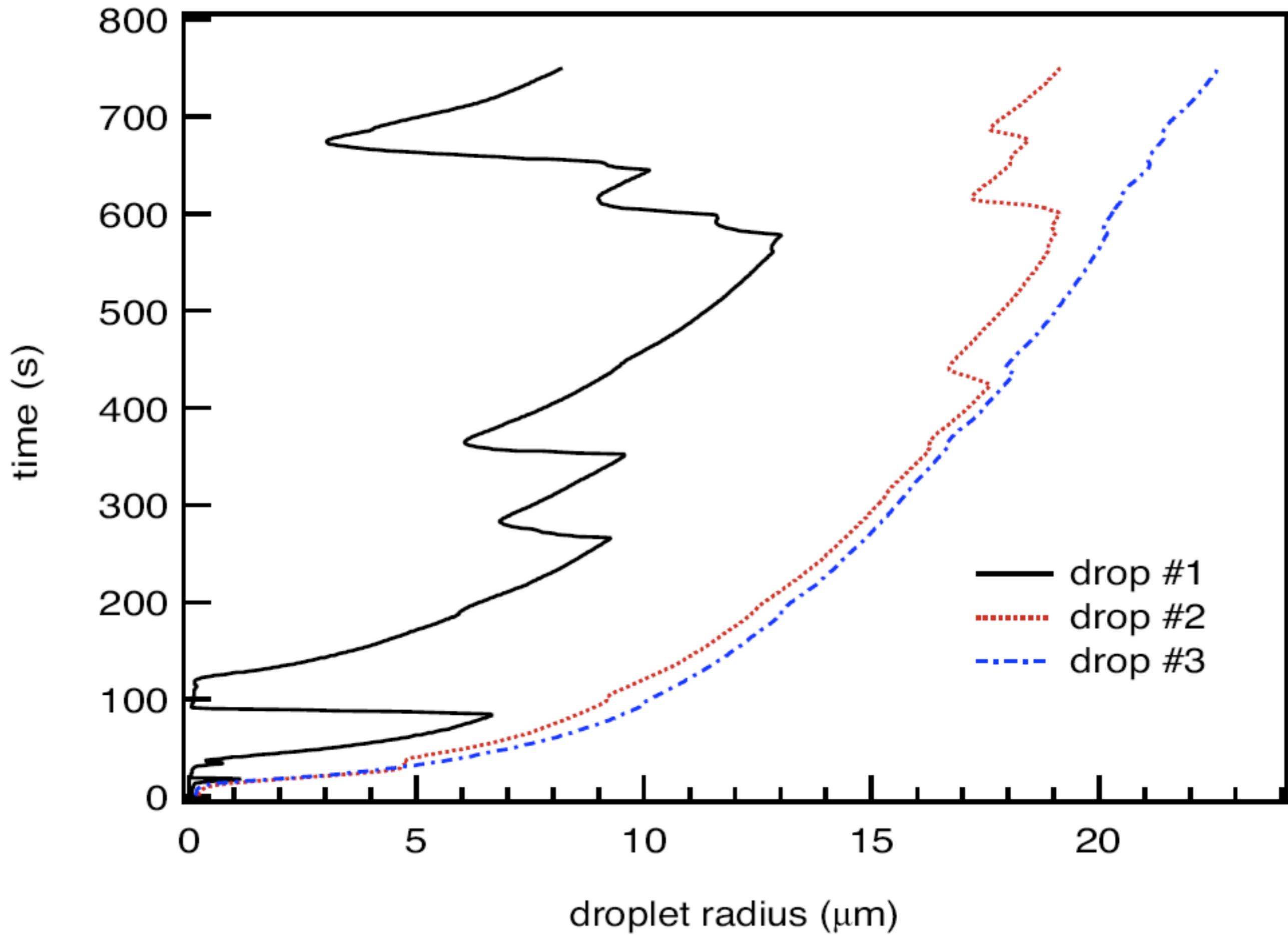
- Entrained blob size,  $d \rightarrow 0$
- Turbulence intensity,  $\epsilon \rightarrow \infty$

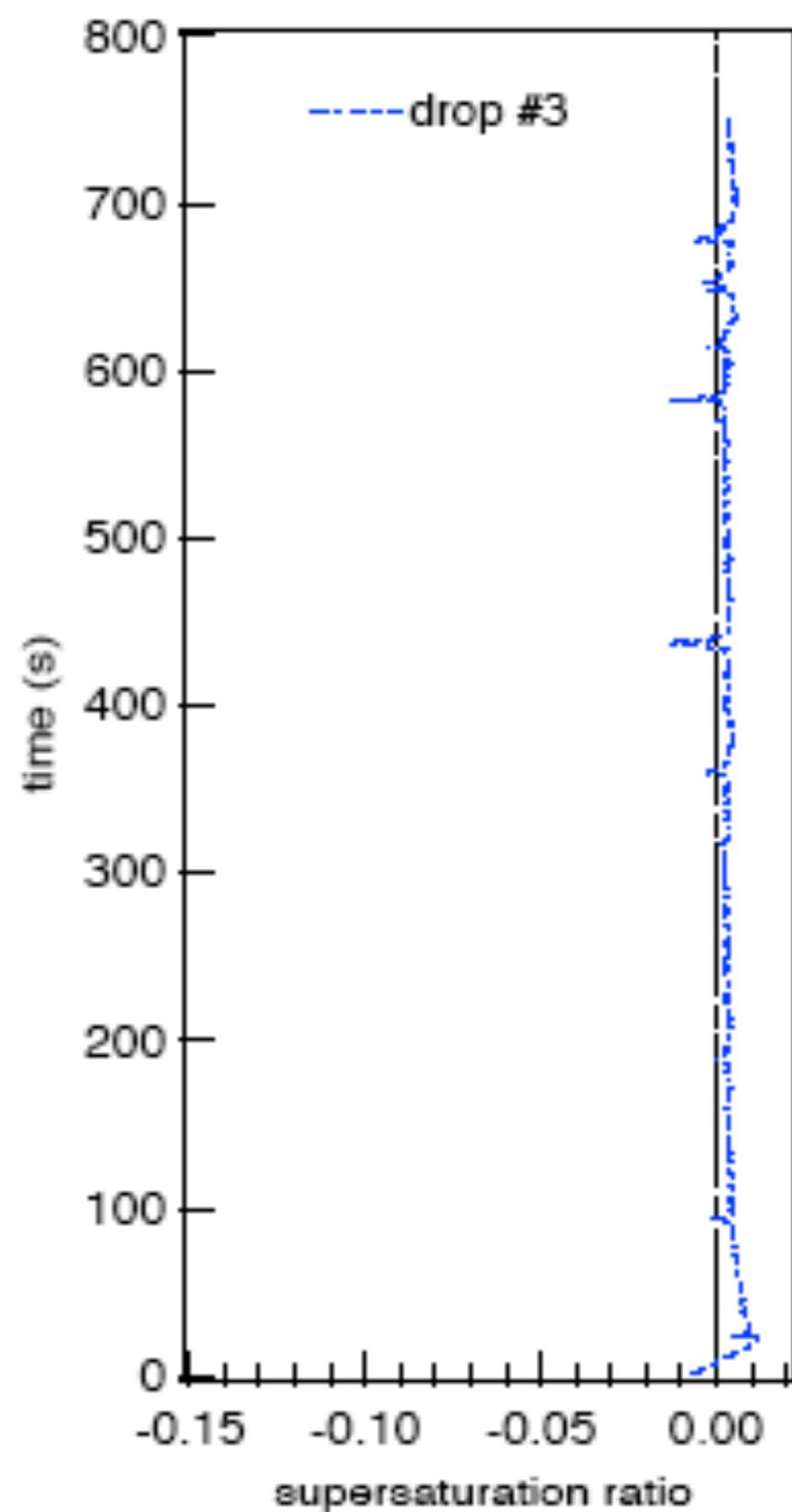
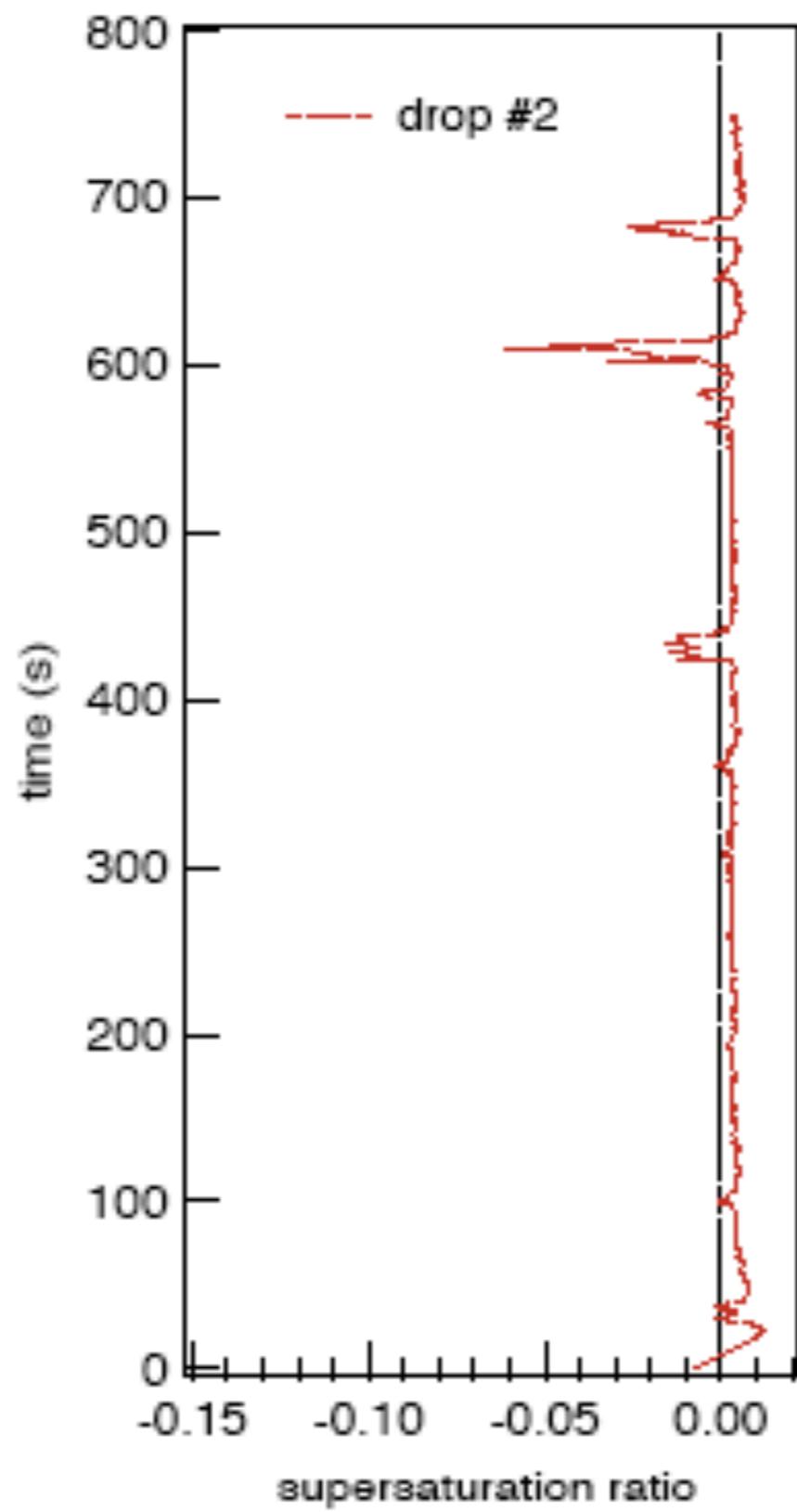
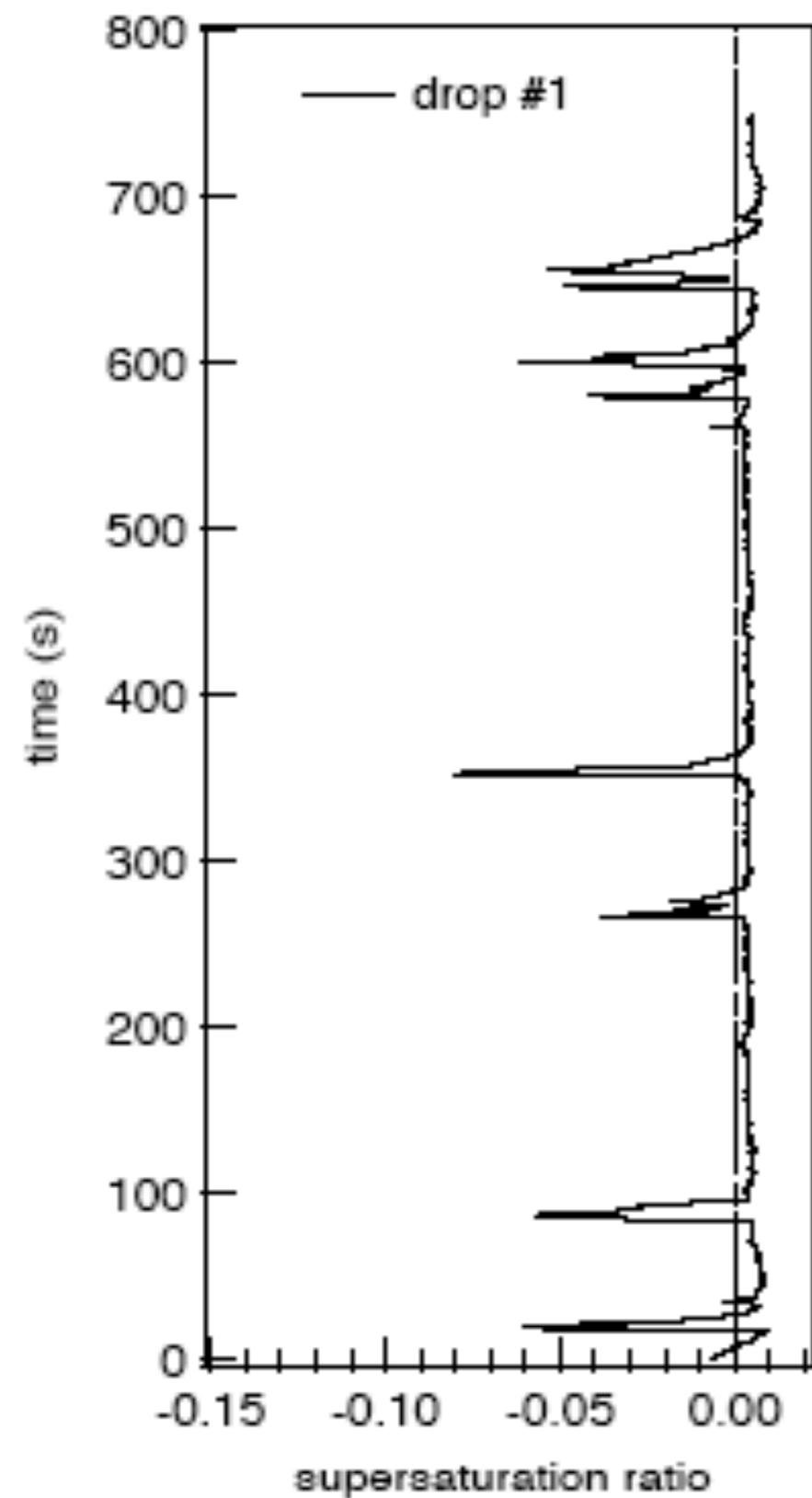
**TOP:** 1 segment of subsaturated air 0.25 m in length in a 1D domain 20 m in length, with a dissipation rate of  $10^{-2} \text{ m}^2 \text{ s}^{-3}$ .

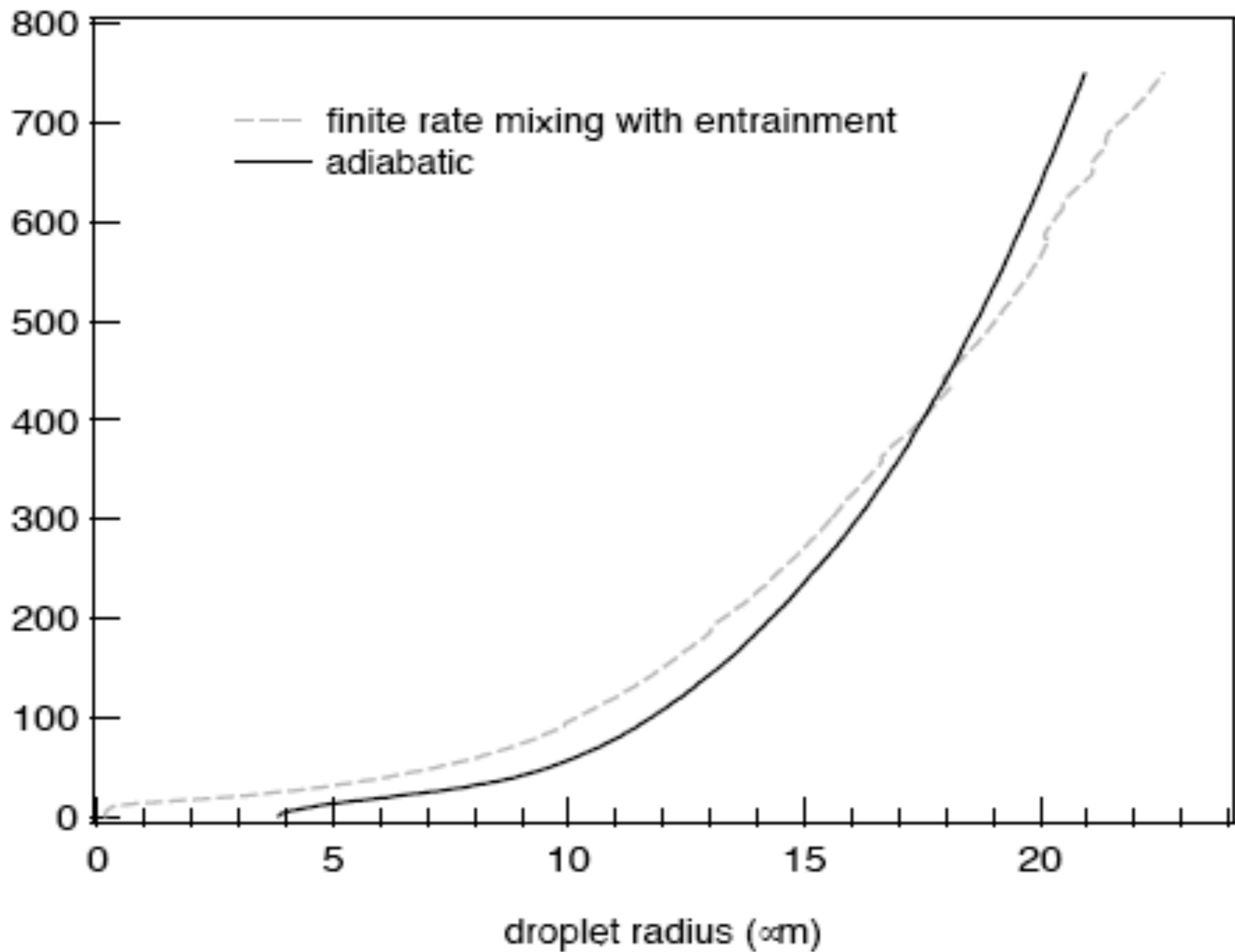


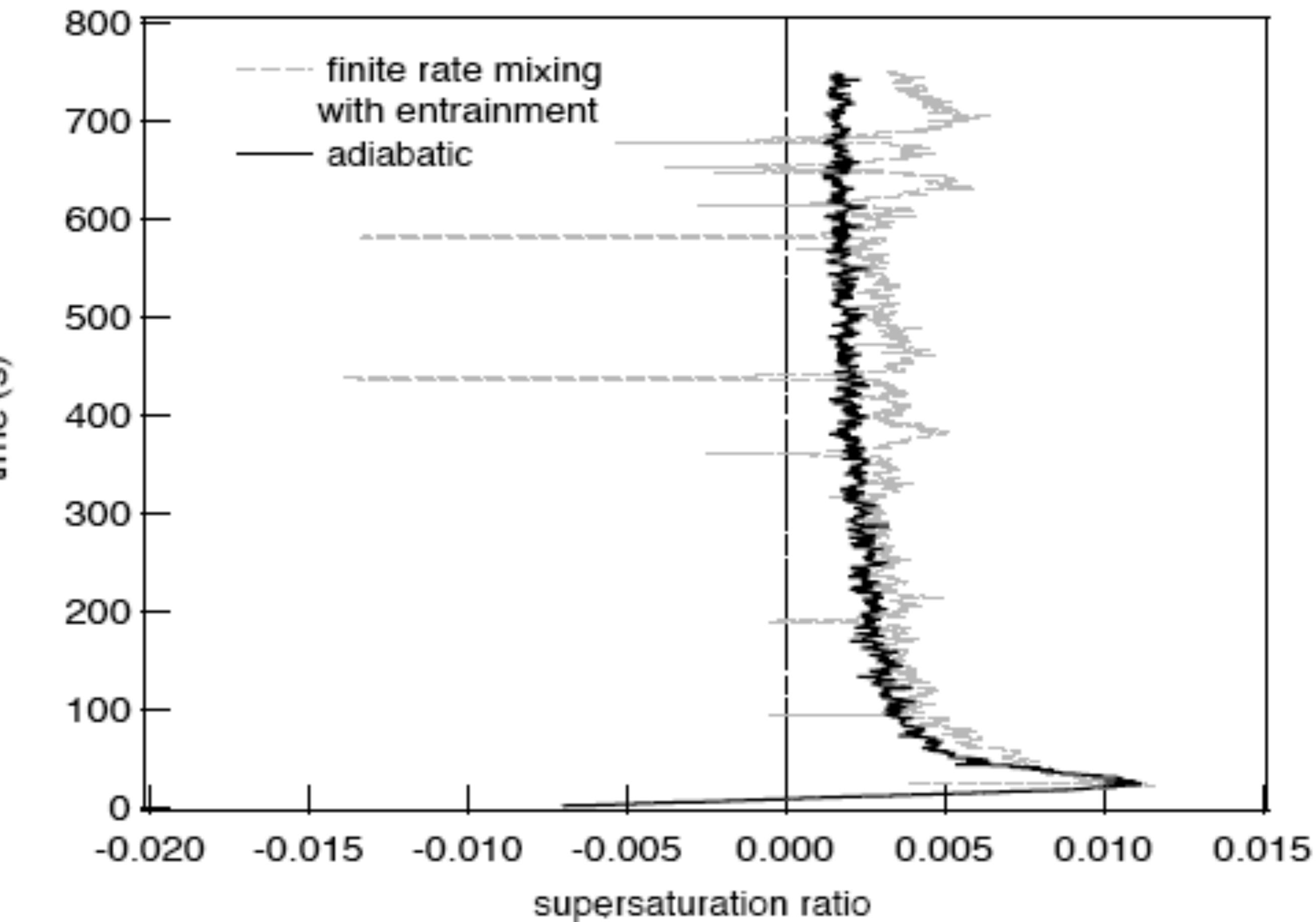
**BOTTOM:** 5 segments of subsaturated air, each 10 m in length, in a 1D domain 100 m in length.

# **Large Droplet Production due to Entrainment and Mixing**



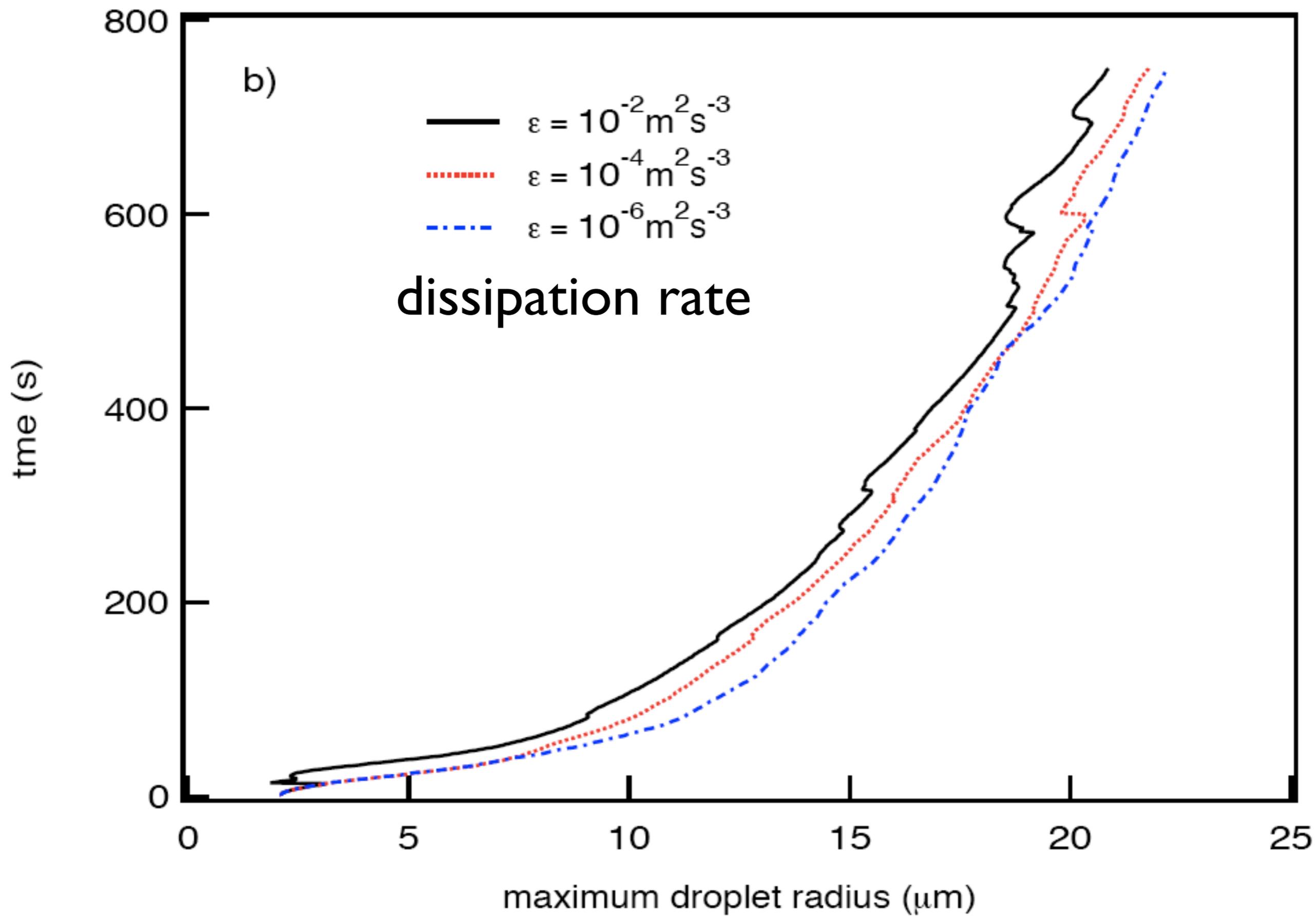


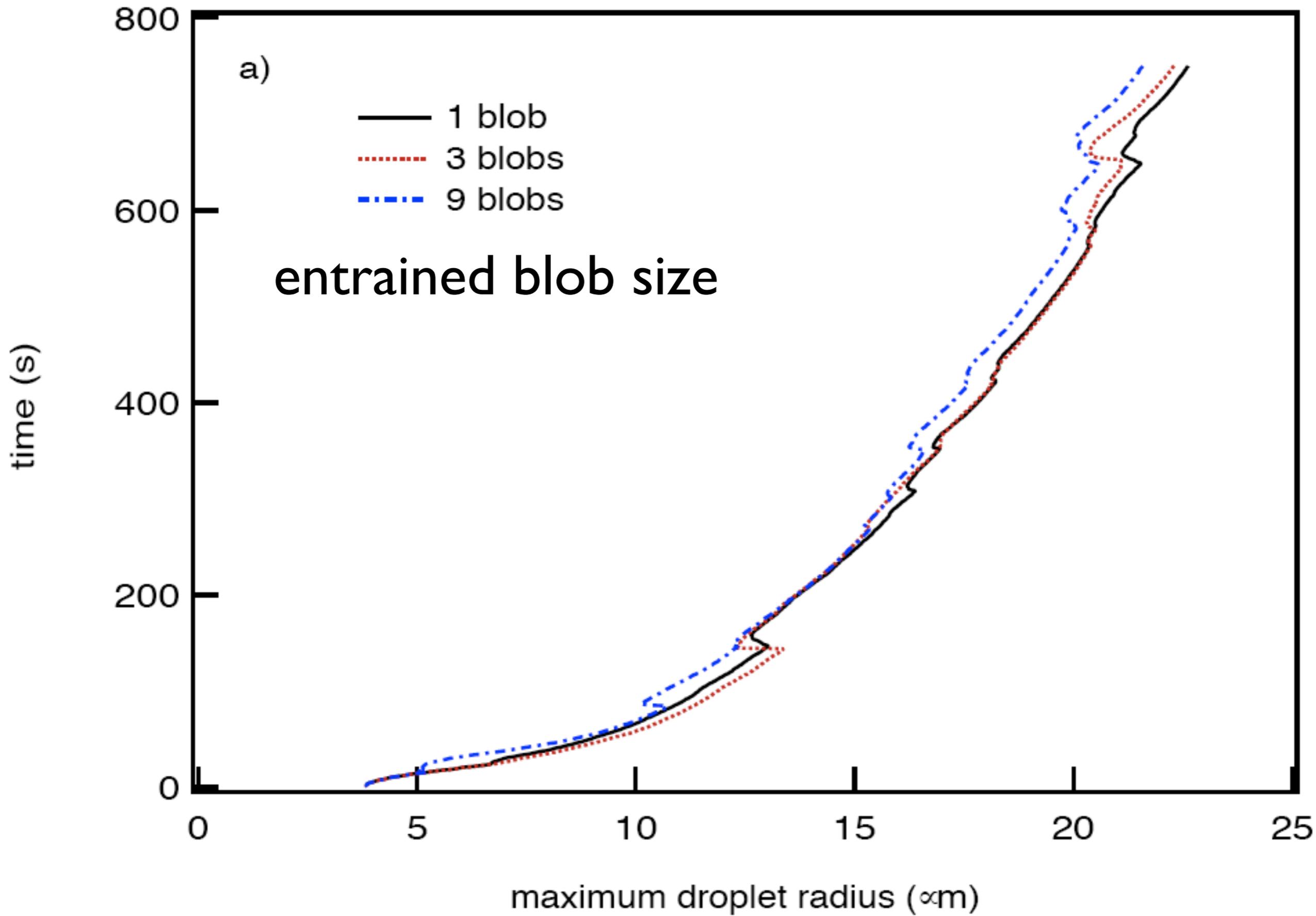


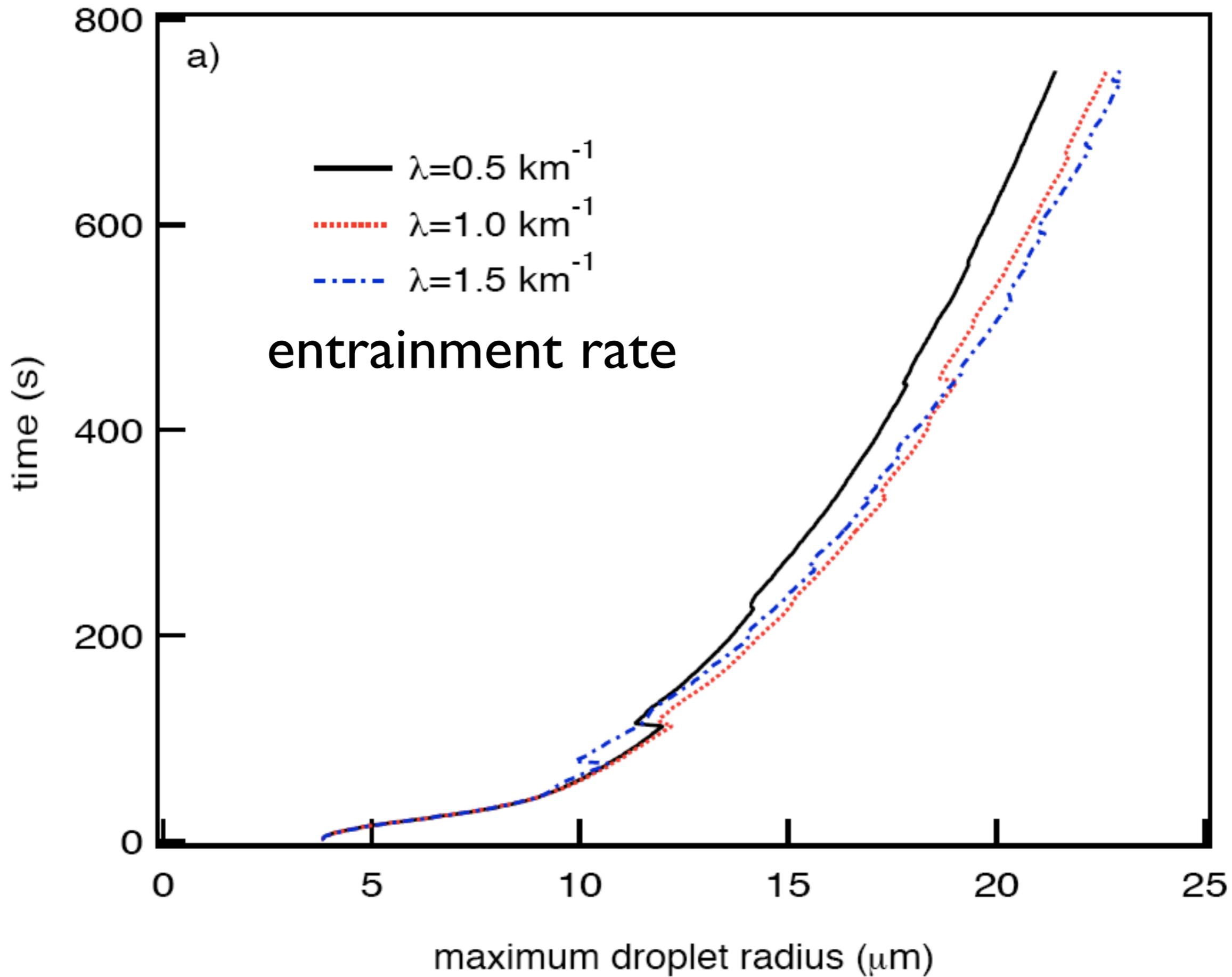


## **Some factors that affect large droplet production**

- Turbulence intensity (dissipation rate)
- Entrained blob size
- Entrainment rate
- Relative humidity of entrained air

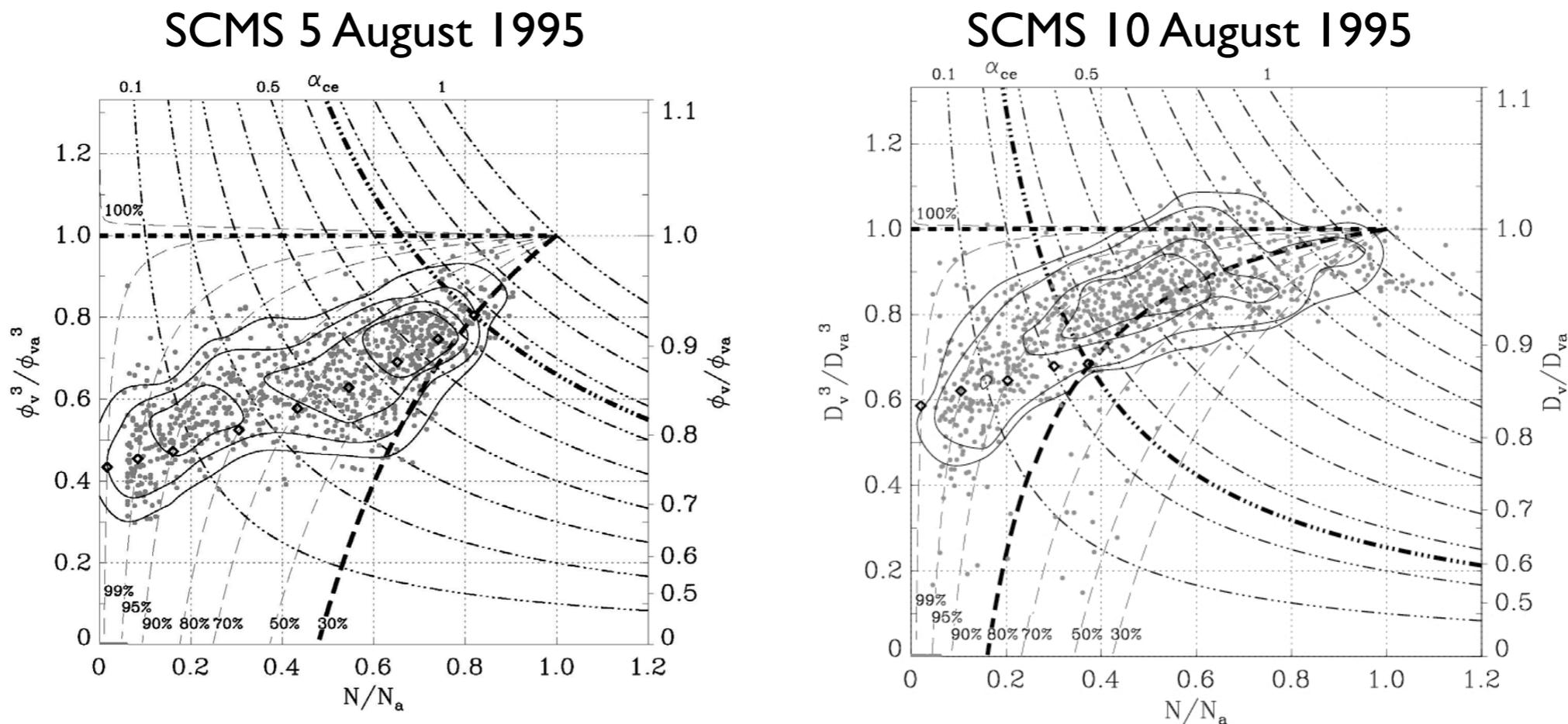






**How entrainment and mixing  
scenarios affect droplet spectra in  
cumulus clouds**

What can aircraft observations of joint frequency distributions of cloud droplet number concentration ( $N$ ) and mean droplet volume ( $V$ ) tell us about entrainment and mixing scenarios in cumulus clouds?



Burnet and Brenguier 2006

Can we use the Explicit Mixing Parcel Model (EMPM) to determine entrainment and mixing scenarios that could produce N-V distributions similar to those observed?

To explore the range of potential N-V distributions that might be encountered in cumulus clouds and to relate them to cloud processes, we applied the EMPM to a variety of realistic entrainment and mixing scenarios.

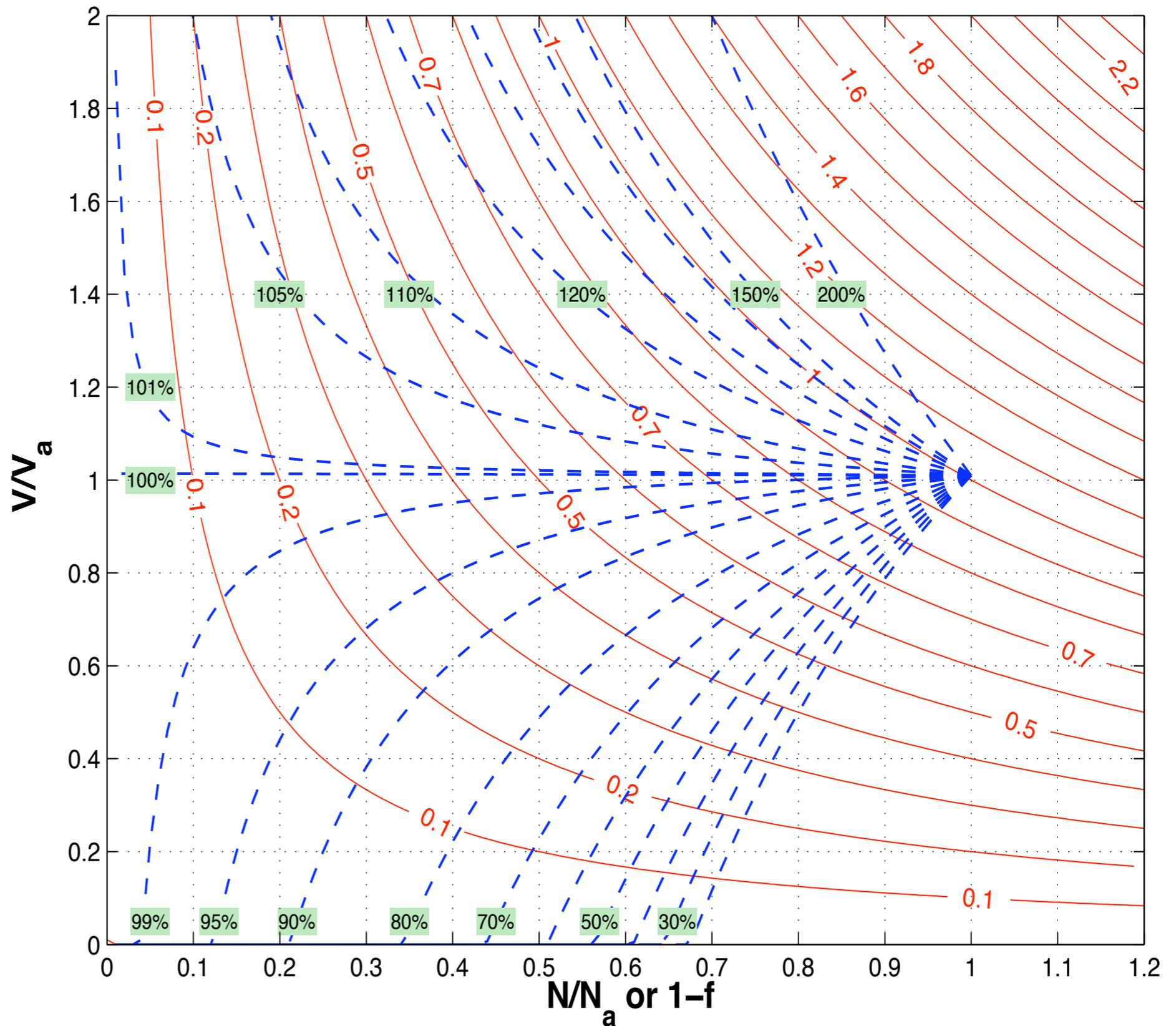
The consequences of ***parcel trajectory*** (isobaric vs ascending) and ***entrained CCN concentration*** (zero vs cloud base) on N-V distributions in entraining, nonprecipitating cumulus clouds as predicted by the EMPM will be presented.

# N-V diagram, part I

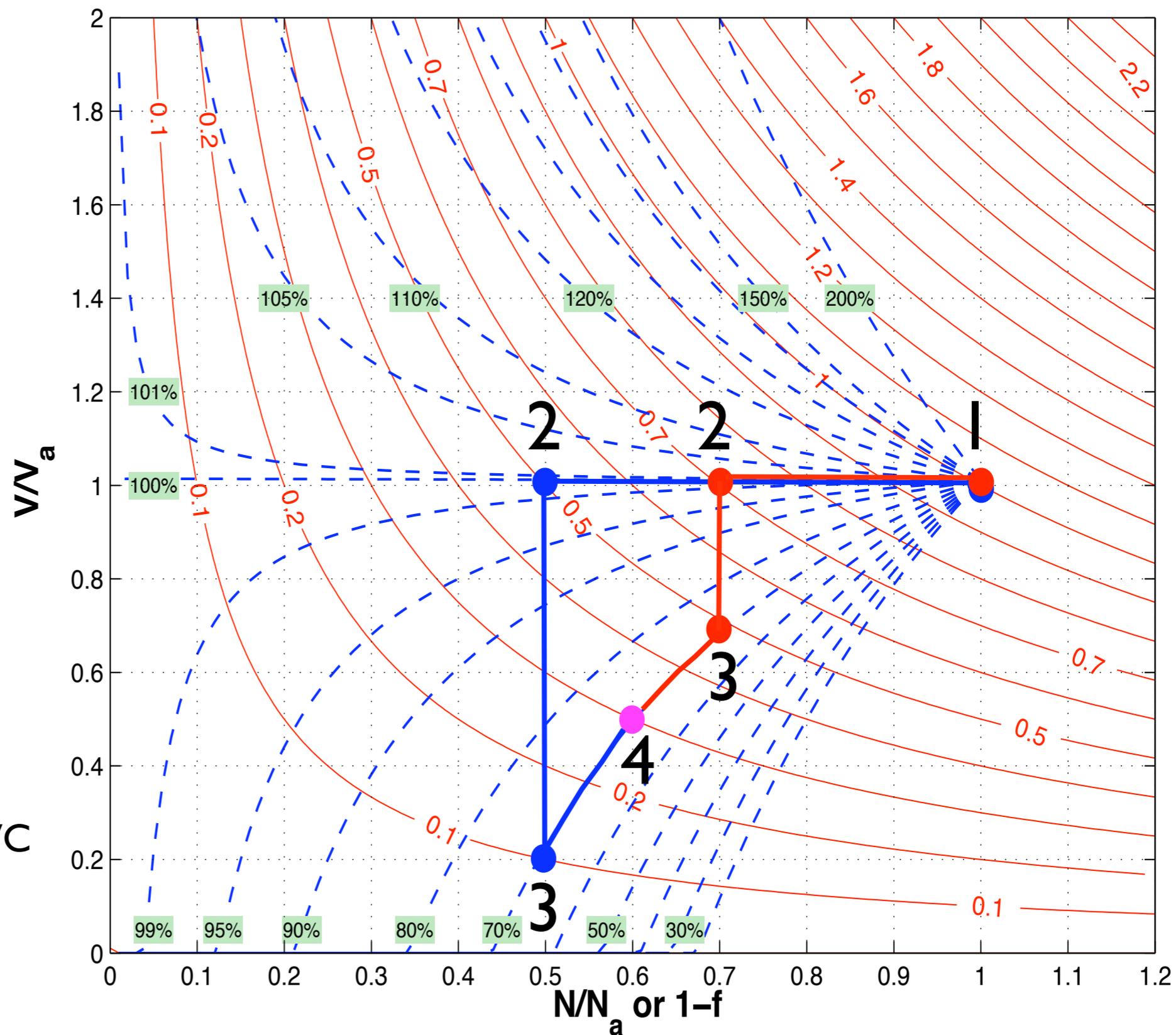
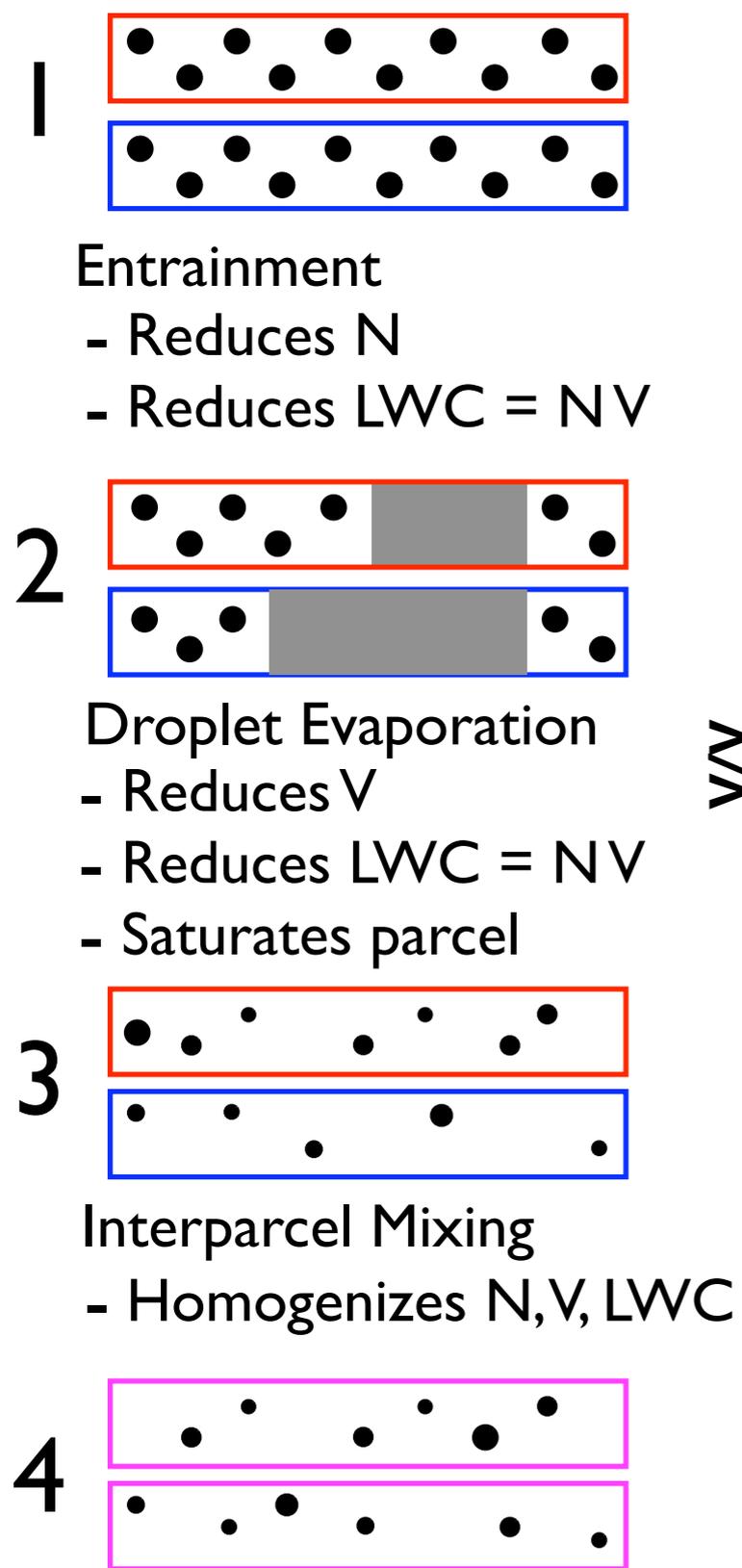
x:  $N/N_a = \text{CDNC}$   
normalized by  
adiabatic value

y:  $V/V_a = \text{mean}$   
droplet volume  
normalized by  
adiabatic value

red curves:  
 $\text{LWC}/\text{LWC}_a =$   
 $(N V) / (N_a V_a)$   
 $= \text{LWC}$   
normalized by  
adiabatic value



# Entrainment, Evaporation, and Mixing

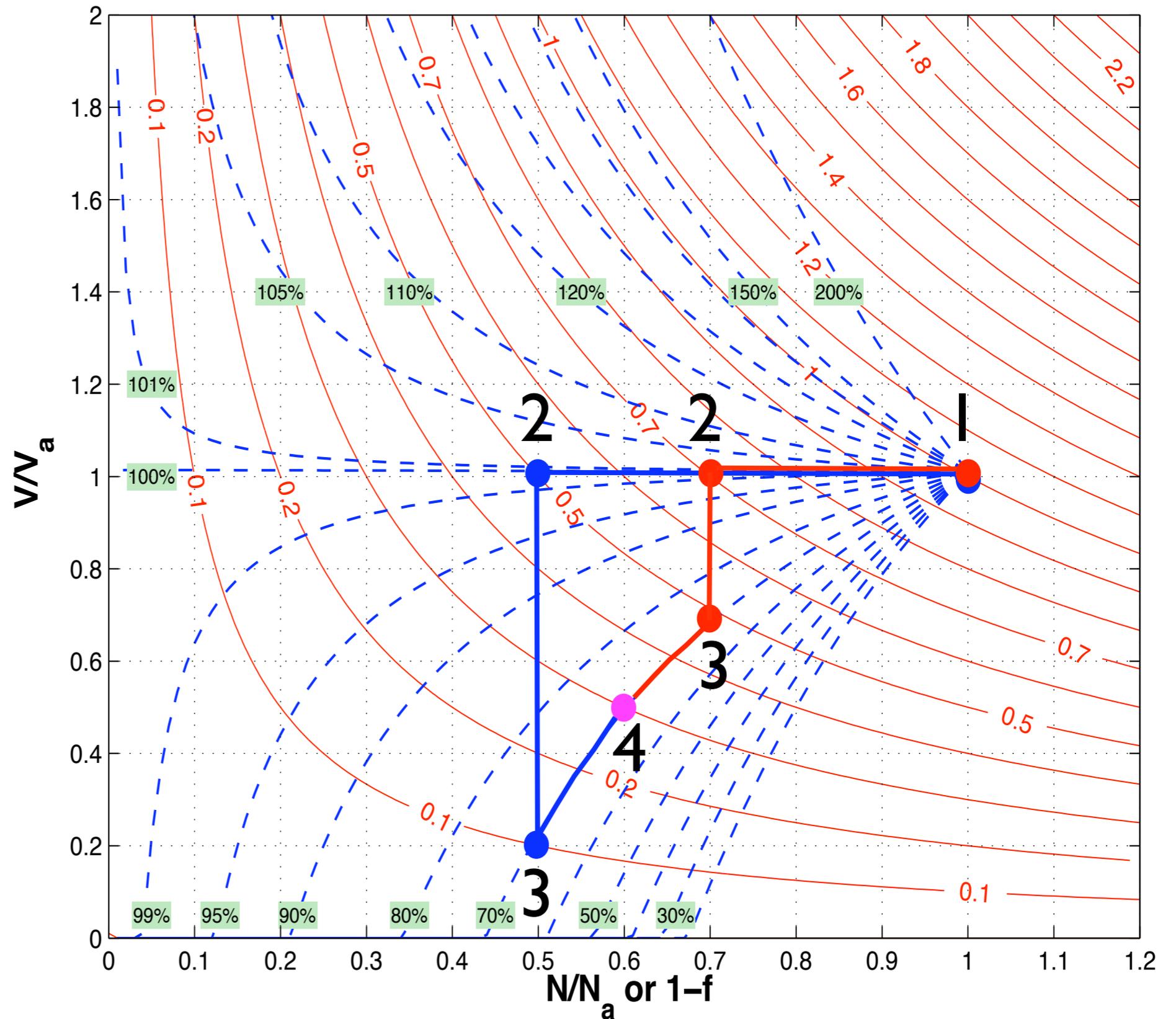


# N-V diagram, part 2

The decrease in LWC required to saturate the mixture is determined by the fraction,  $f$ , RH, and  $T$  of the entrained air.

If no droplets totally evaporate,  $N/N_a = 1-f$ .

The *blue curves* are then specified, and labeled with  $RH_e$ , (for a specific  $T_e$ ).



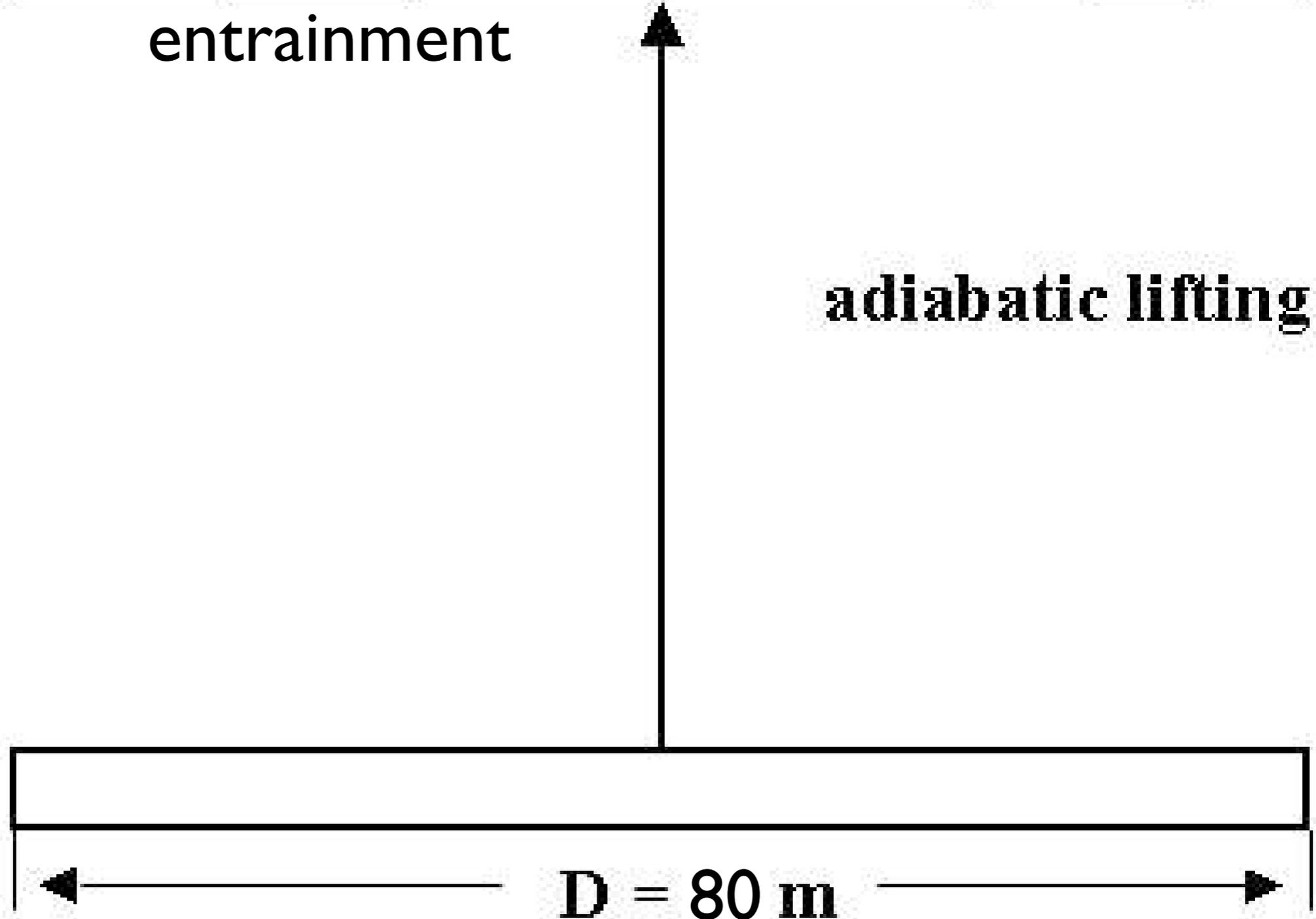
# Use *EMPM* to study mixing scenarios

**isobaric  
mixing**



**entrainment**

**adiabatic lifting**



# Isobaric mixing in the EMPM after one entrainment event

l-m averages from 80-m domain

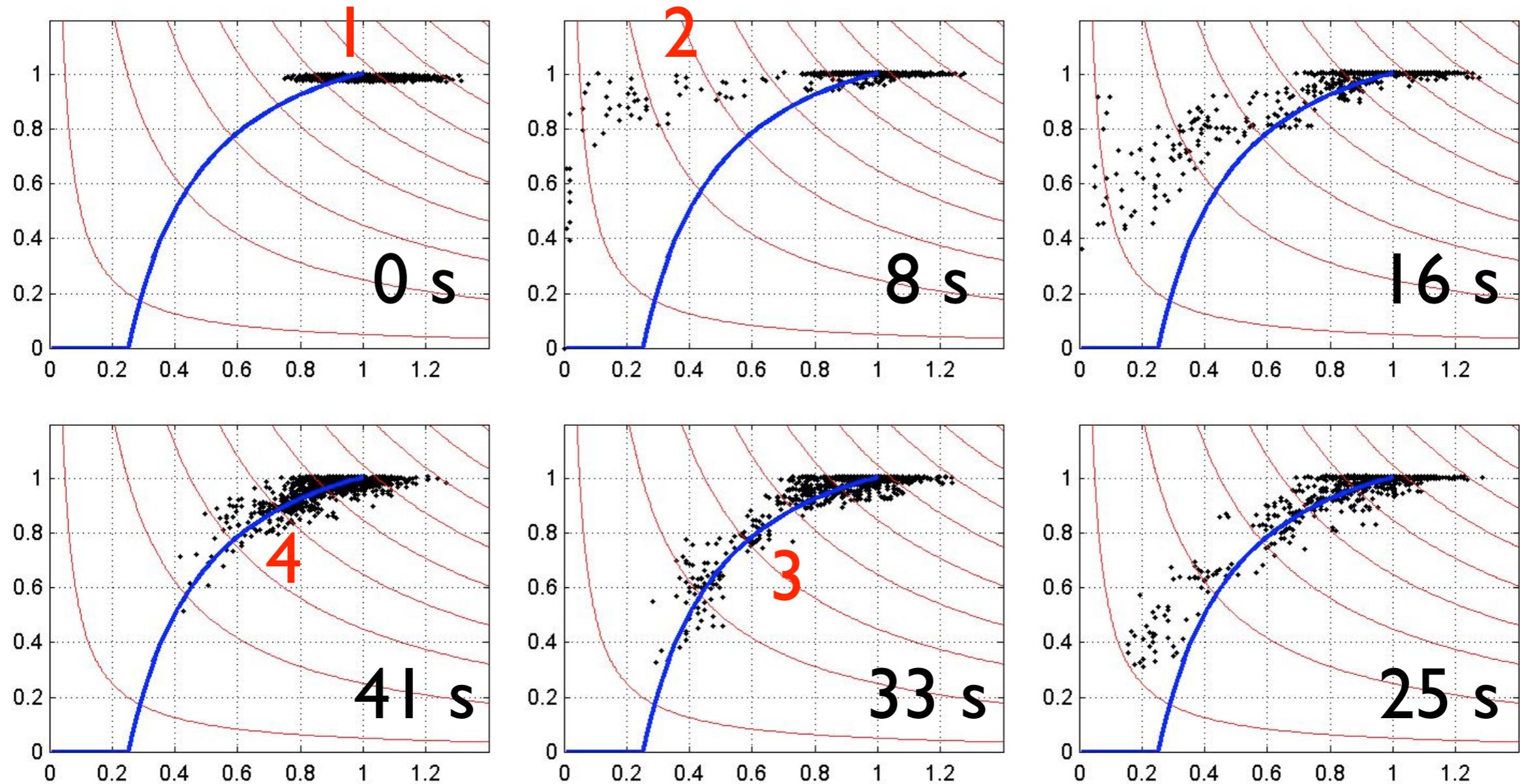


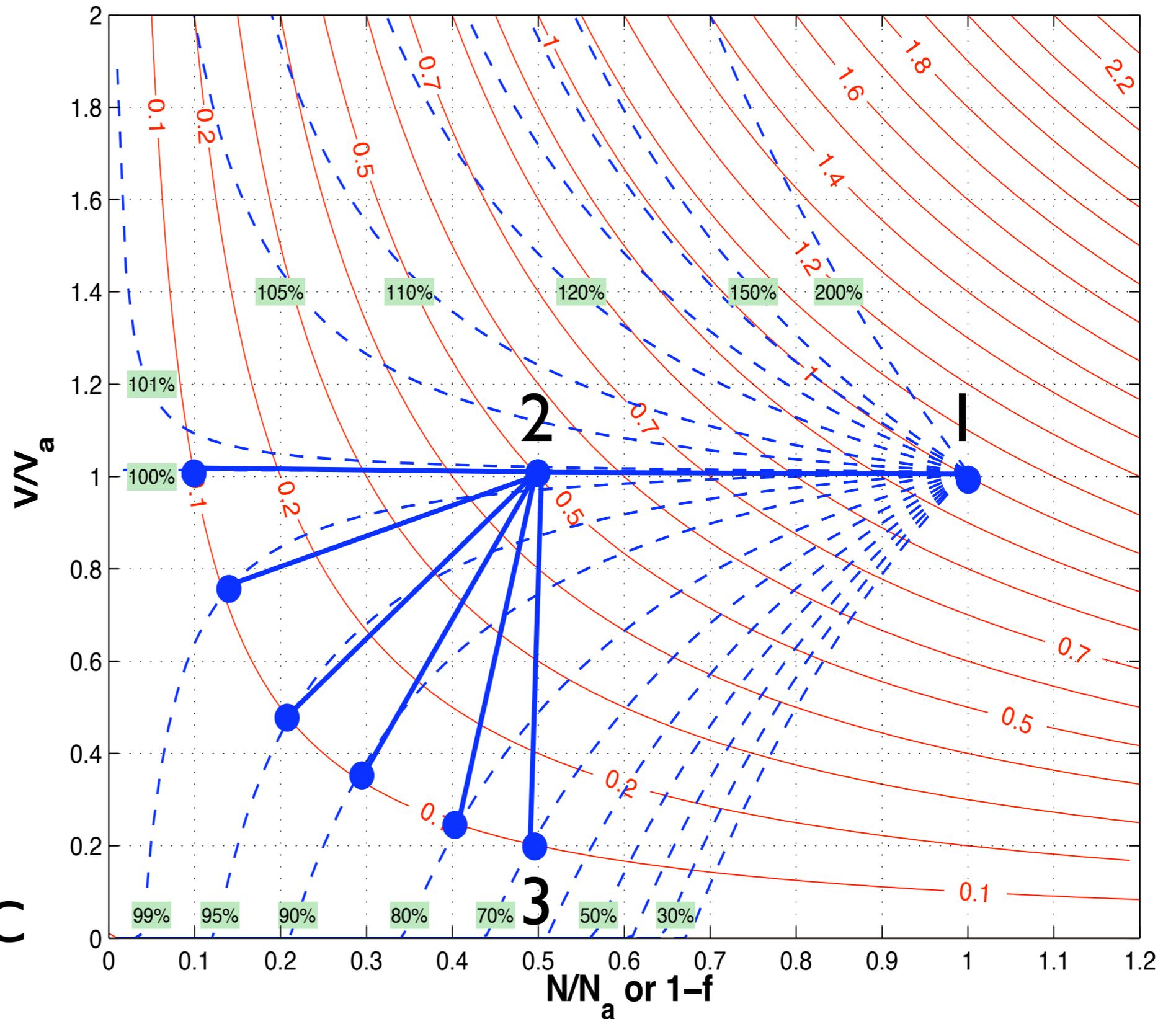
Figure 2:  $N-V$  diagram for isobaric mixing in the EMPM after an entrainment event. Each point is a 1-m average. Plotted in each panel are points from 11 “traverses” of the 80-m EMPM domain during an 8.25-s interval. Time increases clockwise from the upper left panel.

# Total evaporation of some droplets

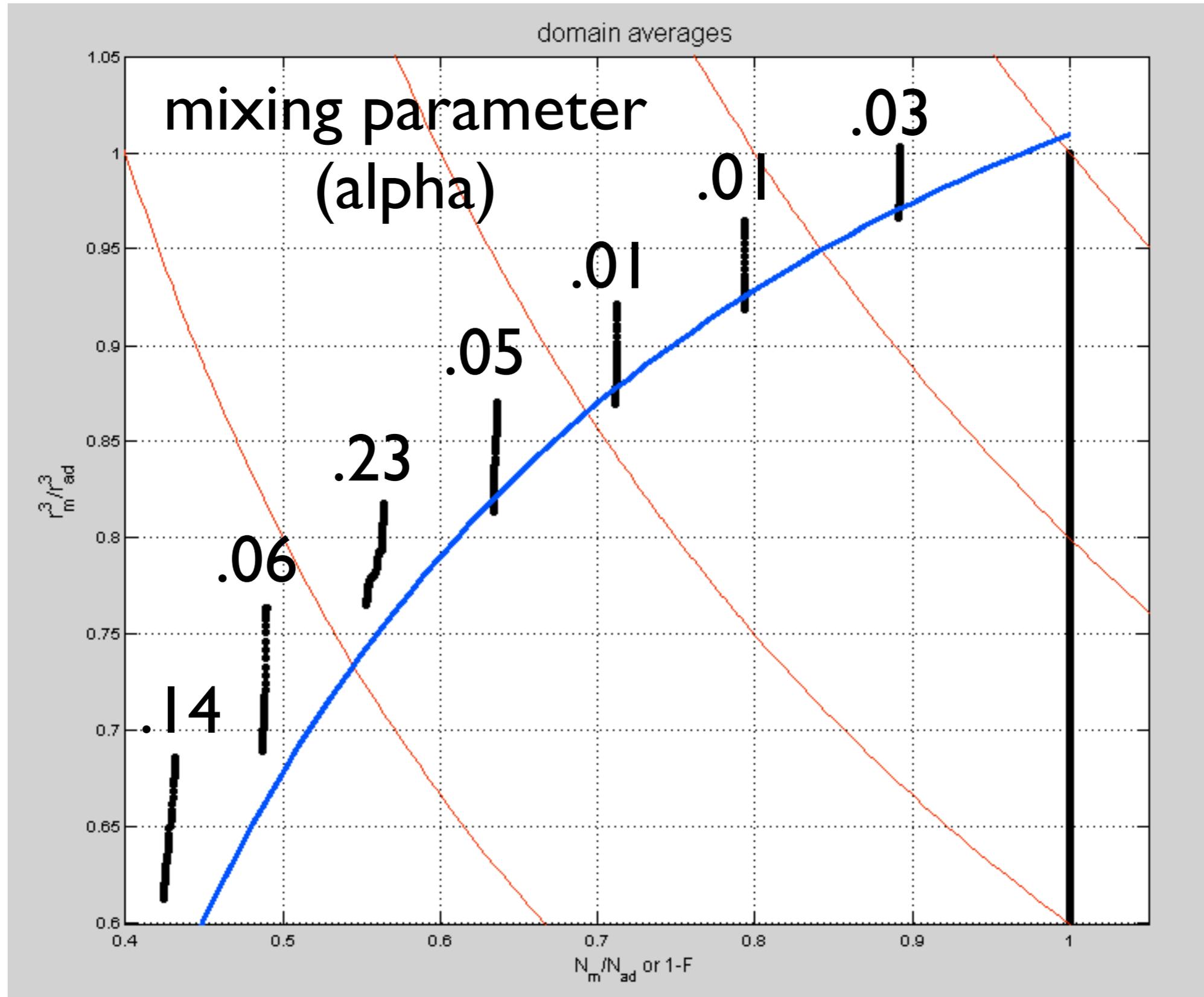
N decreases  
*after*  
entrainment.

If V does not  
change, we  
have “extreme  
inhomogenous  
mixing.”

Mixture is  
saturated at  
endpoint, on LWC  
contour.



# EMPM: entrainment and isobaric mixing, 80-m domain

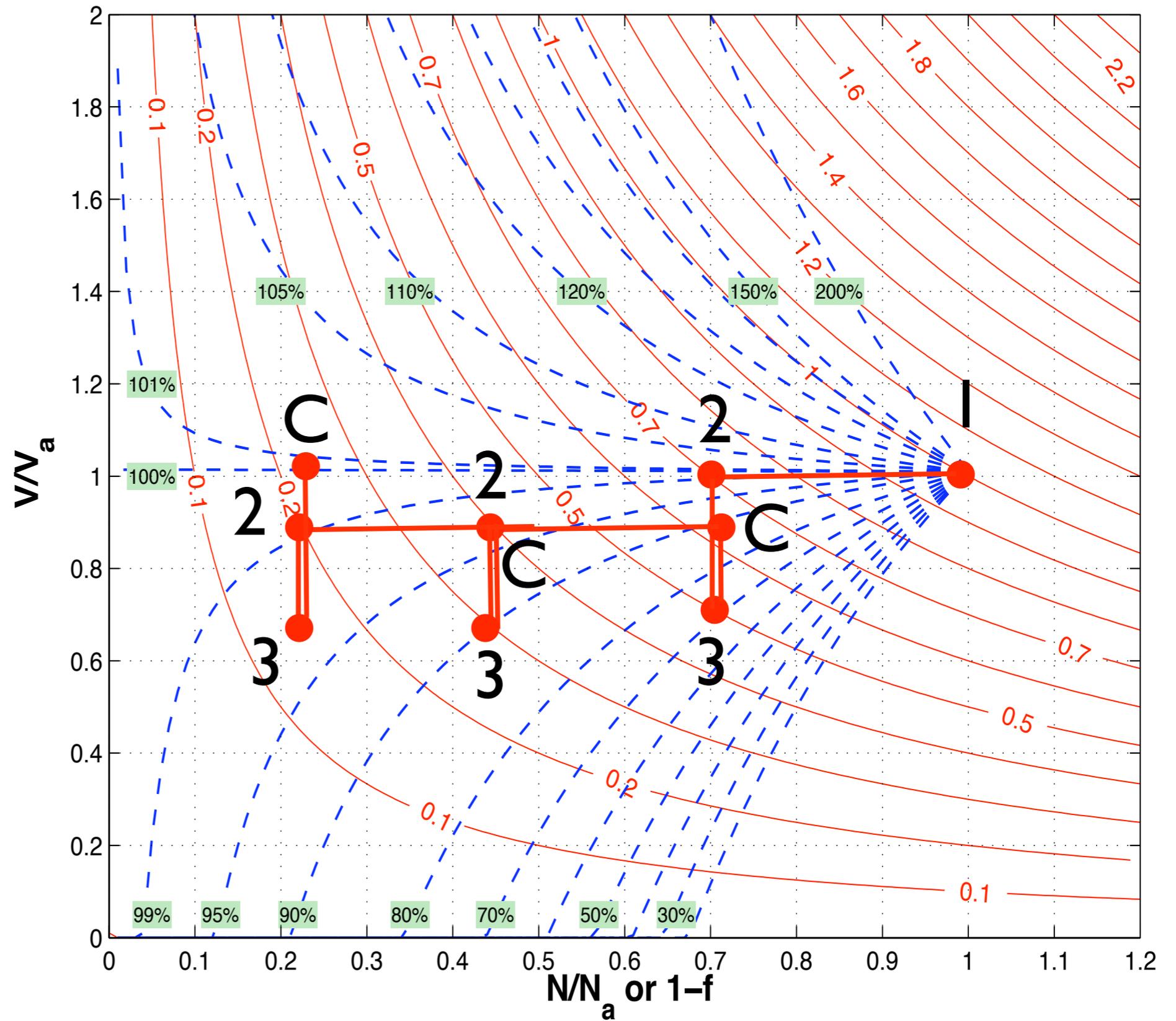


# Condensation due to Ascent

## Condensation:

- $N = \text{const}$
- LWC increases at rate  $\sim dz/dt$ .
- $V$  increases

Entrainment and ascent without activation tends to increase  $V$  towards super-adiabatic values (“weed and feed”).



# Ascent without entrained CCN

## Multiple entrainment events

### 10-m averages from 200-m domain

all levels

one level

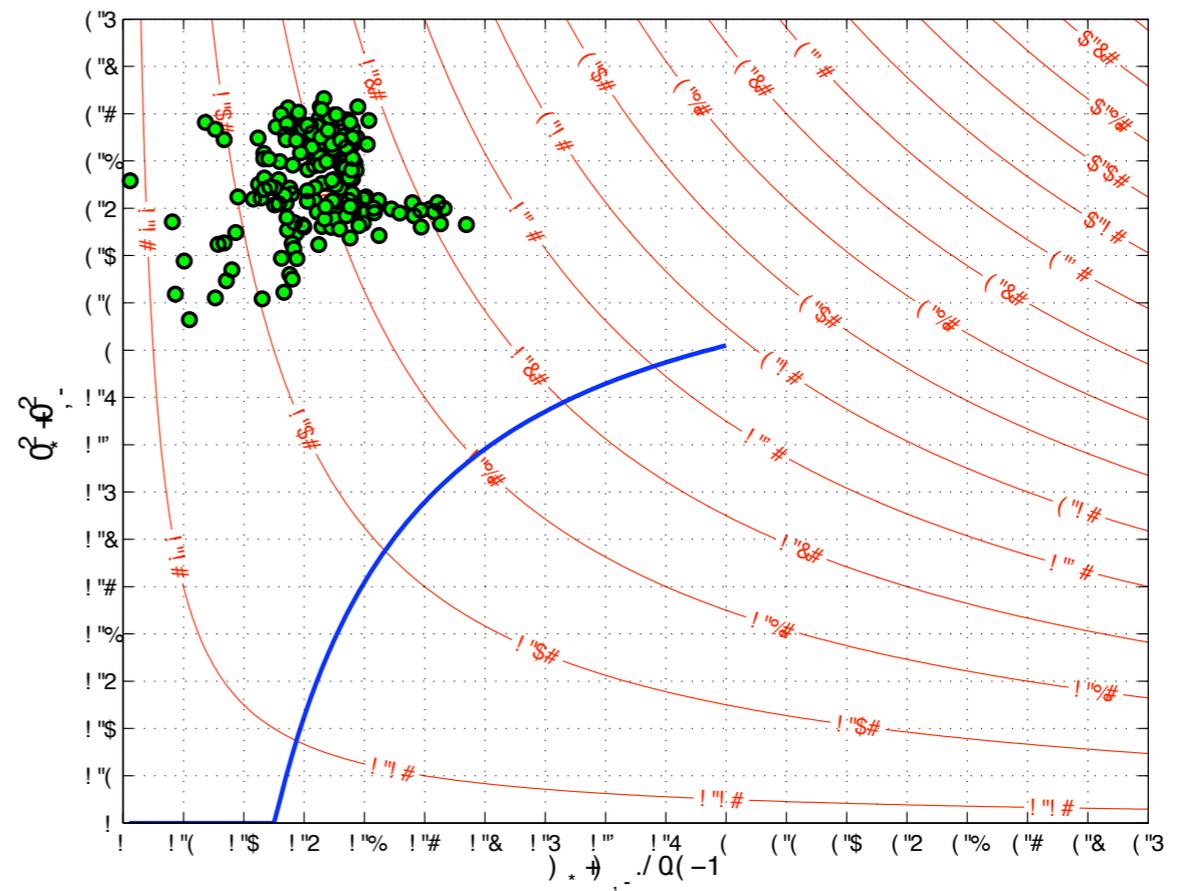
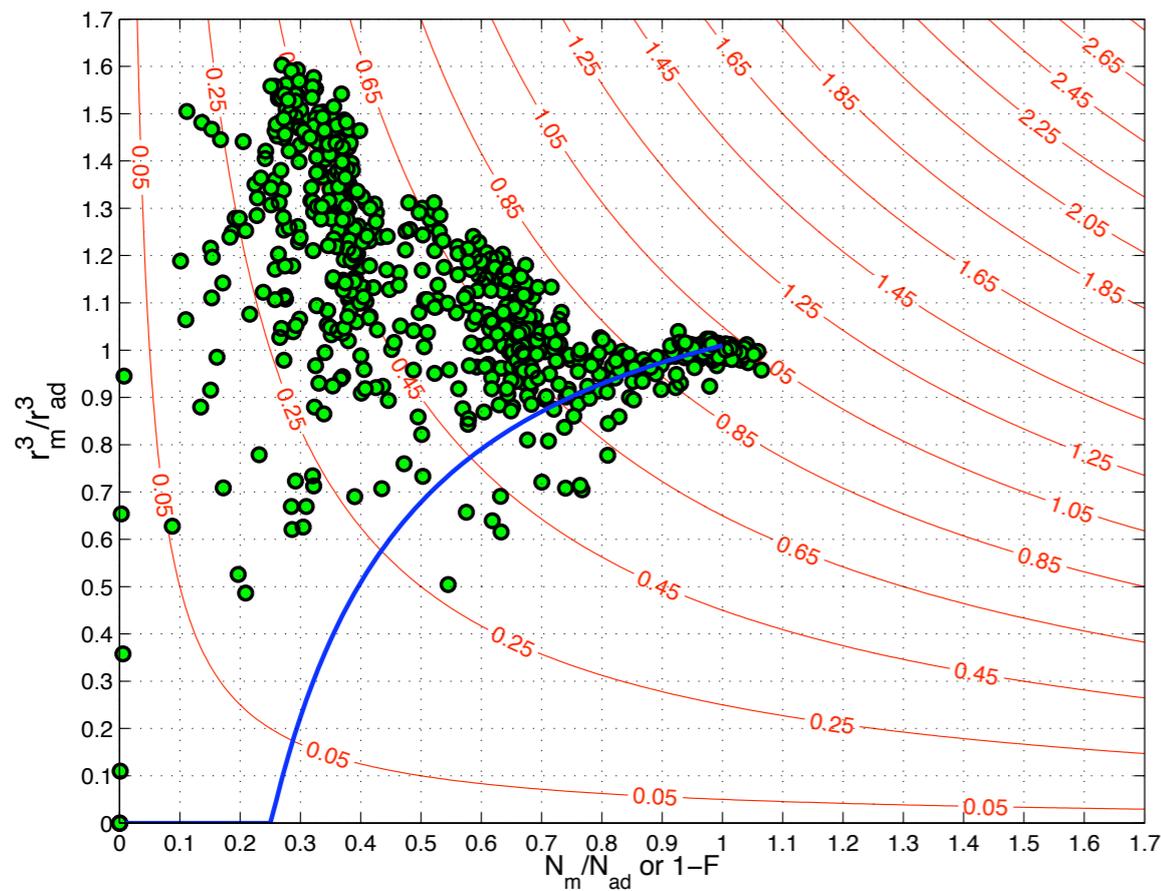


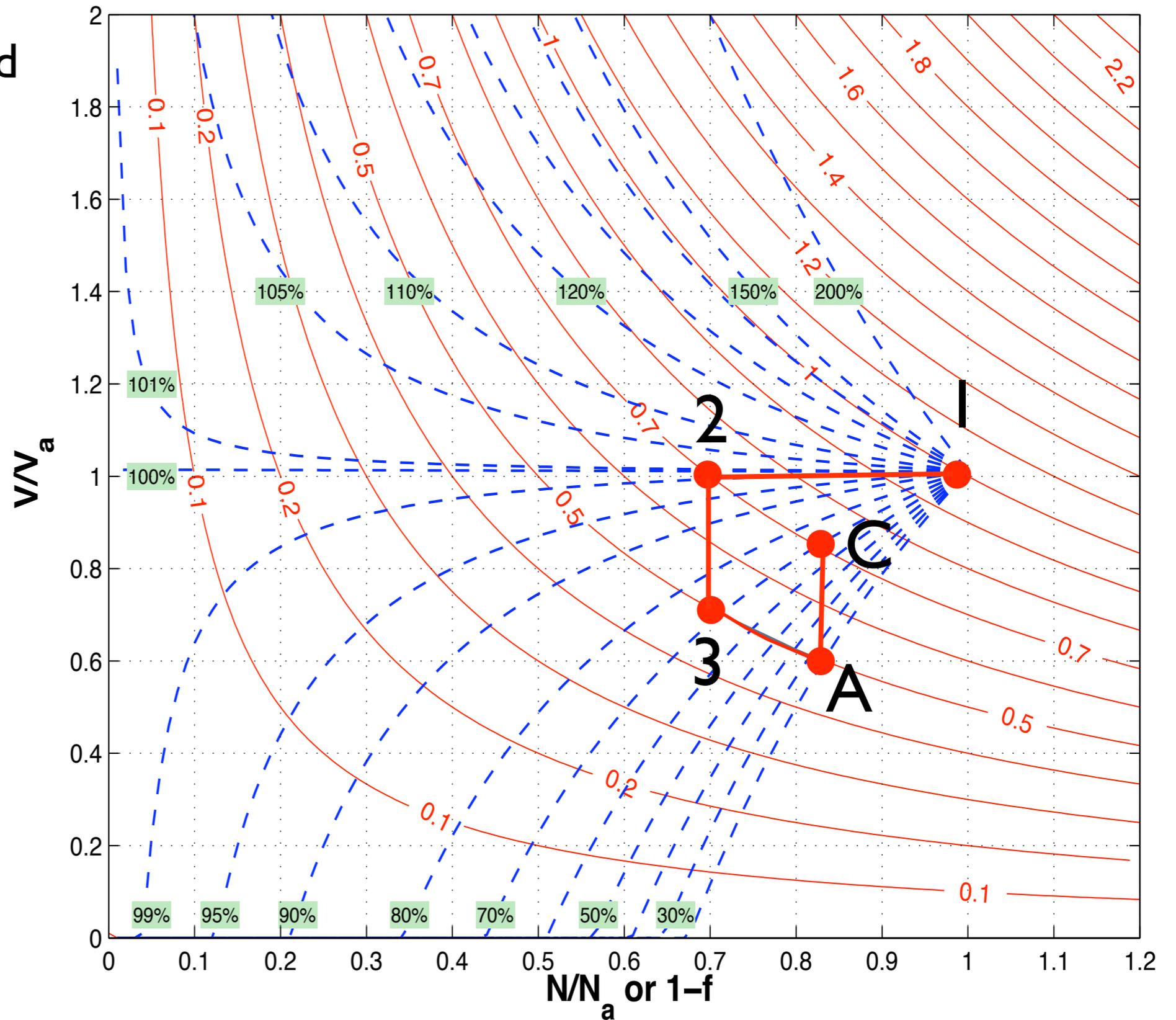
Figure 5: 10-m averages for an EMPM simulation in a 200-m ascending domain without entrained CCN. Left: All values. Right: Values for a short time interval, similar to what would be sampled by an aircraft traverse.

# Activation and Condensation due to Ascent

Activation of entrained CCN may occur.

- $N$  increases.
- $V$  decreases.
- $LWC = \text{const}$

Activation and condensation together tend to counteract entrainment effects and keep  $N$  and  $V$  more nearly constant.



# Ascent with entrained CCN (cloud-base conc.)

## Multiple entrainment events

10-m averages from 200-m domain

all levels

one level

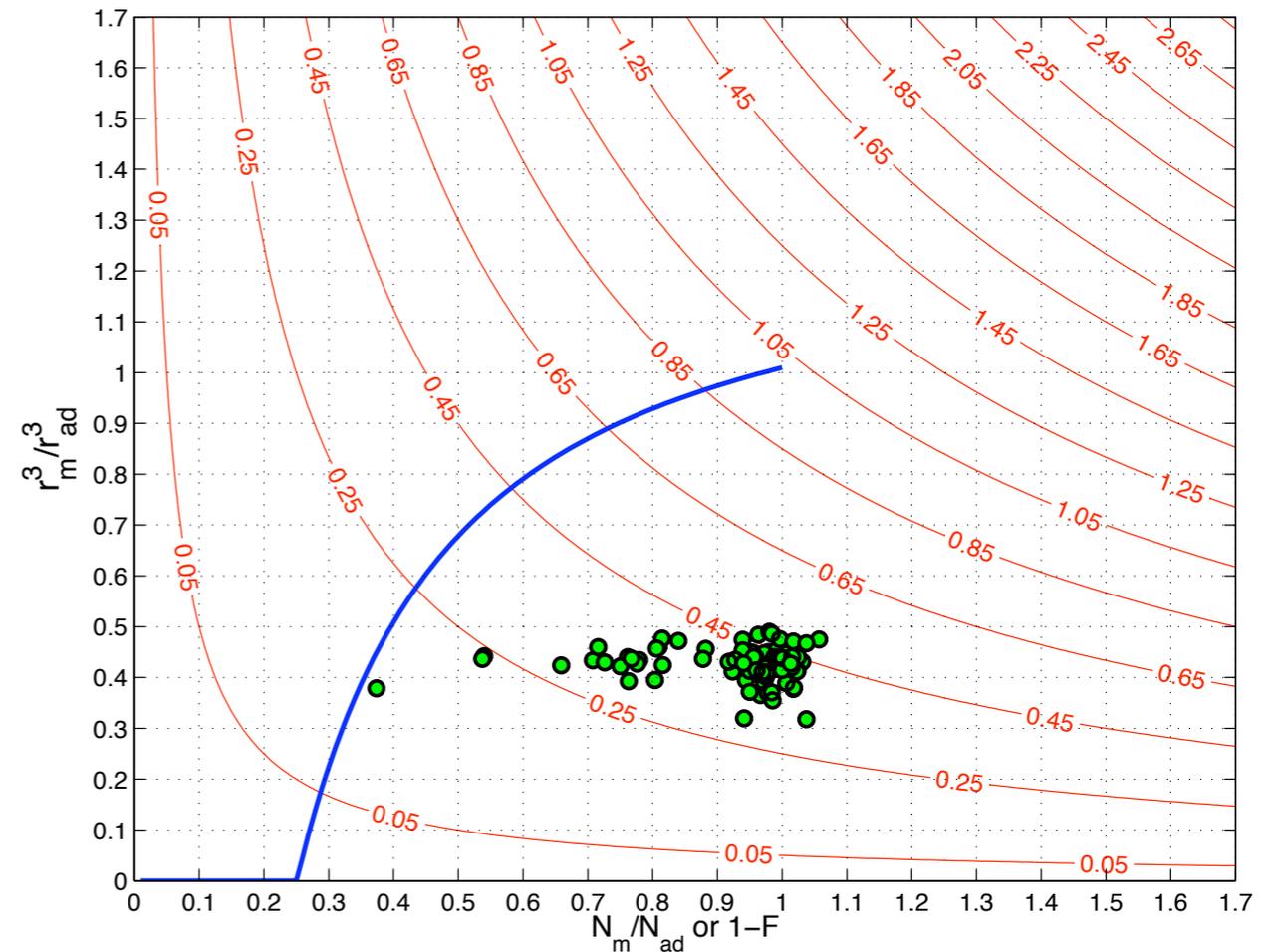
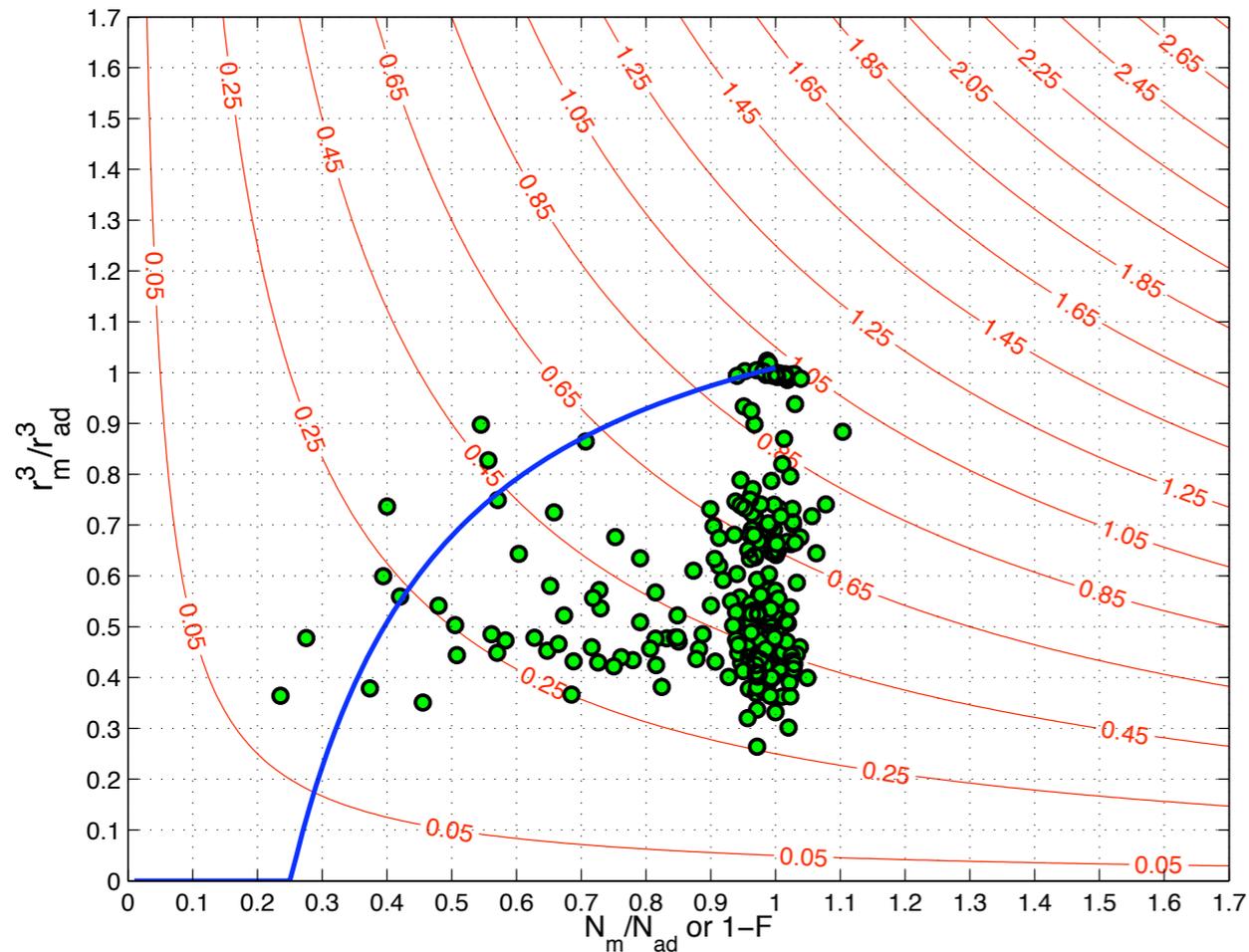
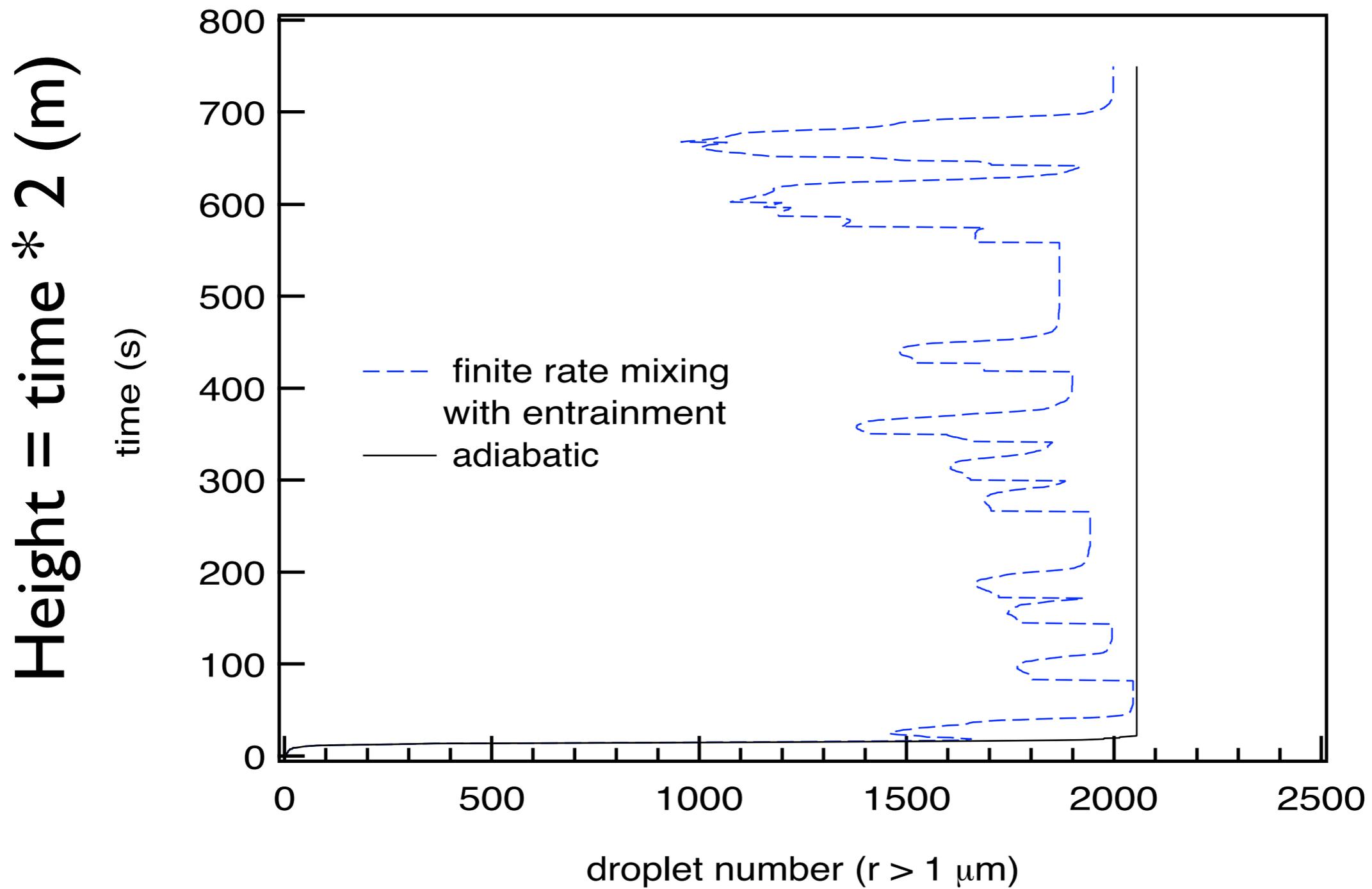


Figure 7: Like Fig. 5 but for entrained CCN at cloud base concentrations.

# N(z) with entrained CCN



Su 1997

# Ascent with entrained CCN (half cloud-base conc.)

## Multiple entrainment events

10-m averages from 200-m domain

all levels

one level

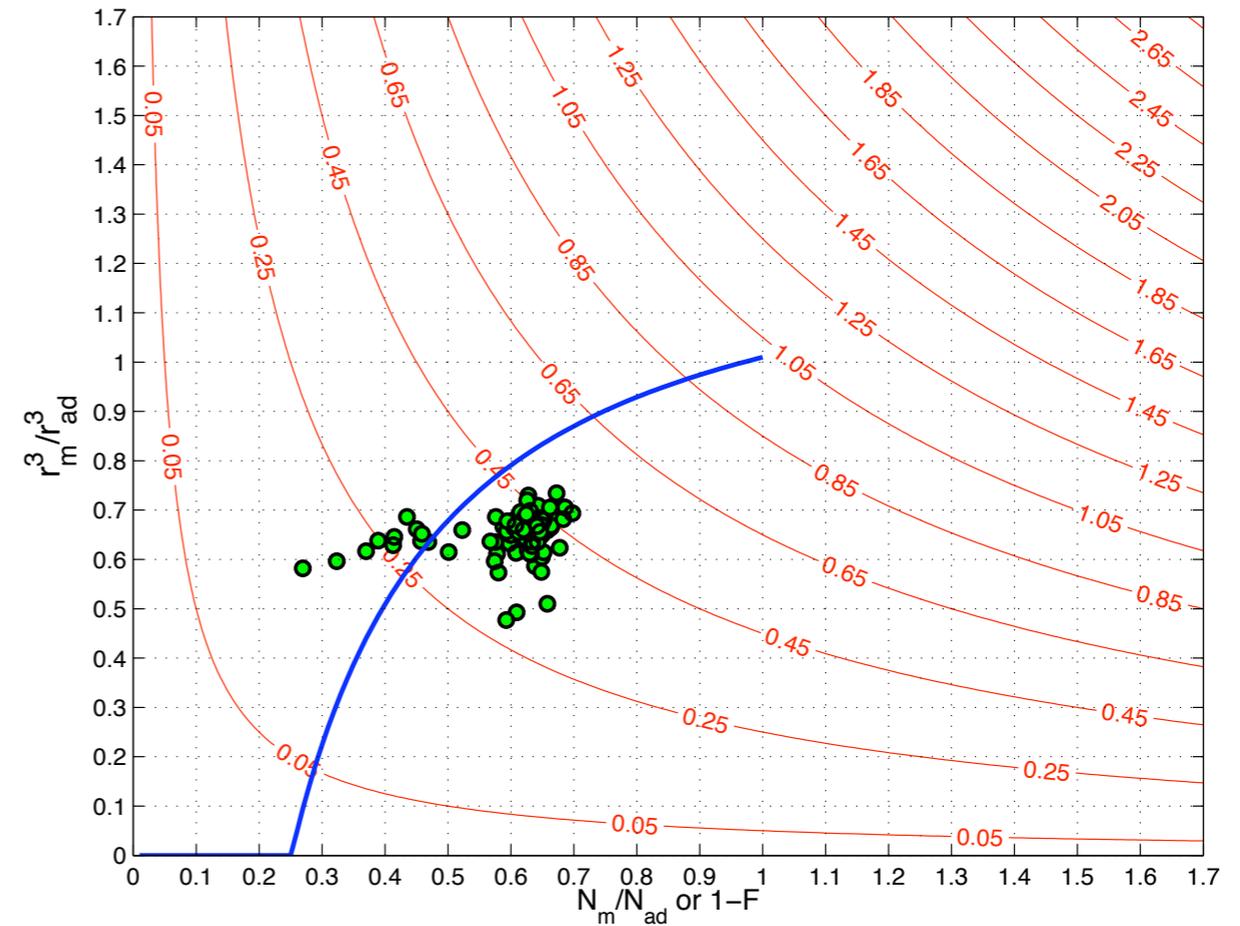
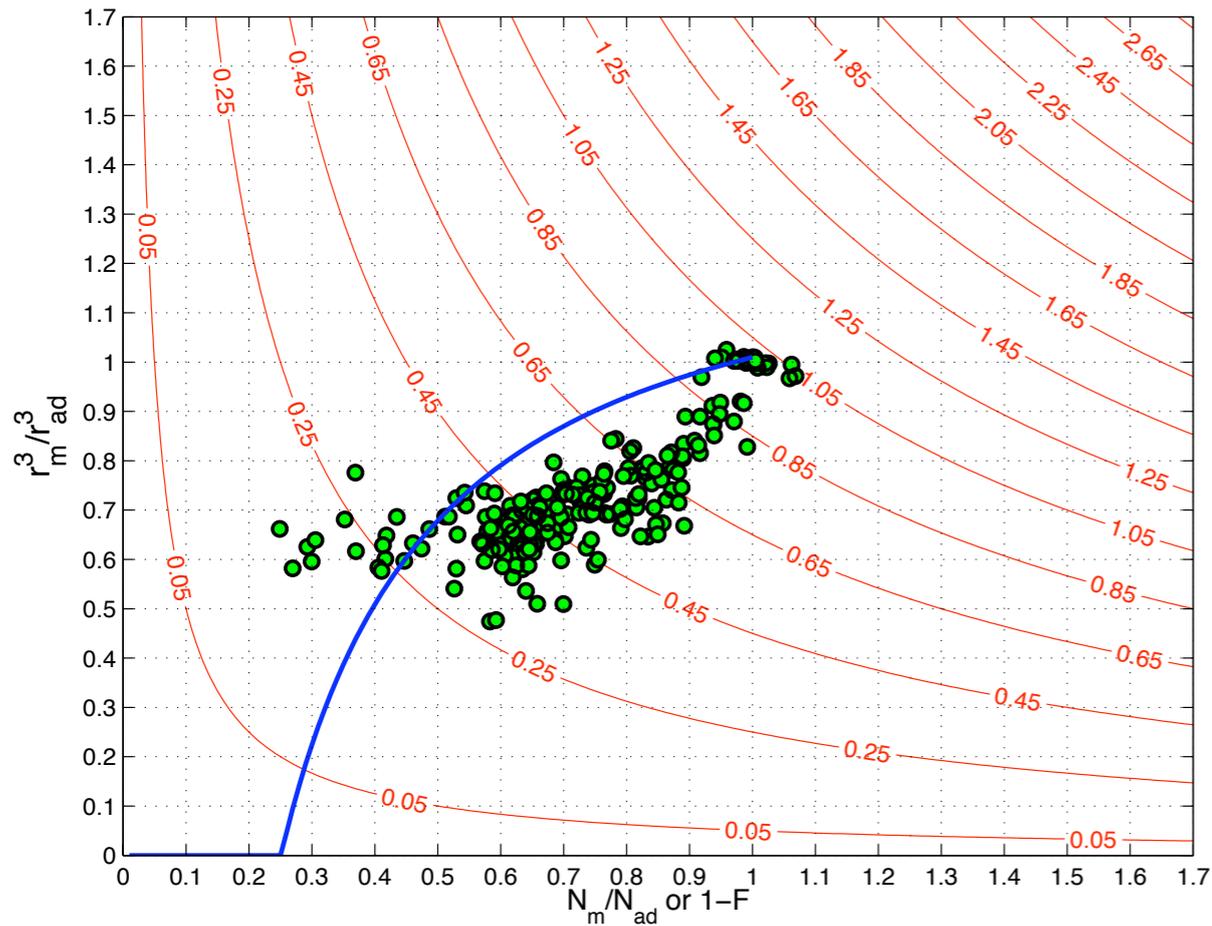
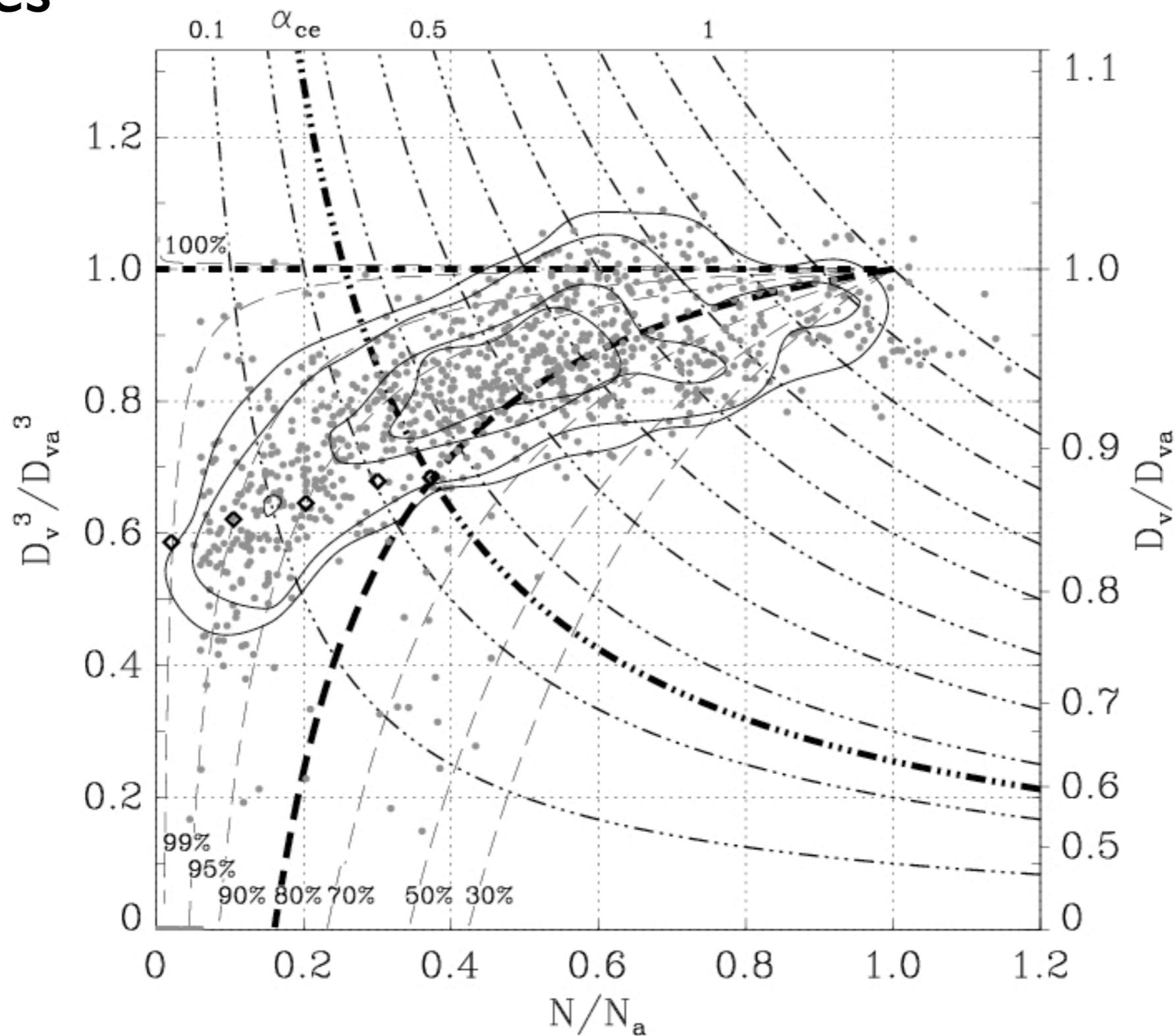


Figure 6: Like Fig. 5 but for entrained CCN at one half of cloud base concentrations.

10-m  
averages

SCMS 10 August 1995



Burnet and Brenguier 2006

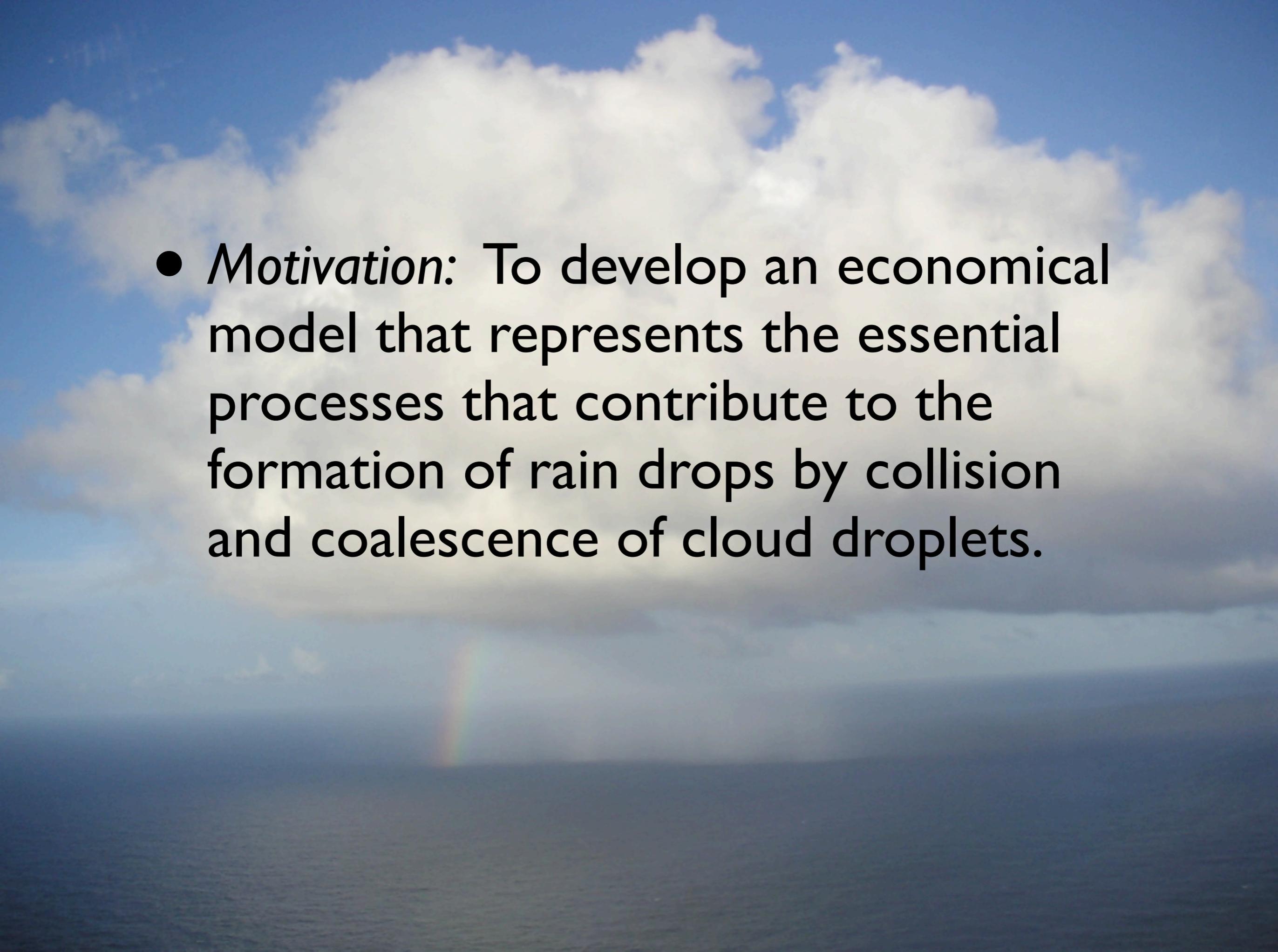
# Conclusions

- Without entrained CCN,  $V$  is too large and  $N$  is too small in the EMPM compared to the SCMS aircraft observations.
- This suggest that distributions of  $N$  and  $V$  similar to those observed can be produced in an ascending parcel by entraining air with intermediate CCN concentrations.
- Better discrimination between possible scenarios would be possible if subsaturated and saturated parcels could be identified by aircraft measurements.

- Bridging the LES-DNS gap
- Large-eddy simulation (LES)
- Parcel model
- Linear Eddy Model (LEM)
- One-Dimensional Turbulence (ODT)
- Explicit Mixing Parcel Model (EMPM)
- **ClusColl (Clustering and Collision Model)**

- **ClusColl (Droplet Clustering and Collision Model)**

- Inertial droplets move in response to Kolmogorov-scale turbulence and gravity.
- Economically evolves 3D droplet positions and detects collisions.
- Can be incorporated into EMPM.

- 
- *Motivation:* To develop an economical model that represents the essential processes that contribute to the formation of rain drops by collision and coalescence of cloud droplets.

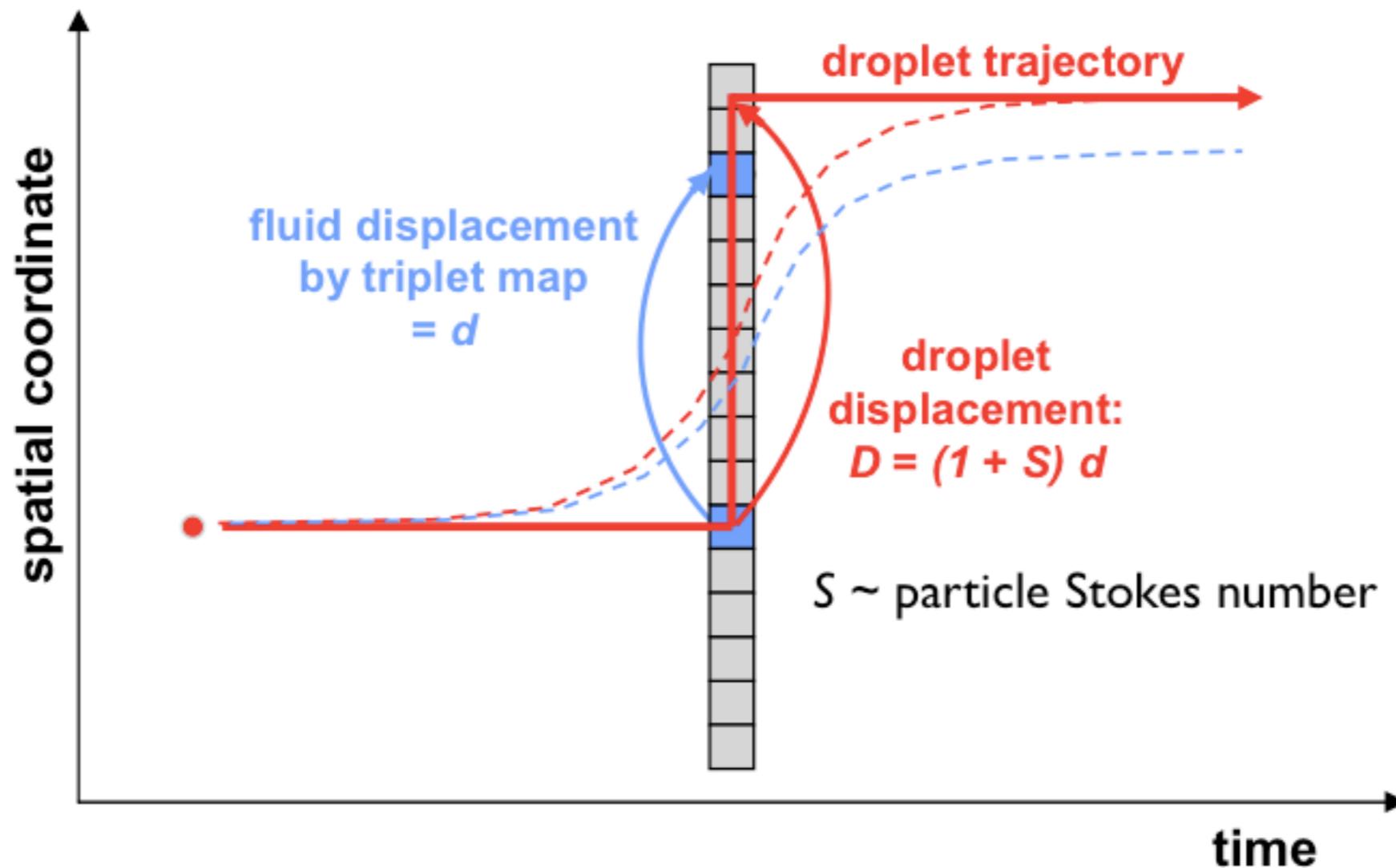
# An Economical Simulation Method for Droplet Motions in Turbulent Flows

Each droplet has a radius and a 3-D position.

- *Radius* changes due to collision and coalescence.
- *Position* changes due to turbulence and sedimentation.
- *Map-based advection* is an efficient tool for capturing the physics that governs droplet motions and collisions in turbulence.

# Turbulent Motion of *Droplets* can also be Represented by Applying I-D Maps

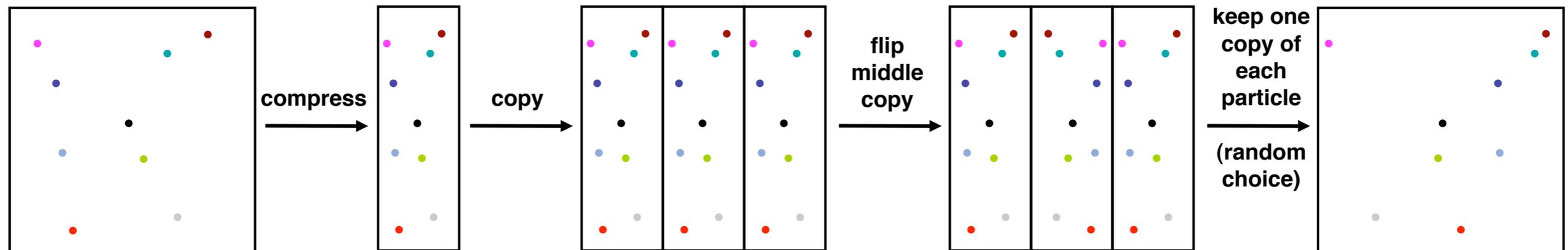
The droplet trajectory model idealizes droplet response to continuum flow (dashed curves: notional continuum fluid streamline and droplet trajectory)



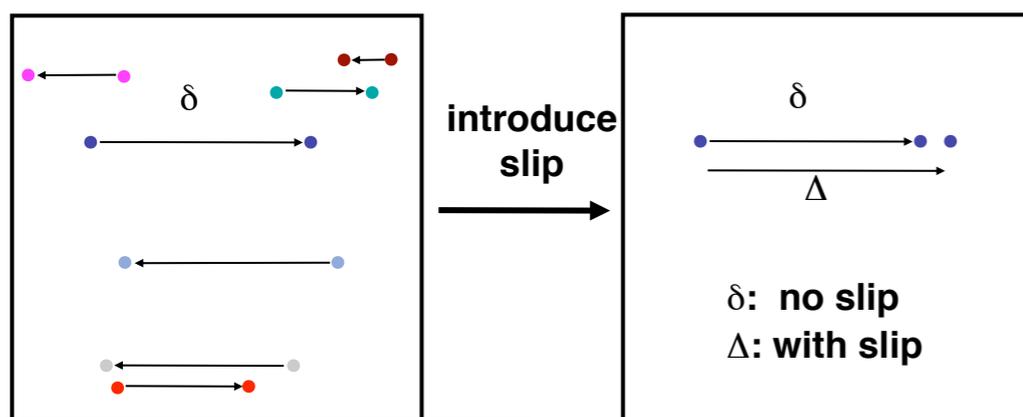
Kerstein, A. R., and S. K. Krueger, 2006: Clustering of randomly advected low-inertia particles: A solvable model. *Phys. Rev. E*, **73**, 025302.

# Using map-based advection, a 3D Lagrangian (grid-free) low-inertia particle advancement model is formulated

## Displacement of slip-free (zero-inertia) particles by a 3D triplet map:



## Fluid displacements $\delta$ are multiplicatively incremented to represent particle inertia:



### Inertia model:

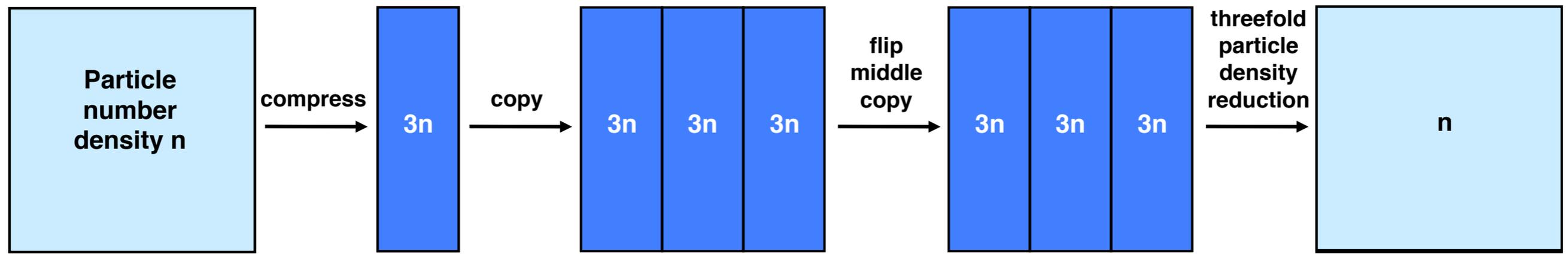
$$\Delta = (1+S) \delta$$

$S \ll 1$  is the model analog of Stokes number,  
 $St = [\text{particle response time}] / [\text{flow time}]$

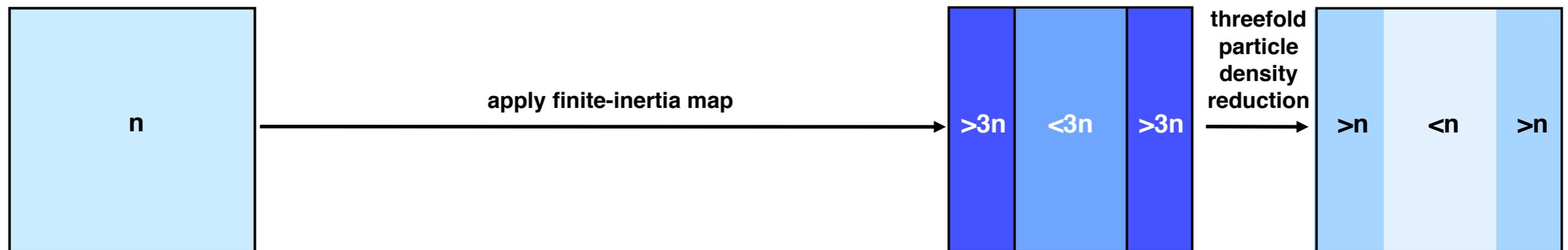
If polydisperse,  $S$  can be different for each particle

# Continuum interpretation: slip induces fluctuations in an initially uniform particle-density field

**Zero inertia: uniform multiplicative compression, compensated by number reduction**



**Non-zero inertia: non-uniform compression, inducing particle-density fluctuations**

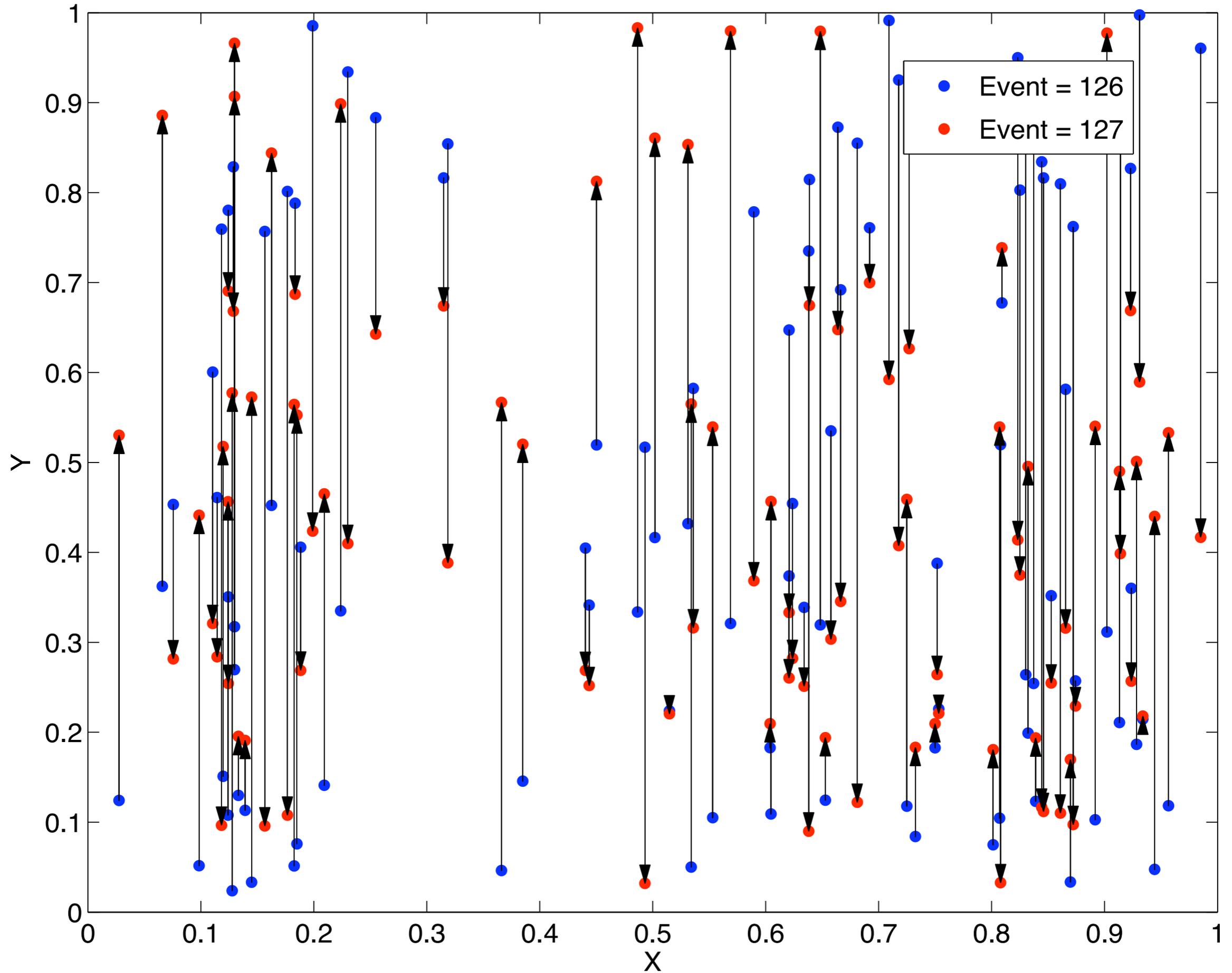


# Triplet Map for Droplets

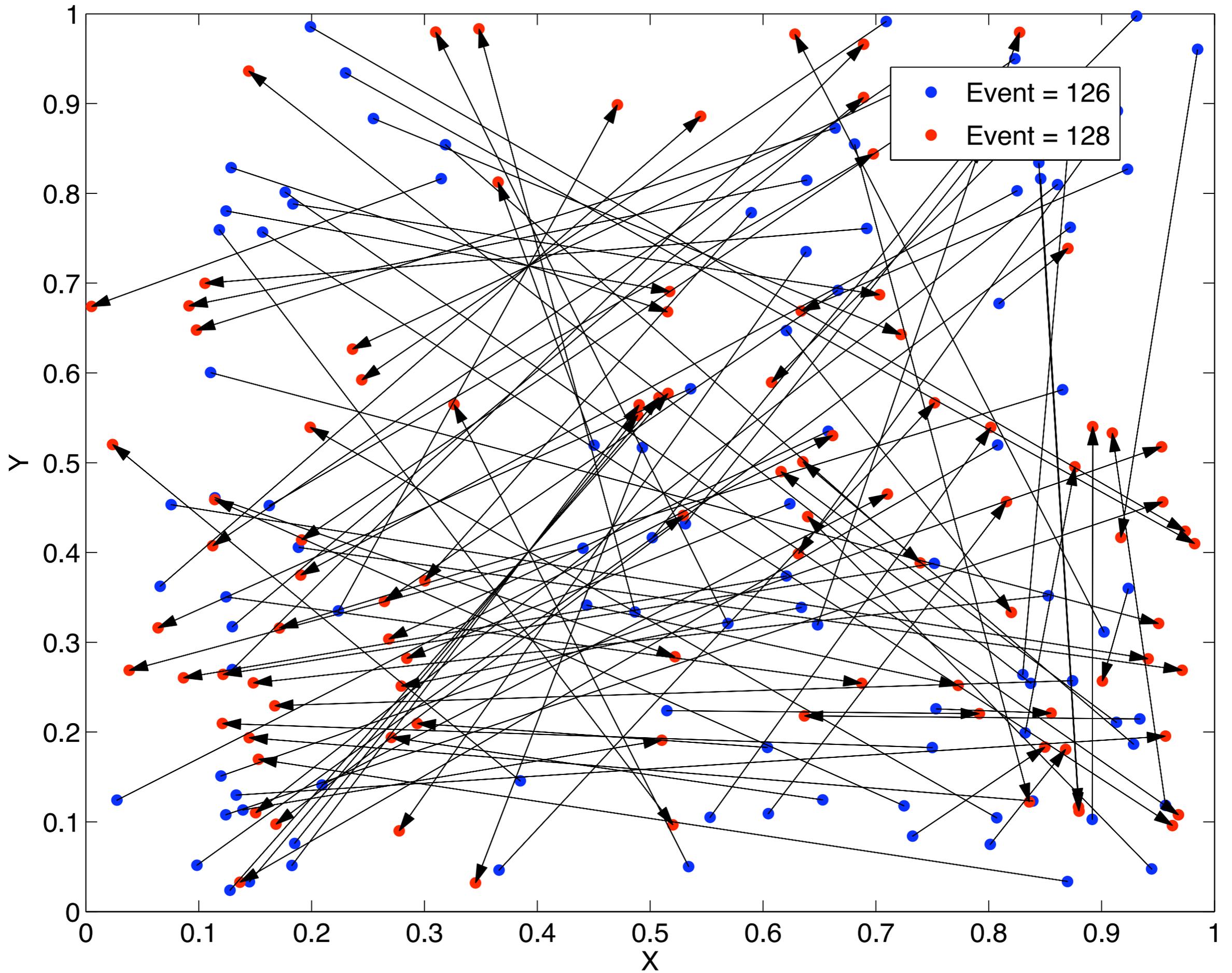
Each triplet map has a location, orientation, size, and time.

- *Location* is randomly chosen.
- *Orientation* is parallel to  $x$ -,  $y$ -, or  $z$ -coordinate and is randomly chosen.
- *Size*  $\sim$  Kolmogorov length scale.
- *Interval between maps*  $\sim$  Kolmogorov time scale.

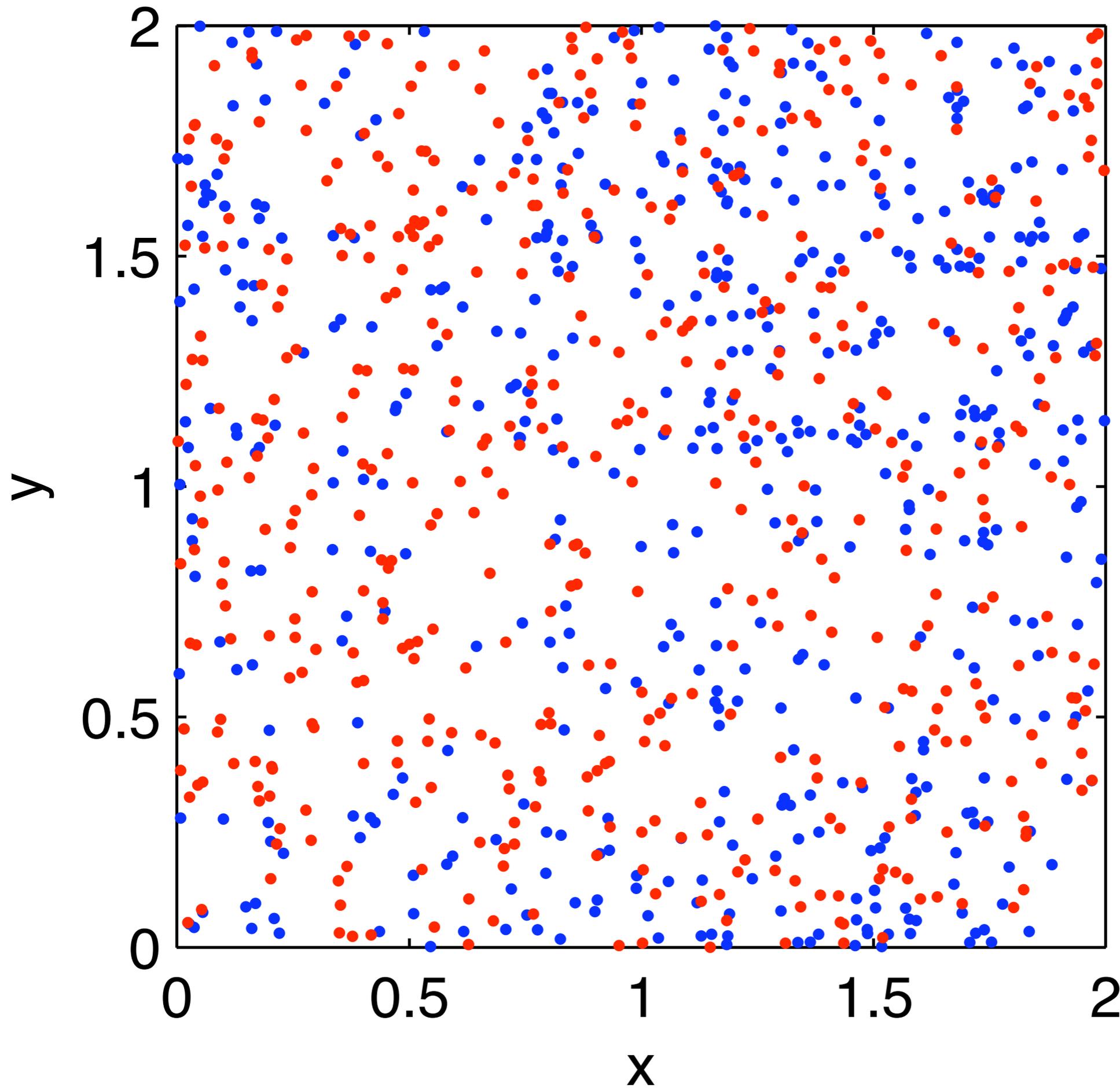
Triplet Map Rearrangement (one axis),  $S = 0.1$



Triplet Map Rearrangement (two axes),  $S = 0.1$



x-y plot for all z



$St = 0$

$St = 0.025$

length unit is  
triplet map  
eddy size =  
20 x  
Kolmogorov  
scale

- To use the triplet map to calculate droplet motions in turbulence we must relate:
  - The model's *map (eddy) size* to the *Kolmogorov length scale*.
  - The *map (eddy) interval* to the *Kolmogorov time scale*.
  - The *ratio of droplet displacement to fluid displacement* ( $S$ ) for each map to the *particle Stokes number* ( $St$ ).

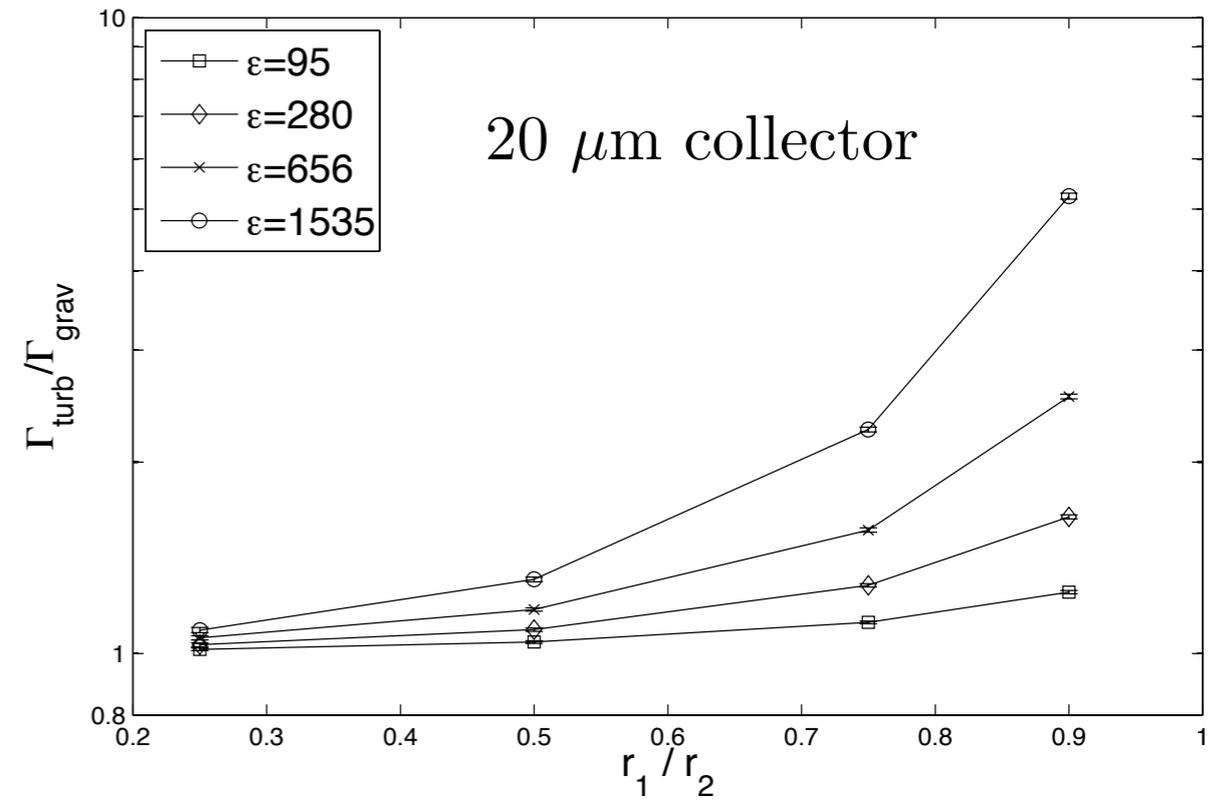
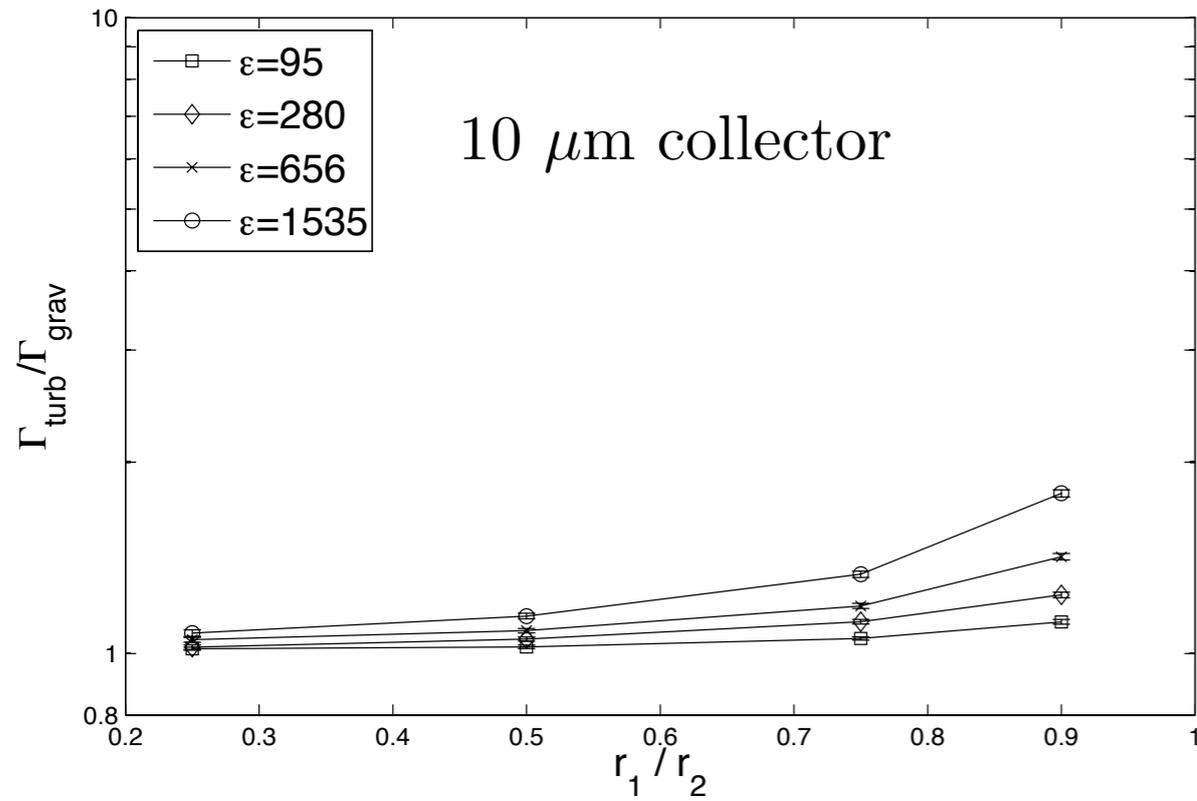
- The first two relationships were determined by comparing our results for inertial bidispersions to the DNS results of Chun et al. (2005).
- The third relationship was determined by comparing our results for zero-inertia monodispersions to the collision rates for the Saffman-Turner regime.
- For details, see:  
Krueger, S. K., J. Oh, and A. R. Kerstein, 2008: Enhancement of Coalescence due to Droplet Inertia in Turbulent Clouds. *Proceedings of the 15th Conference on Clouds and Precipitation, Cancun, Mexico, July 2008*

# Collision Kernels

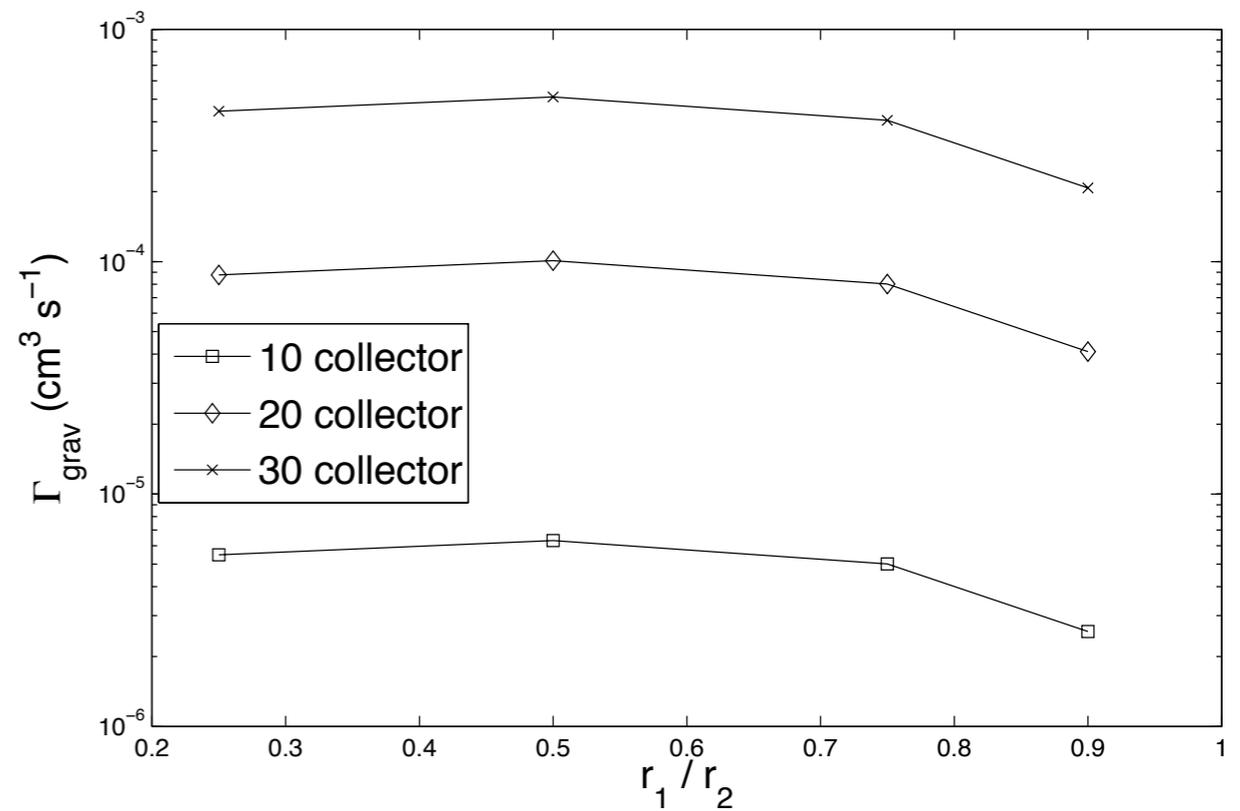
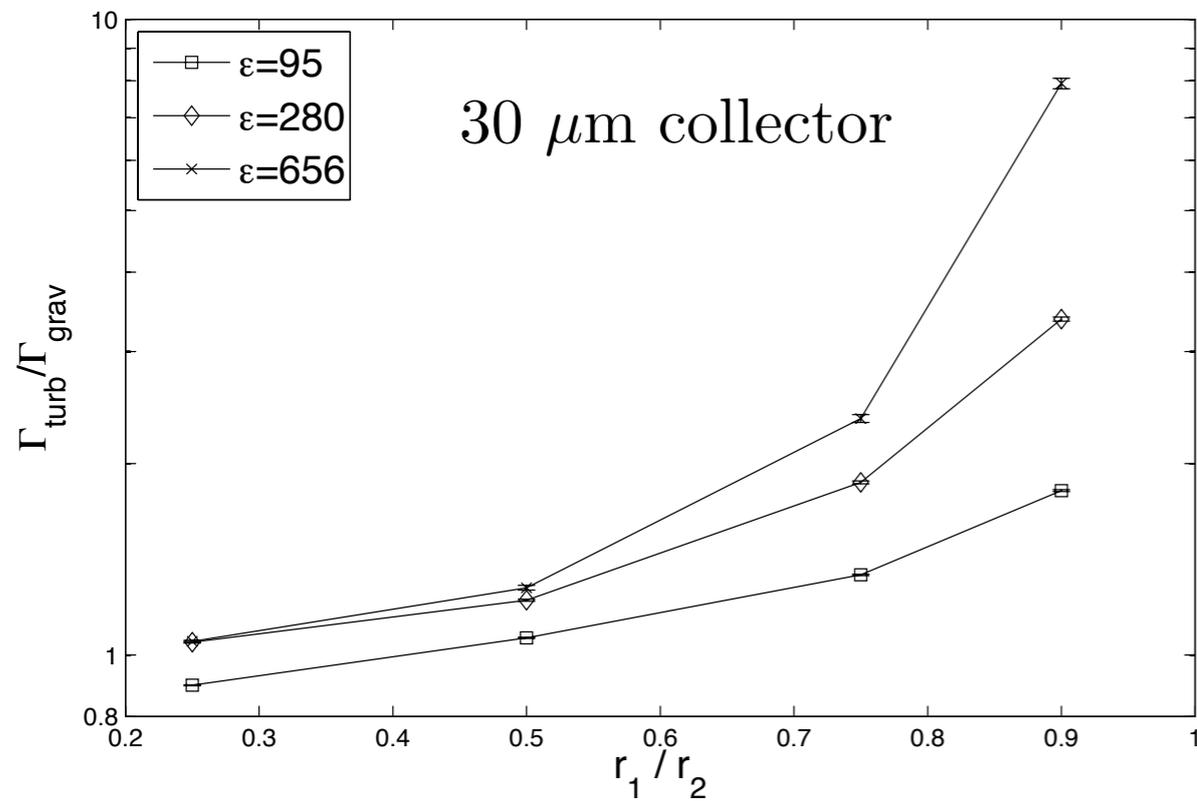
We implemented an efficient collision detection code and compared our *collision kernels* of

- bidispersions with inertia and gravity with those from DNS by Franklin et al. (2005).
- monodispersions with inertia and gravity with those from DNS by Ayala et al. (2008).

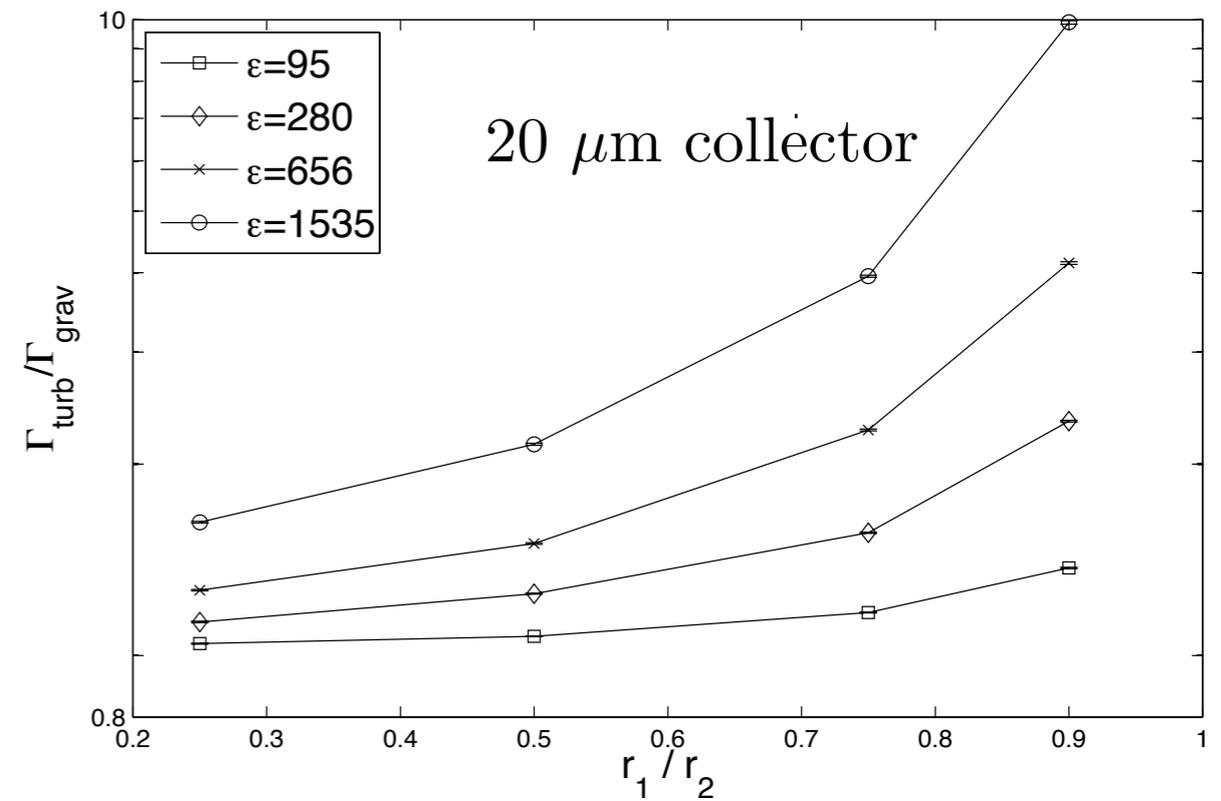
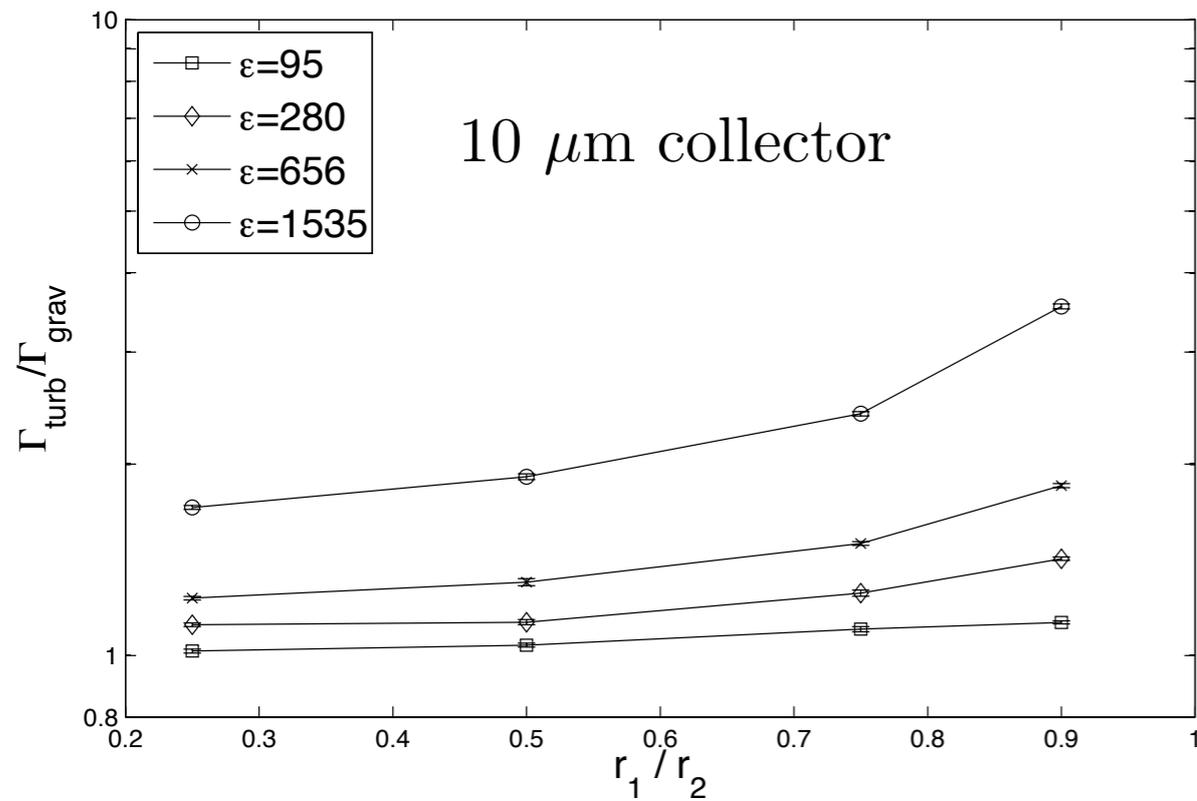
# Normalized Collision Kernels



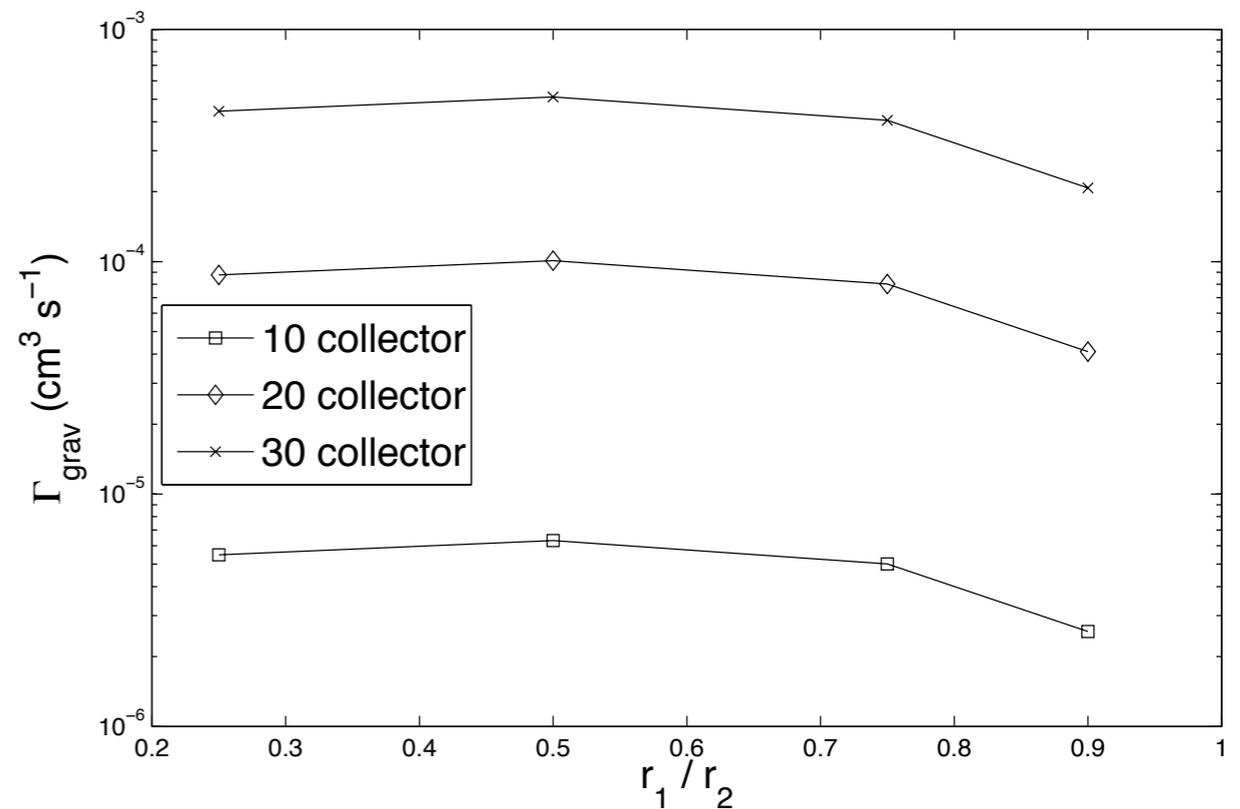
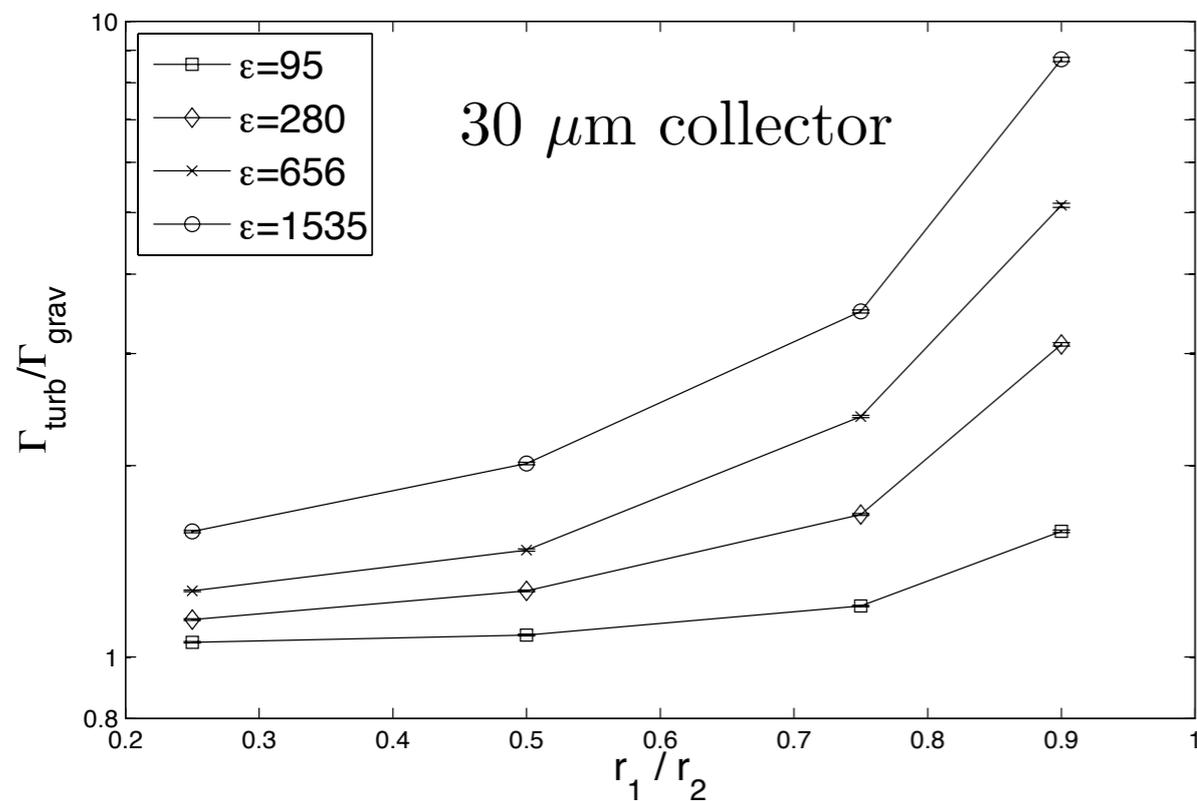
## Triplet Map for Droplets



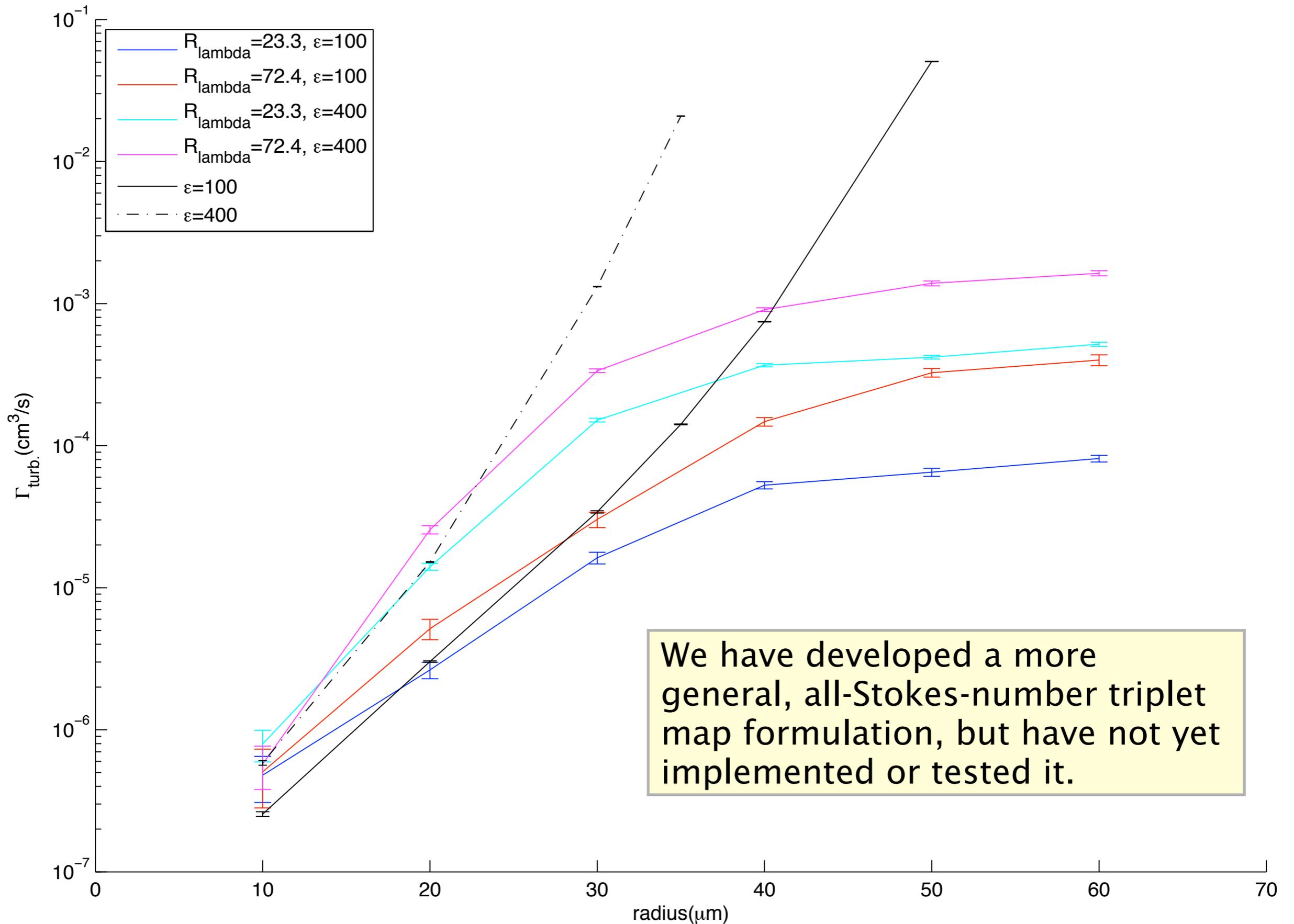
# Normalized Collision Kernels



DNS by Franklin et al. (2005)



# Monodisperse Collision Kernels vs Ayala et al. (2008)



# Collision and Coalescence Calculations

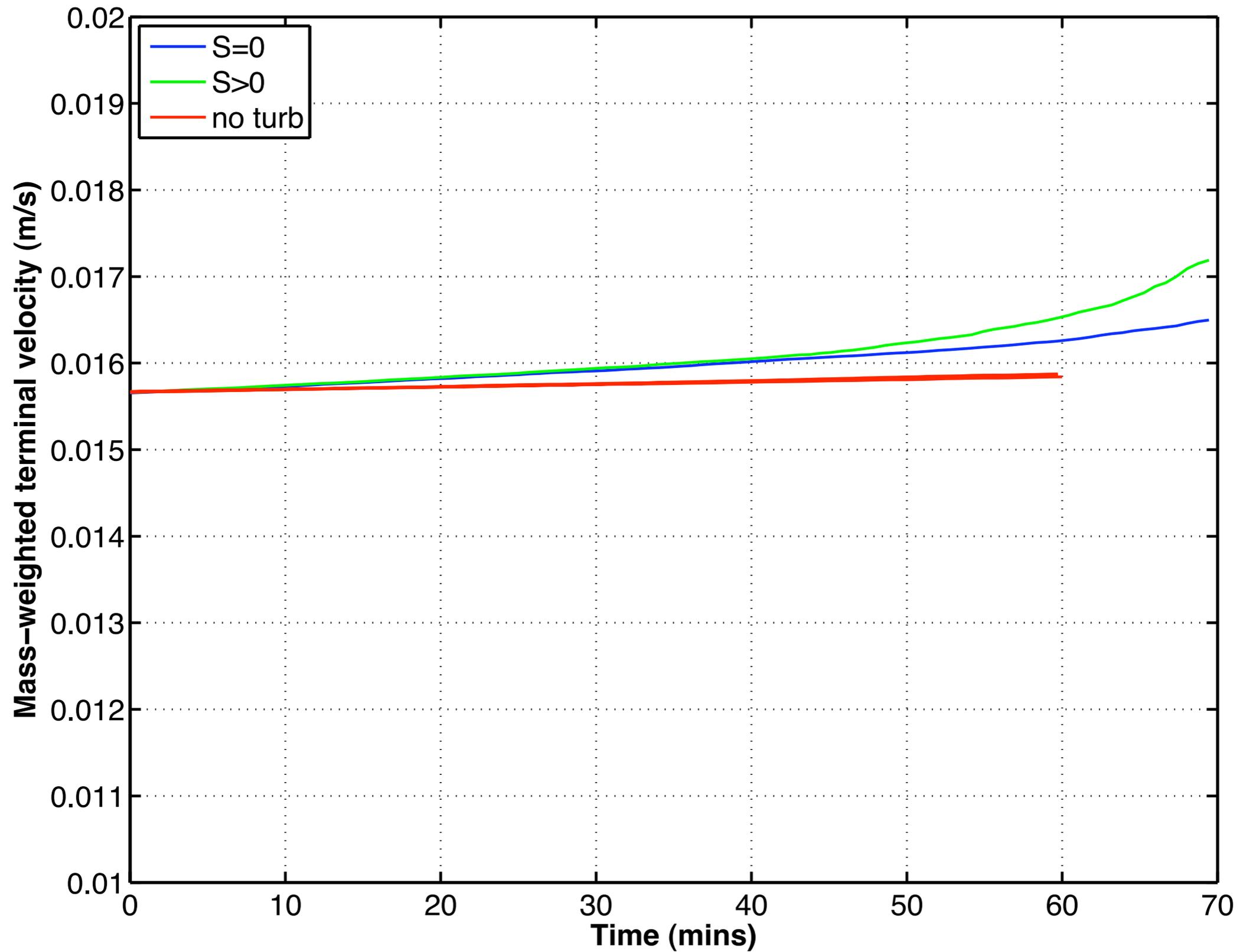
- Dissipation rate =  $100 \text{ cm}^2 \text{ s}^{-3}$ .
- Droplet number concentration =  $100 \text{ cm}^{-3}$ .
- LWC = 0.6 to  $1.6 \text{ g m}^{-3}$ .
- We used *collision efficiencies* from Hall (1980) for laminar flows.
- The results are sensitive to the choice of collision efficiencies, *and are preliminary*.
- $St < 0.8$  enforced.

# Collision and Coalescence Calculations

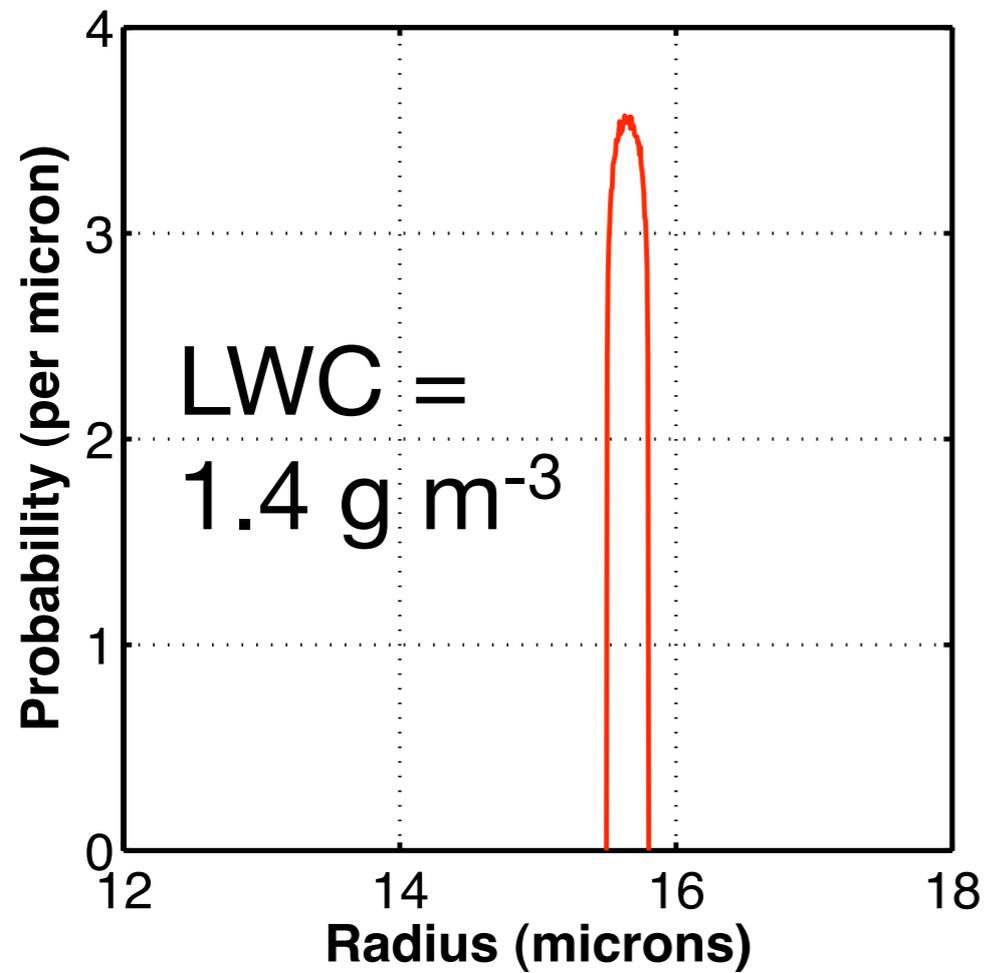
- *Case 1*: Uniform DSD from 10 to 12.6 microns.  
LWC =  $0.6 \text{ g m}^{-3}$ .
- *Case 2*: Narrow DSD from 15.5 to 15.8 microns.  
LWC =  $1.6 \text{ g m}^{-3}$ .
- *Case 3*: Wide DSD from 12 to 16.5 microns.  
LWC =  $1.4 \text{ g m}^{-3}$ .

# Case 1

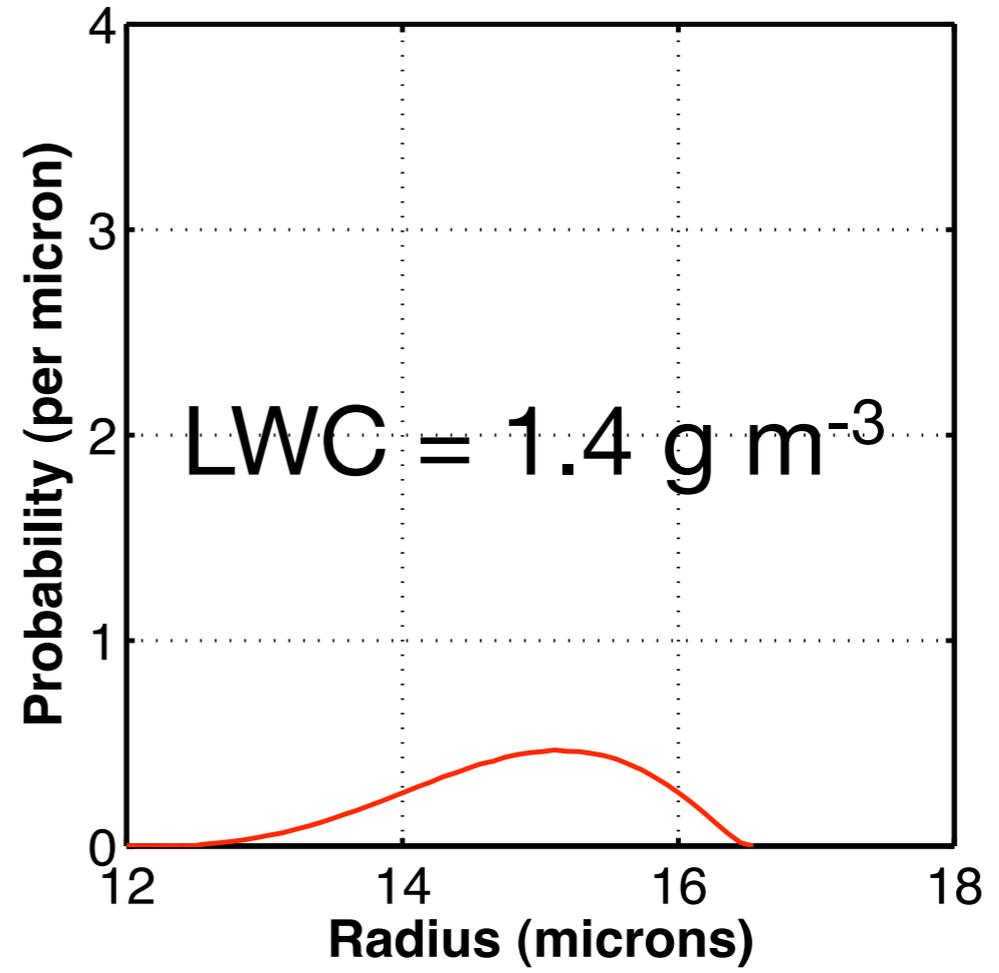
Uniform DSD 10 to 12.6 microns



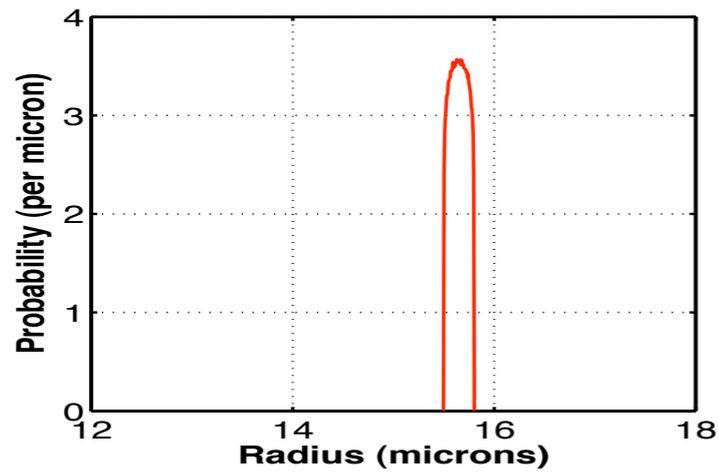
*Case 2*  
adiabatic



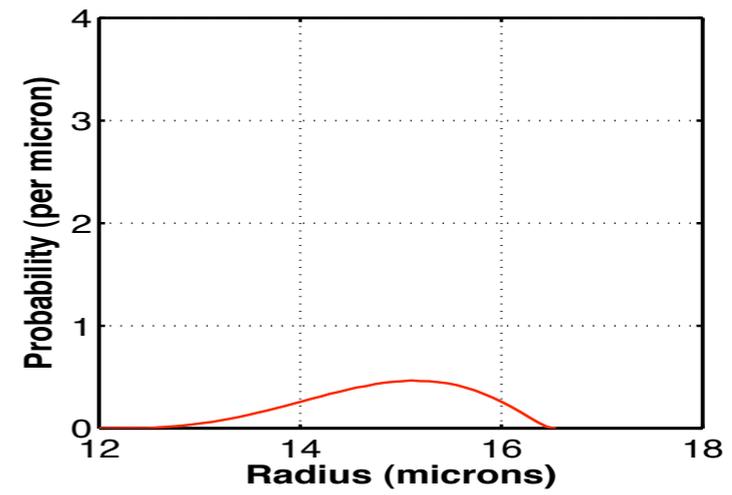
*Case 3*  
diluted



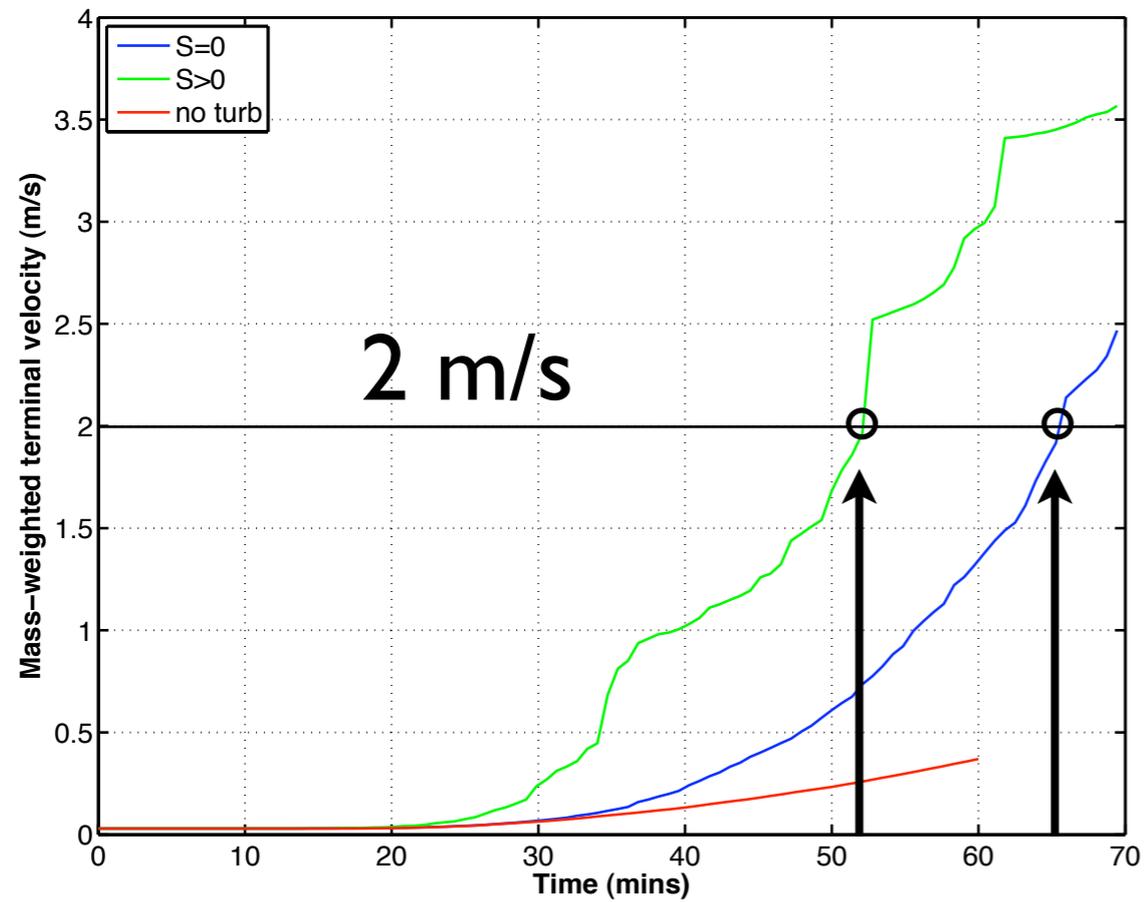
# adiabatic



# diluted

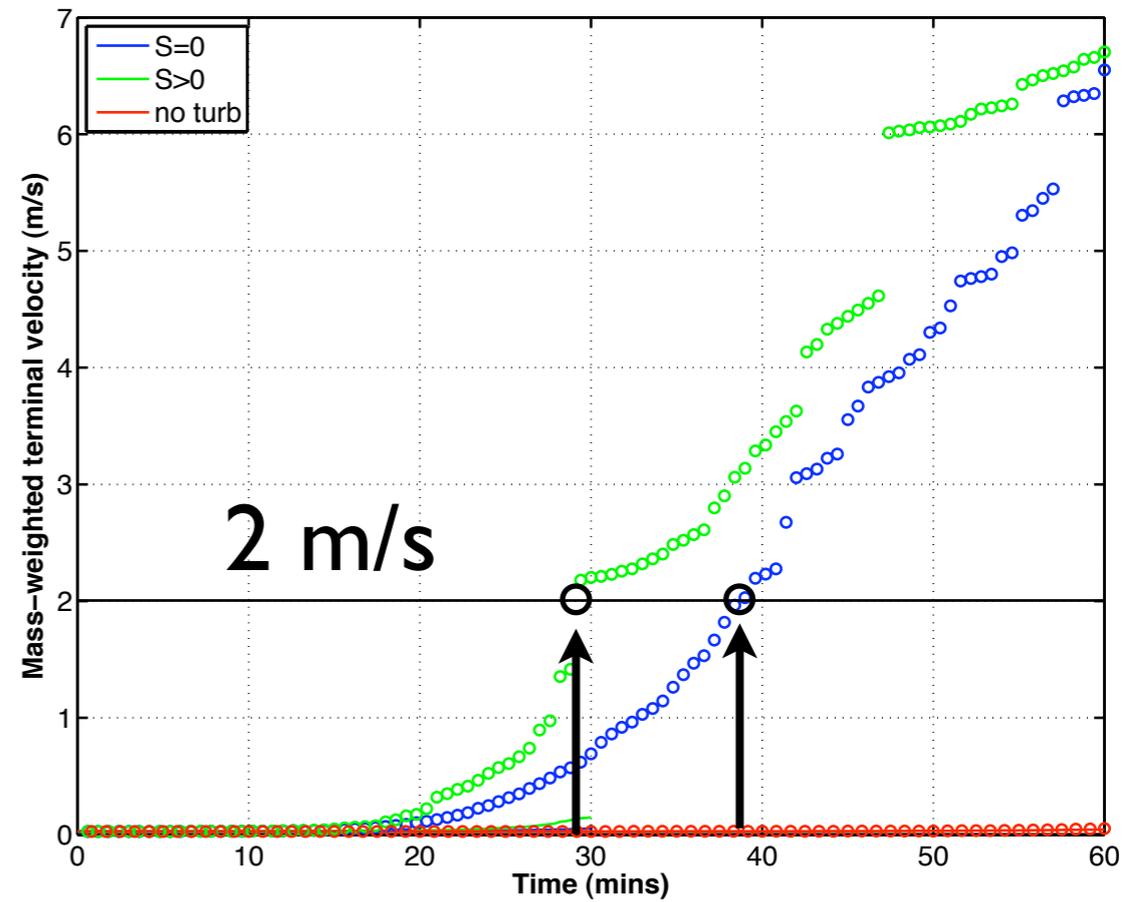


Uniform DSD 15.5 to 15.8 microns



52 65

Nonuniform DSD 12 to 16.5 microns



29 39

# Collision and Coalescence Calculations

- Without turbulence (and without condensational growth), neither DSD produces rain.
- In turbulence, rain forms about 10 minutes sooner with inertial compared to zero-inertia droplets.
- In turbulence, rain forms about 25 minutes sooner with the broader DSD.

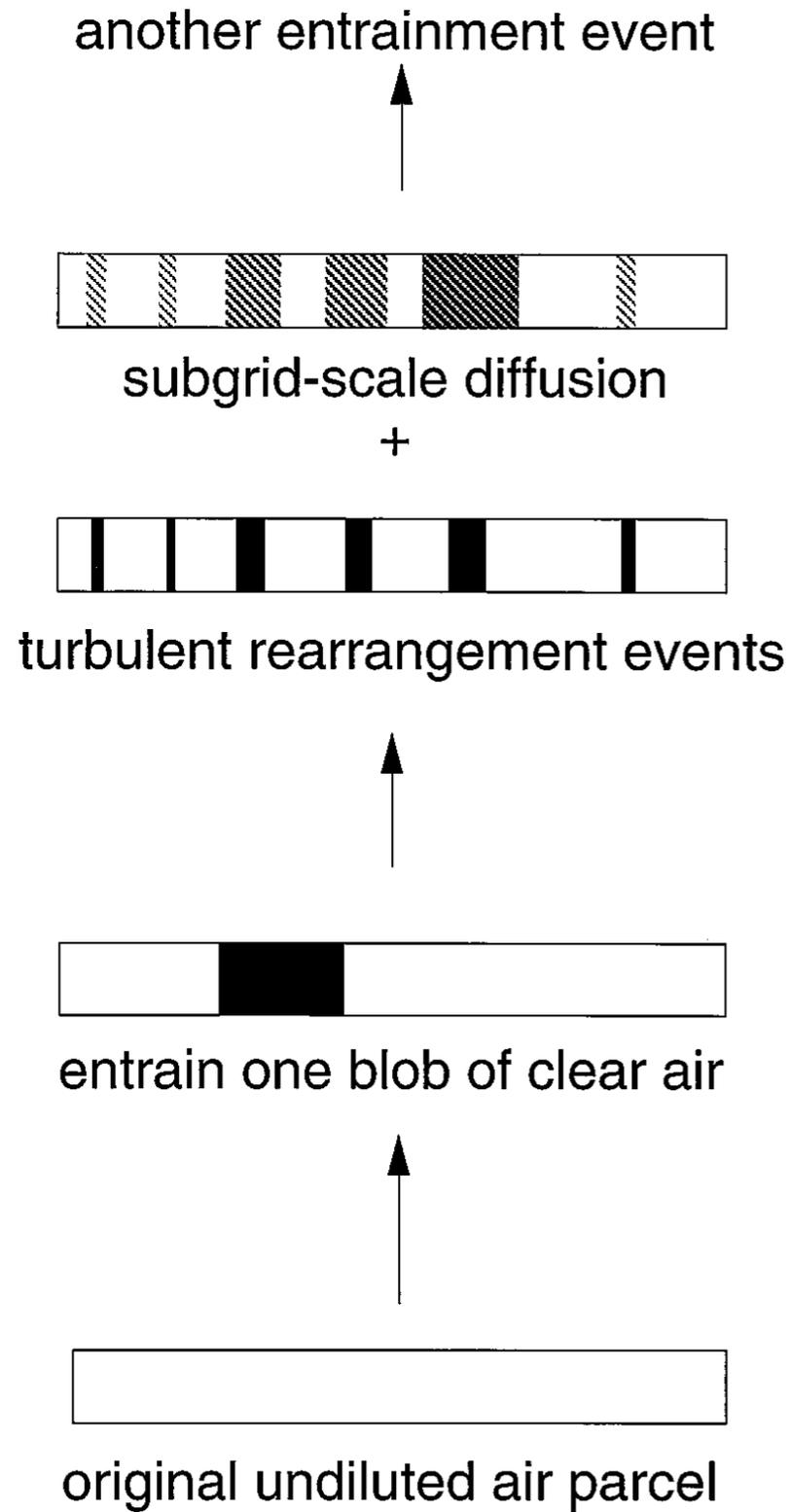
# Other Potential Applications

- Study the impacts of entrainment and mixing on:
  - *coalescence growth* (using a larger domain).
  - *condensational and coalescence growth* (by including condensational growth and a larger domain).
- Study the growth of particles in turbulent mixed-phase clouds.

# Summary

- An economical simulation method for droplet motions in turbulent flows has been developed.
- We compared collision kernels to DNS results.
- Model appears to be valid for  $St \ll 1$ .
- Some preliminary collision and coalescence calculations have been performed.
- They suggest that turbulence can significantly accelerate rain formation by droplet clustering and by spectral broadening due to entrainment and mixing.

# Explicit Mixing Parcel Model (EMPM)



# LES with 1D subgrid-scale model

