

# Stronger Local Dissipation for Stronger Waves In *Fluid and MHD Turbulence*

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Pouquet et al., *Atmosphere* **14**, 1375 (2023)

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# A few questions on energy dissipation in turbulence, with or w/o waves: Rotating stratified flows, MHD, solar wind, galaxies ...

- 1- How much **global** energy dissipation in a turbulent fluid (with or w/o magnetic field) ?
  - *Role of exact laws to measure dissipation, and role of bi-directional cascades*
- 2- How much **global** energy dissipation in **wave** turbulence ?
- 3- How much **local** energy dissipation in a turbulent fluid in the presence of waves (*intensity vs. localization*) ?
- 4- Statistical properties of kinetic energy dissipation: a link between its third- and fourth- order moments

# Equations & definitions: rotating stratified flows

$$\partial_t \mathbf{u} = -(\mathbf{u} \cdot \nabla) \mathbf{u} - N\theta \mathbf{z}^* + \nu \nabla^2 \mathbf{u} + 2\mathbf{u} \times \boldsymbol{\Omega} - \nabla p + \mathbf{F}_u, \quad \nabla \cdot \mathbf{u} = 0$$

$$\partial_t \theta = -(\mathbf{u} \cdot \nabla) \theta + Nw + \kappa \nabla^2 \theta + F_\theta$$

$$\varepsilon_V = D_t \langle u^2 \rangle = -p \langle u^3 \rangle + \dots$$

$$\varepsilon_\theta = D_t \langle \theta^2 \rangle = -q \langle u \theta^2 \rangle + \dots$$

$$Re = \frac{U_{rms} L_{int}}{\nu}, \quad Ro = \frac{U_{rms}}{f L_{int}}, \quad Fr = \frac{U_{rms}}{N L_{int}}; \quad R_\lambda = \frac{U_{rms} \lambda_T}{\nu}, \quad R_B = Re Fr^2, \quad Ri_g = \frac{N(N - \partial_z \theta)}{[(\partial_z u)^2 + (\partial_z v)^2]}$$

$f = 2\boldsymbol{\Omega} \cdot \mathbf{z}^*, \quad \lambda^2 = \langle u^2 \rangle / \langle \omega^2 \rangle, \quad \omega = \nabla \times \mathbf{u}$

Skewness and excess kurtosis (both 0 for a Gaussian distribution):

$$S_V = \langle V^3 \rangle / \langle V^2 \rangle^{3/2}, \quad K_V = \langle V^4 \rangle / \langle V^2 \rangle^2 - 3; \quad K_V(S_V) = a_V S_V^2 + b_V$$

range	Dissipation estimate (Eq.)	symbol	definition	assumption
dissipative sub-range	instantaneous (3)	$\epsilon_0$	$2\nu(s_{ij}s_{ij})$	
	(local) volume average (4)	$\langle \epsilon(\mathbf{x}, t) \rangle_R$	$\frac{3}{4\pi R^3} \iiint_{V(R)} \epsilon_0(\mathbf{x} + \mathbf{r}, t) d\mathbf{r}$	SHI
	(longitudinal) gradient (6)	$\epsilon_G$	$15\nu \left\langle \left( \frac{\partial u'_1(\mathbf{x})}{\partial x_1} \right)^2 \right\rangle$	SHI
	2nd-order SF (dissipation range) *	$\epsilon_{D2}$	$15\nu D_{LL}(r)/r^2$	SHI, $r \lesssim \eta_K$
inertial sub-range	zero-crossings *	$\epsilon_+$	$15\pi^2 \nu \langle u_1'^2 \rangle N_L^2$	SHI
	4/5 law * ((7), $n = 3$ )	$\epsilon_{I3}$	$-5/4 \underline{D_{LLL}(r)}/r$	SHI, K41
	2nd-order SF (inertial range) ((7), $n = 2$ )	$\epsilon_{I2}$	$(D_{LL}(r)/C_2)^{3/2}/r$	SHI, K41
	spectral (9)	$\epsilon_S$	$\left( \frac{\kappa_1^{5/3} E_{11}(\kappa_1)}{18/55 C_K} \right)^{3/2}$	SHI, K41
	cutoff filter *	$\epsilon_C$	$\left( \frac{2}{3} \frac{2\langle u_G'^2 \rangle}{18/55 C_K (\kappa_{1,low}^{-2/3} - \kappa_{1,up}^{-2/3})} \right)^{3/2}$	SHI, K41
energy injection scale	scaling argument (10)	$\epsilon_L$	$C_\epsilon \sigma_{u'}^3 / L_{11}$	SHI
	global mean (5)	$\langle \epsilon \rangle$	$\lim_{R \rightarrow \infty} \langle \epsilon_0(\mathbf{x}, t) \rangle_R$	SHI



# Exact Kolmogorov (1941) law

- **Starting point: Invariants** ( $v=\eta=0$ ): total energy, magnetic helicity & cross-helicity in 3D-MHD, ...
- **Assumptions:** homogeneity, stationarity and large Reynolds number, together with finite dissipation  $\epsilon$ , as well as incompressibility and full isotropy (*but not always: Galtier-Banerjee*)
  - Fluids: *Kolmogorov 1941*; *Antonia+ 1997*; 2D: *Lindborg-99*. Passive scalar: *Yaglom, 1949*
  - MHD: *Politano+1998ab*, *Banerjee+ 2016, 2017*; 2D: *Caillol, unpublished*.
  - Compressible: *Banerjee+'13,14*, *Kritsuk 23*, ...
  - **Helical** laws for fluids, MHD & Hall-MHD: *Gomez+ 00*, *Politano+ 03*, *Banerjee+ 16,17*
    - Helical sub-invariants (Alexakis, 2017)
    - Helical MHD case ?
  - **Non-linear models** of small-scale dynamics: EDQNM (fluids: *Briard+17*), *alpha*-models for fluids & MHD: *Graham+ 2006, 2008*.
    - MHD closure case ?

\* *Beyond Hall & e-MHD: 2D-3C; 2-fluid; extended MHD ?*

$$\delta u_L(r) = u(x+r) - u(x)$$

$$S_3(r) = \langle \delta u_L^3(r) \rangle \rightarrow$$

\*

(1): Exact

Seven points

(2), ESS:  
high-order structure  
functions scaling  
with  $r \leftrightarrow Y_H$

$$S_3(\mathbf{r}) = -\frac{4}{5}\epsilon^V r$$

+

(3): cascade direction

(4)

measure  $\epsilon$

(5) +: add terms (model, viscous, force)

(6): Non-Gaussianity, *but not necessarily intermittency (cf. the 2D case)*

(7): More laws when **more invariants?**

\* *How do they inter-connect?*

\* *Role of cross-correlations?*

Laboratory experiments  
on decaying fluid flows

$R_\lambda$  up to 5779

Active grids

*Kuchler+2023*

$$R_\lambda = u\lambda/\nu$$

$$\lambda^2 = \langle u^2 \rangle / \langle \omega^2 \rangle$$

$$\omega = \nabla \times \mathbf{u}$$

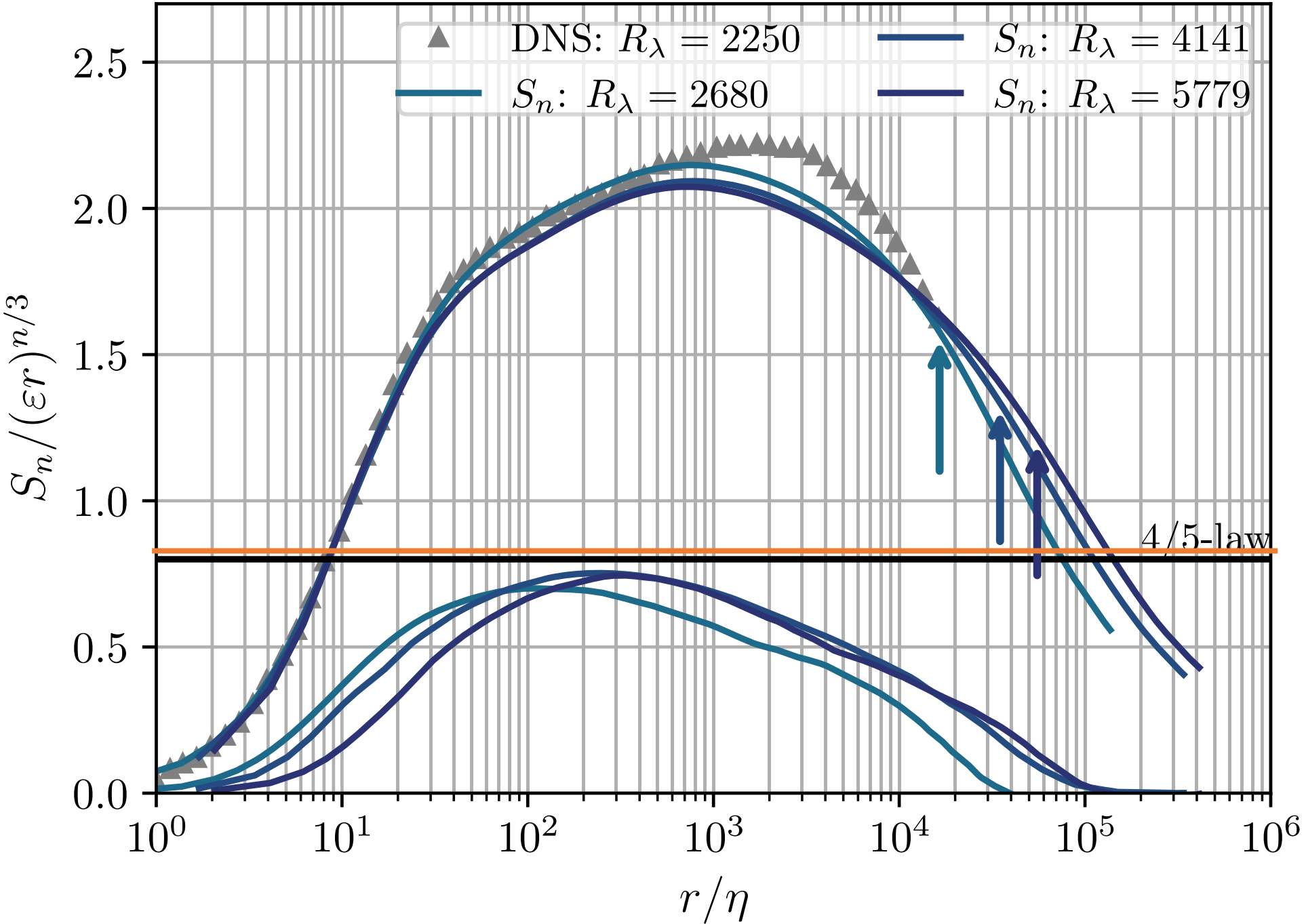
Kolmogorov

dissipation length  $\eta_k$ :

$$\eta = [\epsilon/\nu^3]^{-1/4}$$

Local slope:  $\zeta_2(r)$

$$\delta u^2(r) = r^{\zeta_2}$$

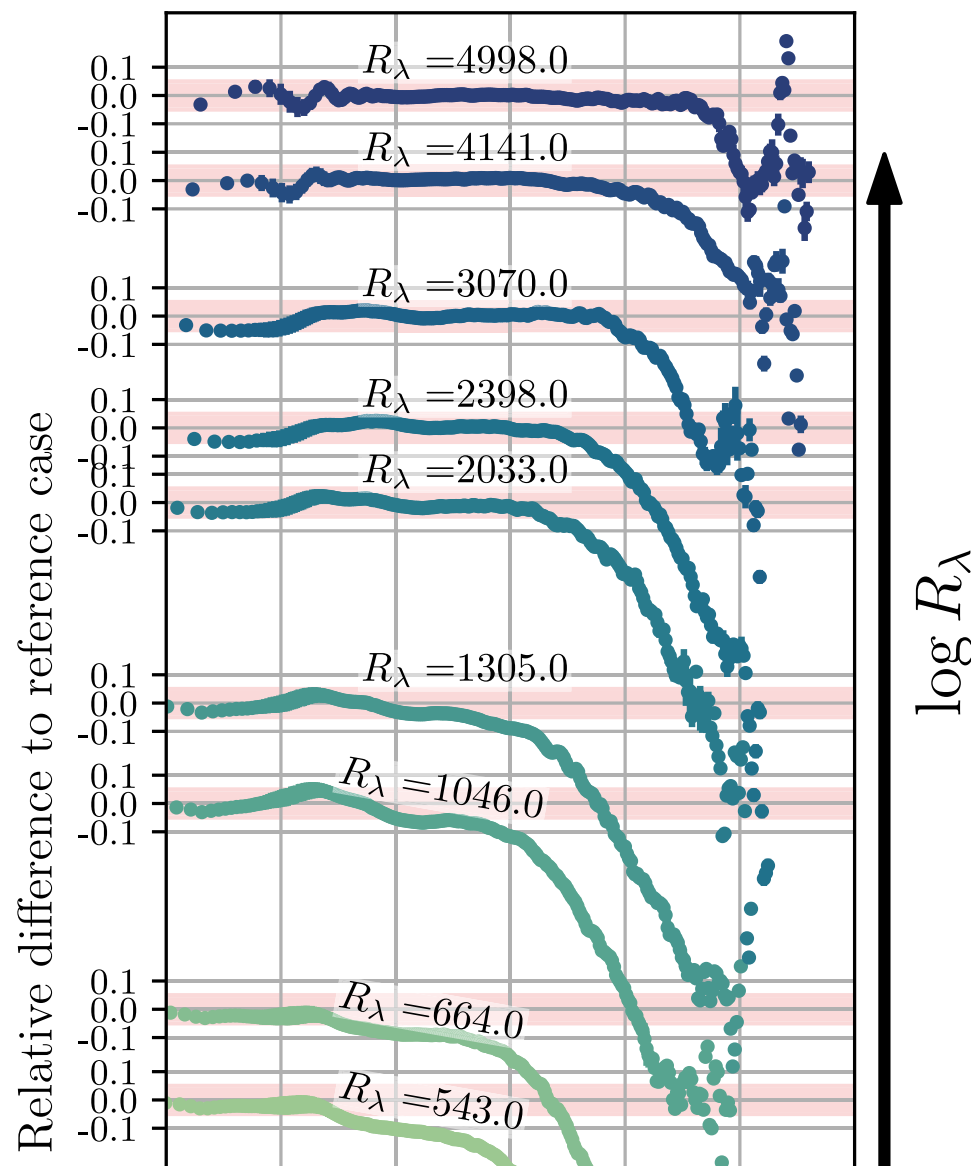
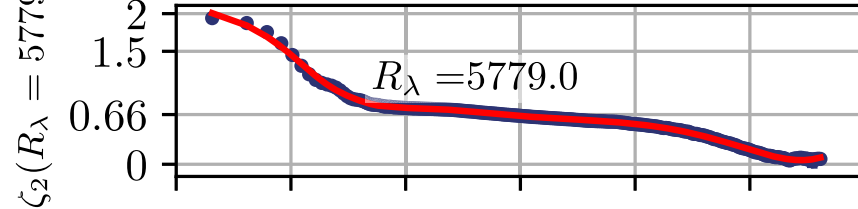


Laboratory experiments  
on decaying fluid flows

$R_\lambda$  up to 5779

*Kuchler+2023*

Compensated energy  
spectra



# Exact laws in MHD

Review: Marino+ 2023

$$\langle \delta \mathbf{v}_L \delta \mathbf{v}_i^2 \rangle + \langle \delta v_L \delta b_i^2 \rangle - 2 \langle \delta b_L \delta v_i \delta b_i \rangle = - (4/d) \epsilon^T r$$

$$- \langle \delta \mathbf{b}_L \delta \mathbf{b}_i^2 \rangle - \langle \delta b_L \delta v_i^2 \rangle + 2 \langle \delta v_L \delta v_i \delta b_i \rangle = - (4/d) \epsilon^C r$$

]

**E<sup>T</sup> AND H<sup>C</sup>**

$$\epsilon^T = - d_t E^T ,$$

$$\epsilon^C = - d_t E^C$$

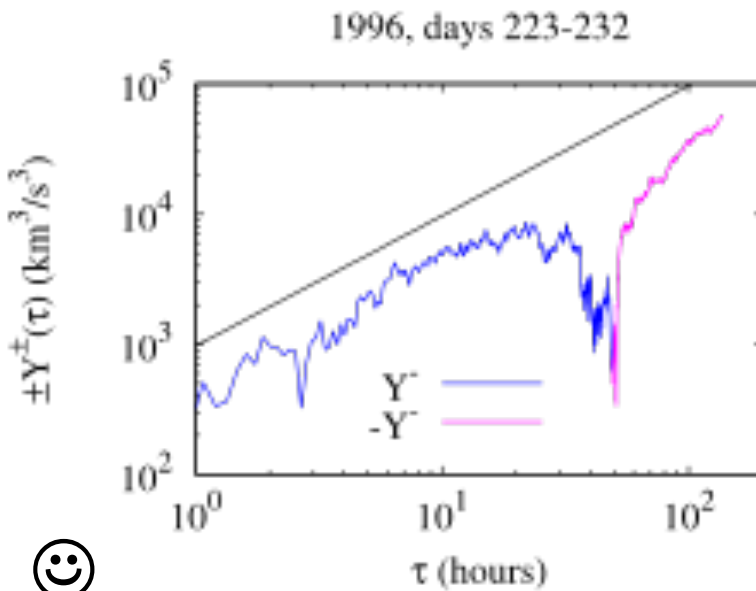
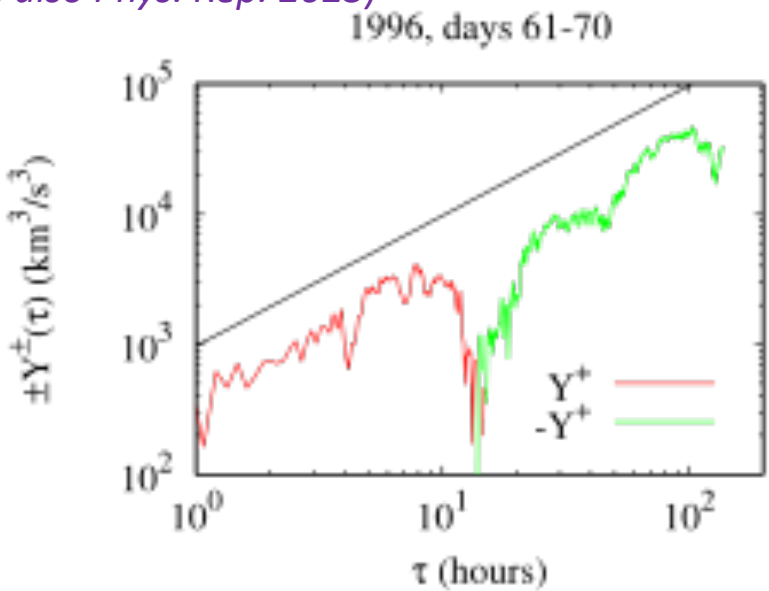
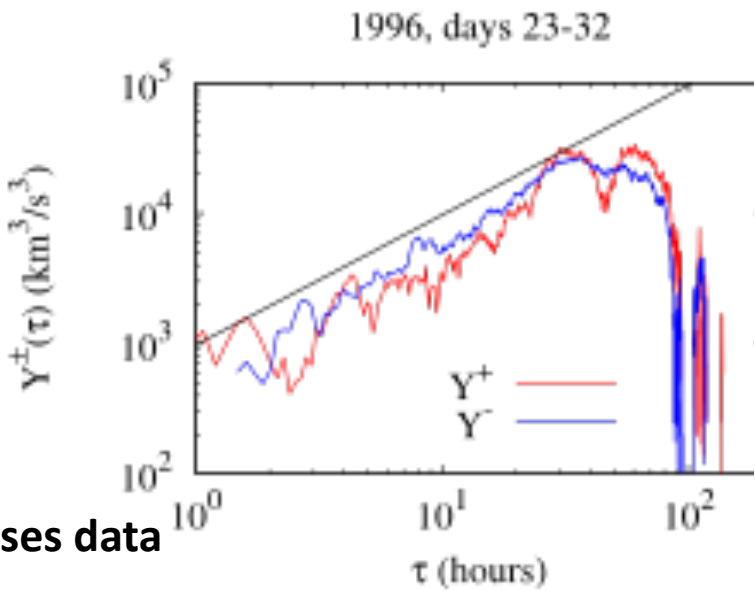
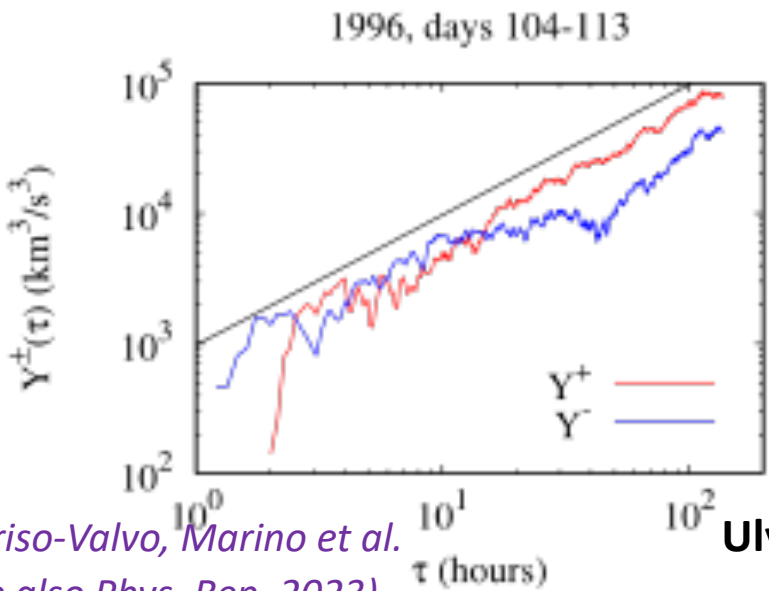
- Also in terms of  $\mathbf{Y}_{\pm}$  fluxes of Elsasser variables, with energy dissipation rates  $\epsilon^{\pm}$

- **Three** regimes: v-dominated vs. B-dominated vs. Alfvénic ( $\mathbf{v} \sim \mathbf{b}$ ) (cf. Ting et al 1986)
- **Dynamical role of the correlation between  $v$  &  $b$  in the mixed regime** (Politano+ GRL 25, 1998; *also Boldyrev, 2006*):  
 $\epsilon^{\pm} = 0$  for exact solutions such as (nonlinear) Alfvén waves.

- When such laws apply, the input/dissipation rates  $\epsilon^{T,C}$  &  $\epsilon^{\pm}$  can be measured, e.g. in the solar wind for different conditions. *What about plasma regimes?*

Politano+1998ab; Banerjee+2017

vKH in MHD: Chandrasekhar 1950



Ulysses data



MacBride+ 2005

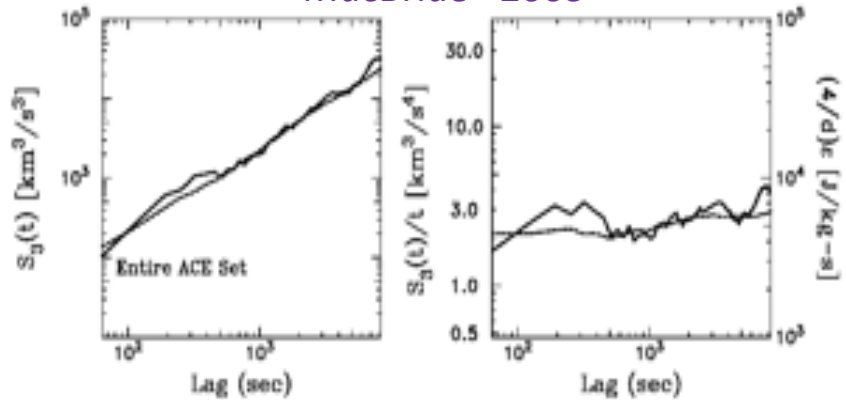


Figure 4. (left) The third moment of fluctuations as a function of lag for the hydrodynamic (dashed) and magnetohydrodynamic (solid) forms spanning years 1998 through 2004. (right) An estimate for  $\epsilon$ .

$$\langle |\Delta \mathbf{z}^{\pm}|^2 \Delta z_{\parallel}^{\mp} \rangle = Y^{\pm}(r) = -\frac{4}{3} \epsilon^{\pm} r$$

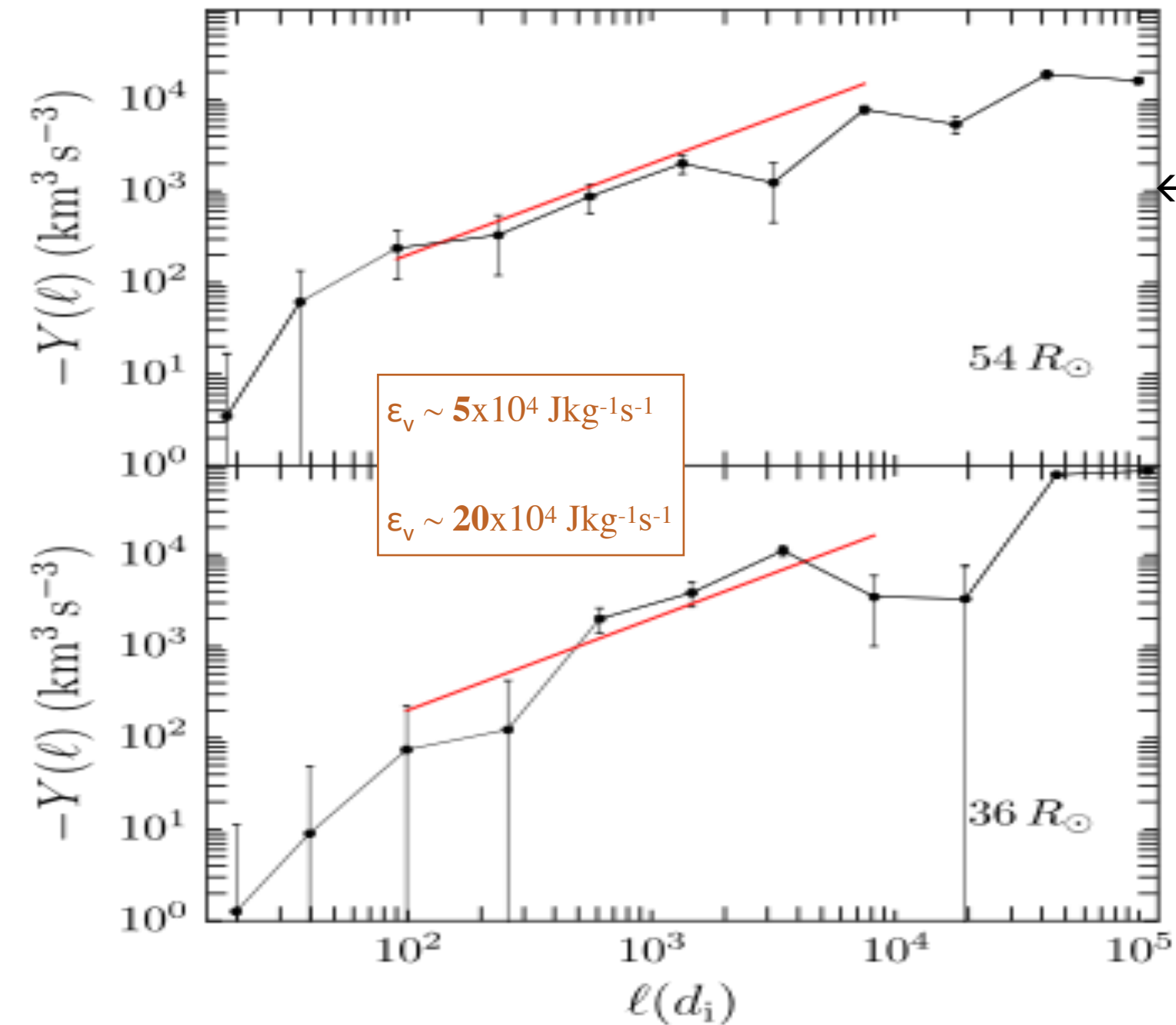
$\mathbf{z}^{\pm} = \mathbf{v} \pm \mathbf{b} \ , \ Z_{\parallel} = \mathbf{z} \cdot \mathbf{r} / |\mathbf{r}|$

$\Delta Z(r) = Z(\mathbf{x} + \mathbf{r}) - Z(\mathbf{x})$

Note:

$\delta z^+ \delta z^+ \delta z^- \sim (\delta v + \delta B)^2 (\delta v - \delta B) \sim \epsilon$

But it does not always work ....



*Bandyopadhyay+ 2020*

Parker Solar Probe, inner heliosphere

←  $Y = [Y^+ + Y^-]/2$

$$\langle |\Delta \mathbf{z}^{\pm}|^2 \Delta z_{\parallel}^{\mp} \rangle = Y^{\pm}(r) = -\frac{4}{3} \epsilon^{\pm} r$$

+ Hall terms

$$\mathbf{Z}^{\pm} = \mathbf{v} \pm \mathbf{b}$$

$$Z_{\parallel} = \mathbf{Z} \cdot \mathbf{r} / |\mathbf{r}|$$

$$\Delta Z(\mathbf{r}) = Z(\mathbf{x} + \mathbf{r}) - Z(\mathbf{x})$$

Larger energy dissipation rate  
closer to the Sun

# Vorticity dynamics for fluid turbulence

$$D_t \omega = \underbrace{\partial_t \omega + \mathbf{v} \cdot \nabla \omega}_{\text{advection}} = \underbrace{\omega \cdot \nabla \mathbf{v}}_{\text{stretching by velocity gradients}} + \underbrace{\nu \nabla^2 \omega}_{\text{+ dissipation}} + \underbrace{\nabla \times \mathbf{F}}_{\text{+ forcing}}$$

**Model:**

$$D_t \omega = \omega \cdot \nabla \mathbf{v} \quad \rightarrow \quad \nabla \mathbf{v} \text{ is } O(1) \text{ at early times: exponential growth of vorticity}$$

*Buaria+2018-22, strain-vorticity amplification in [12k]<sup>3</sup> DNS*

But:  $\omega \sim \nabla \mathbf{v}$  so:

$$D_t \omega = |\omega|^2 : \text{explosive growth}$$

Is there a role for the geometry of structures?

# Vorticity, strain and dissipation in fluid turbulence

$$\omega = \nabla \times \mathbf{u}, \quad \epsilon = 2\nu S_{ij}S_{ij}, \quad \text{where } S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \quad 2\Omega_{ij} = \partial_j u_i - \partial_i u_j$$

$$\frac{D\omega_i}{Dt} = \omega_j S_{ij} + \nu \nabla^2 \omega_i,$$

$$\frac{DS_{ij}}{Dt} = -S_{ik}S_{kj} - \frac{1}{4}(\omega_i\omega_j - \omega_k\omega_k\delta_{ij}) - \Pi_{ij} + \nu \nabla^2 S_{ij},$$

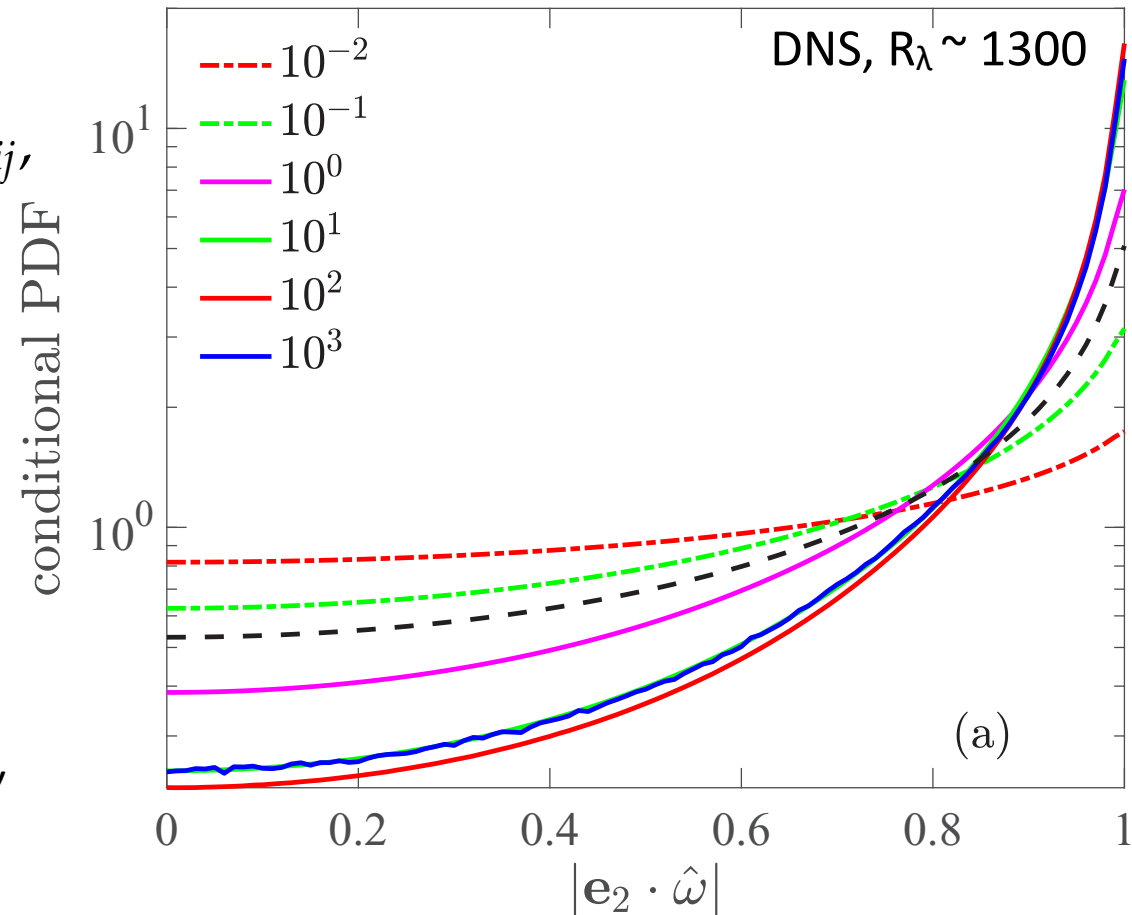
Betchov 1956:  $\langle |\mathbf{S}^2| \rangle = \langle \omega^2 \rangle / 2$

Buaria+ 2018-2022: strain-vorticity amplification in  $12k^3$  DNS



Bradshaw+2019, Johnson 2020, Rafner+2021

Strain and vorticity, local and nonlocal, amplitude and geometry





# Vorticity dynamics: geometry

$$\partial_t \omega = \nabla \times (\mathbf{v} \times \omega) + \nu \nabla^2 \omega + \nabla \times \mathbf{F}$$

$$D_t \omega = \partial_t \omega + \mathbf{v} \cdot \nabla \omega = \underbrace{\omega \cdot \nabla \mathbf{v}}_{\text{advection}} + \underbrace{\nu \nabla^2 \omega}_{\text{stretching by velocity gradients}} + \underbrace{\nabla \times \mathbf{F}}_{\text{+ dissipation + forcing}}$$

**Model:**

$$D_t \omega = \omega \cdot \nabla \mathbf{v}, \quad \text{with } \nabla \mathbf{v} = O(1) : \text{exponential growth of vorticity at early times}$$

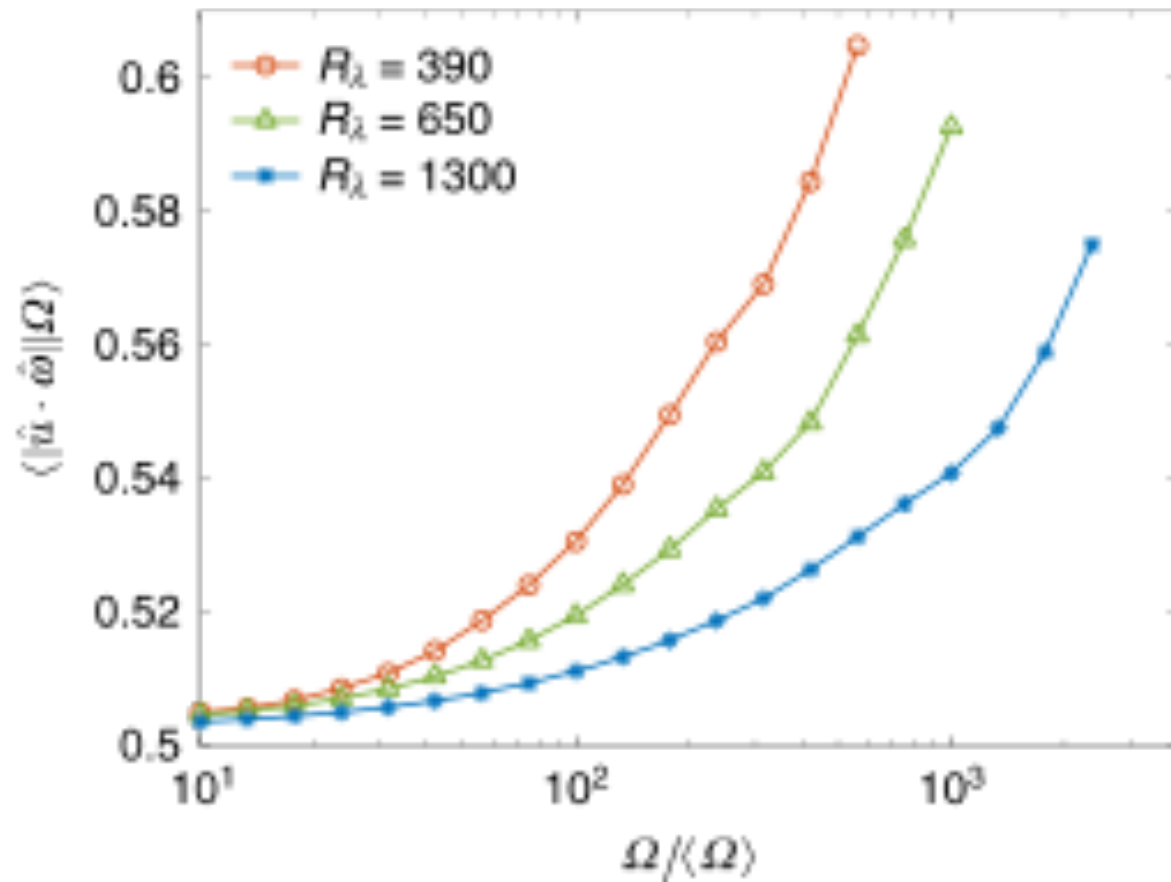
**But.**  $\omega \sim \nabla \mathbf{v}$  so:

$$\underline{D_t \omega = |\omega|^2} : \text{explosive growth}$$

**Is there a role for the geometry of structures?**

**Yes when  $\mathbf{v} \parallel \omega$  (vortex filament)**

**→ locally weak nonlinearities and long-lived coherent structures**

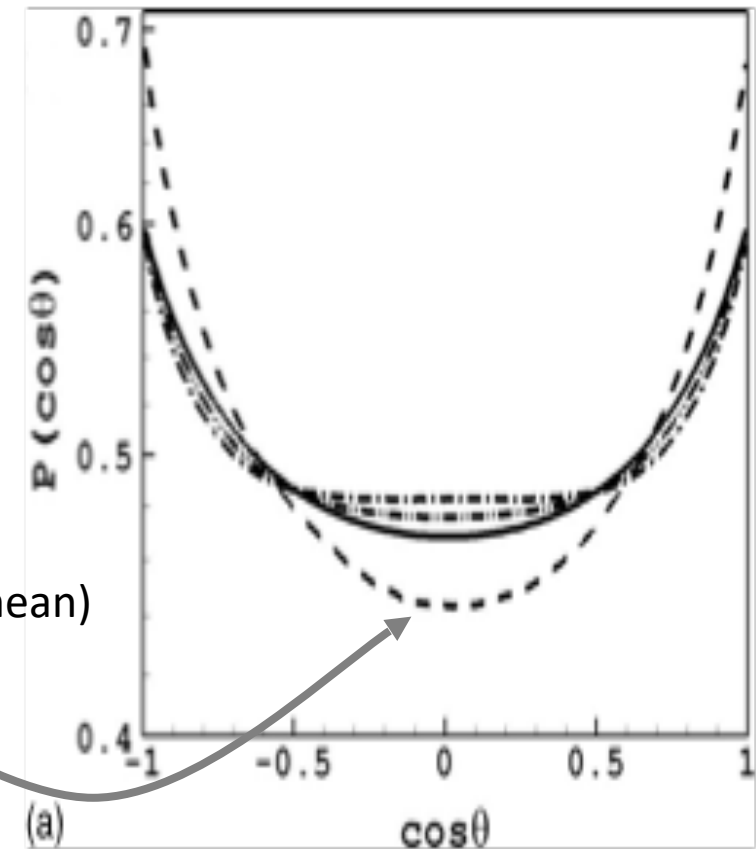


*Buaria+ 2020*

Lamb vector  $\mathbf{u} \times \boldsymbol{\omega}$  alignment

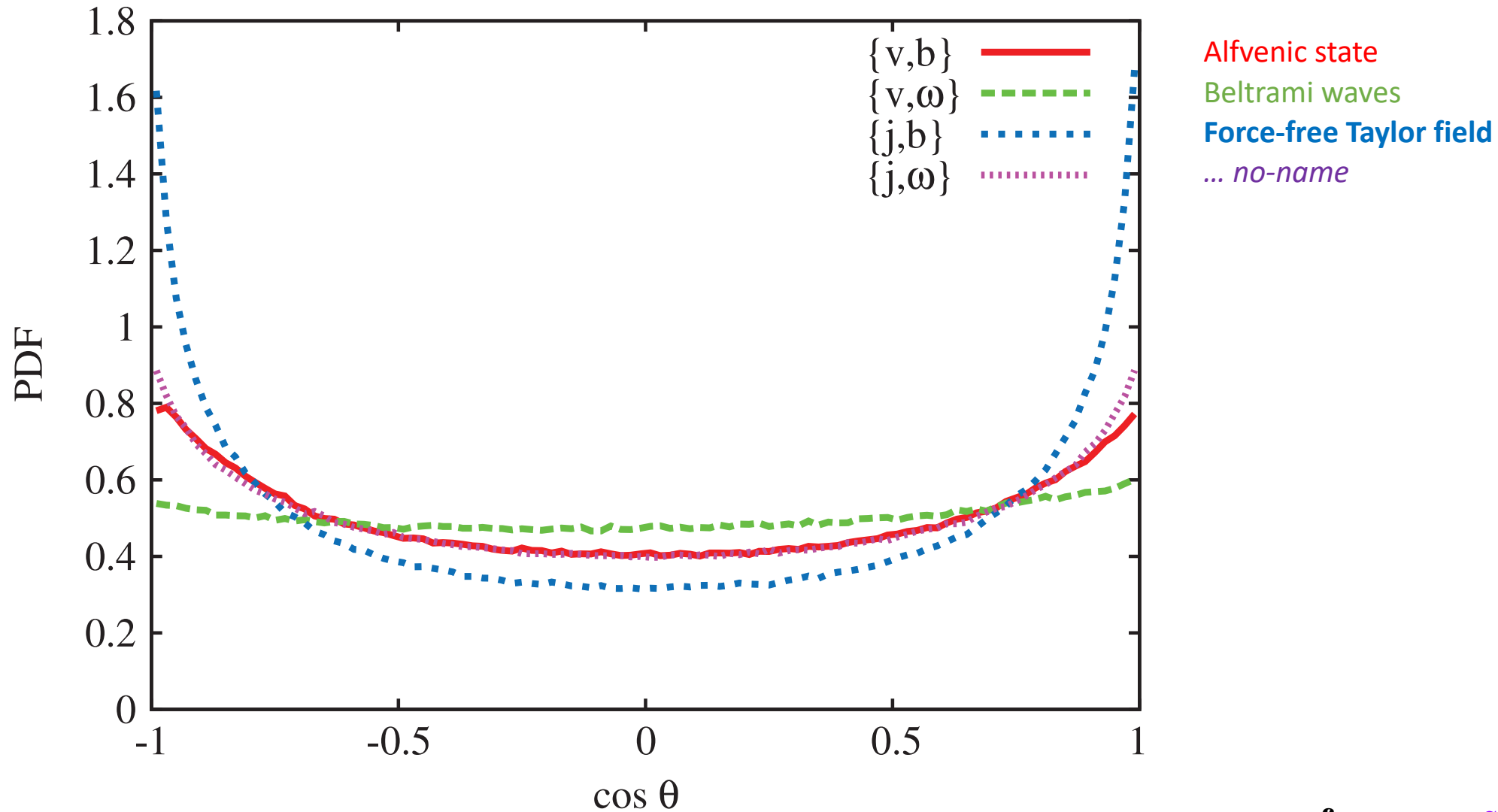
*Choi+09*

Relative helicity  
conditioned by  
level of enstrophy:  
Total (solid), and  
1-4; 4-16;  
16-64 (dash, in terms of mean)



# Other alignments

MHD,  $128^3$  grids, flat initial distributions

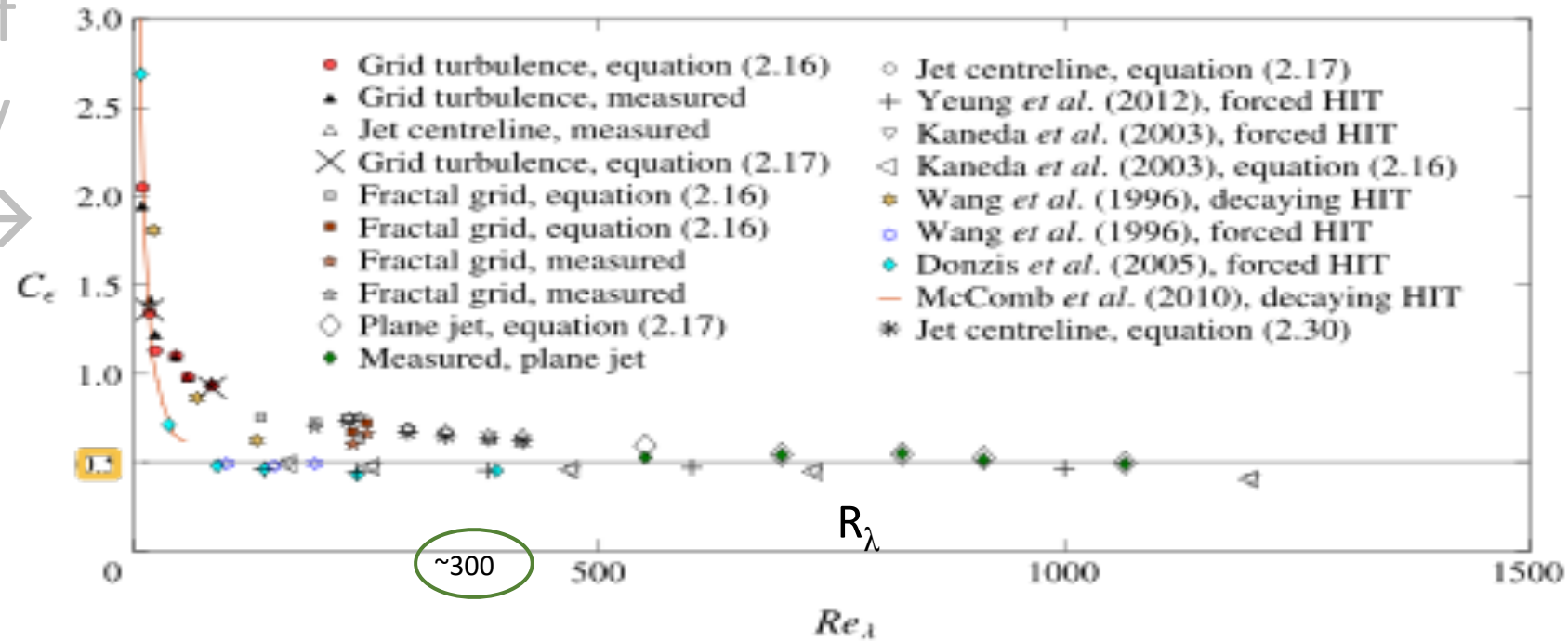


$$\cos \theta = \frac{\mathbf{f} \cdot \mathbf{g}}{|\mathbf{f}| |\mathbf{g}|}$$

*Servidio+ PRL 2008*

Saturation with  $Re$  of  
dissipation efficiency  
in fluids (Djenidi 2017) →

On the normalized dissipation parameter  $C_\epsilon$  in decaying turbulence



Saturation with Re of  
dissipation efficiency  
in fluids (Djenidi 2017) →

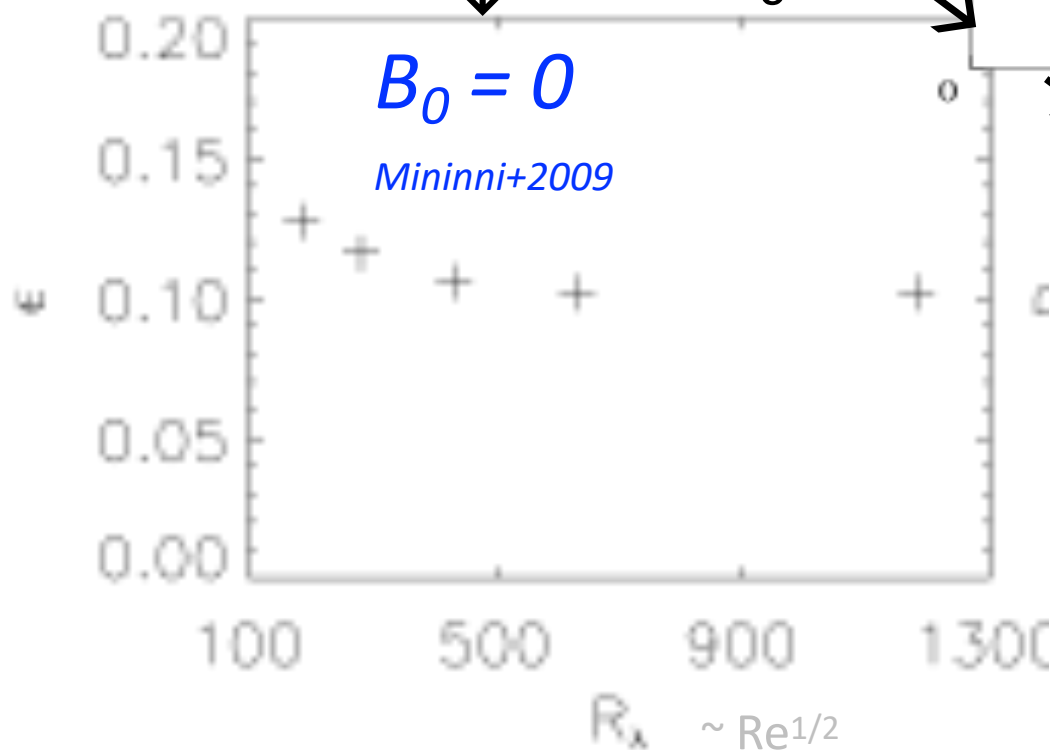
And 3D-MHD,  $B_0=0$



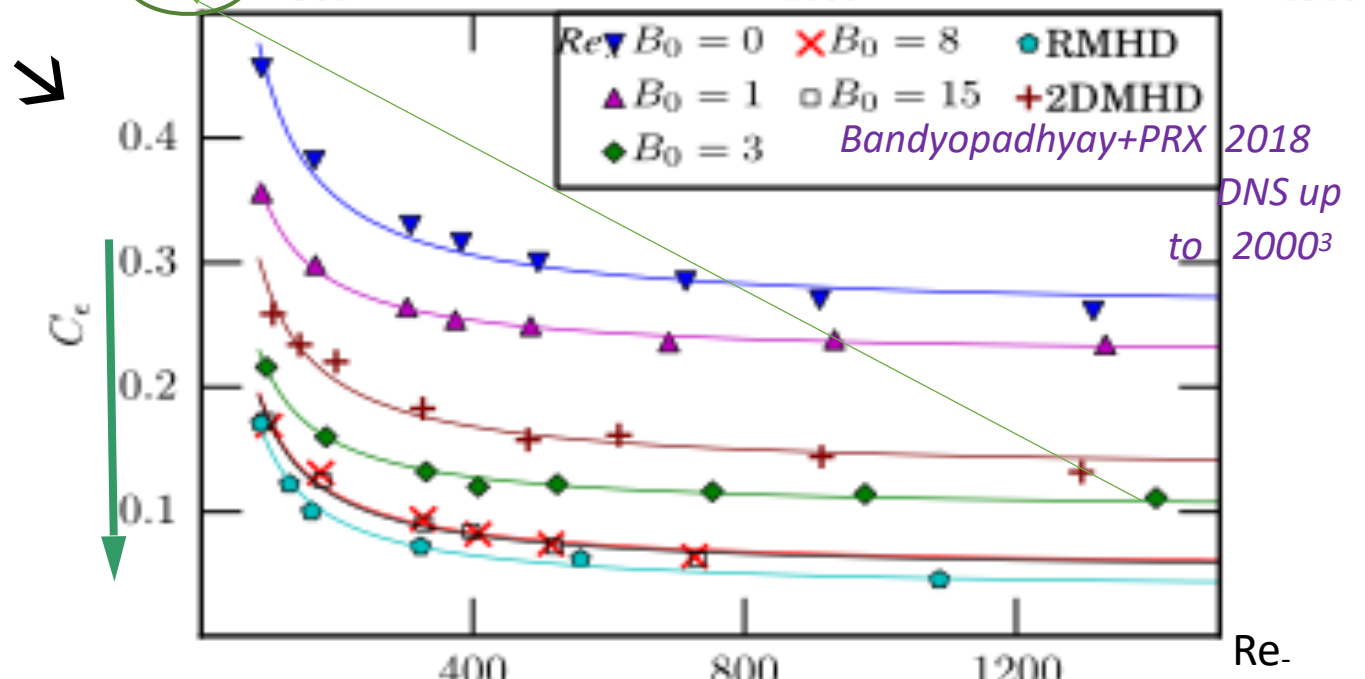
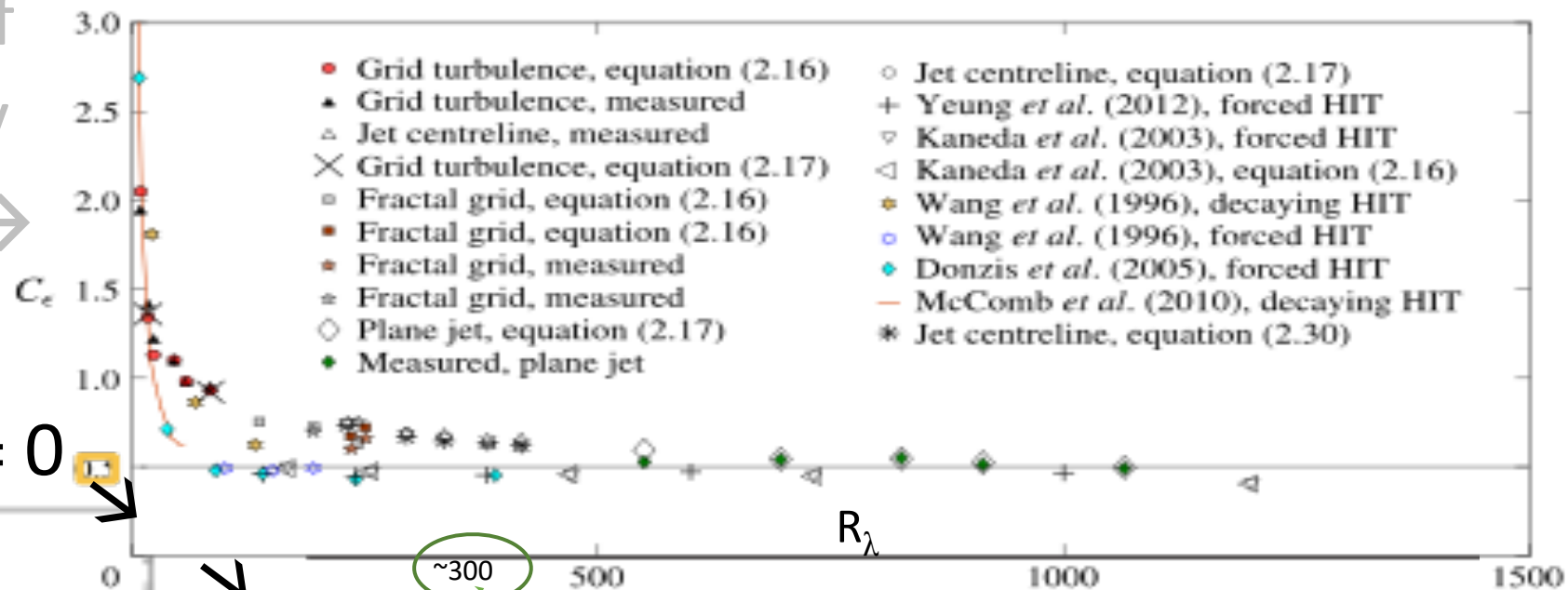
or  $B_0 \neq 0$

$B_0 = 0$

Mininni+2009



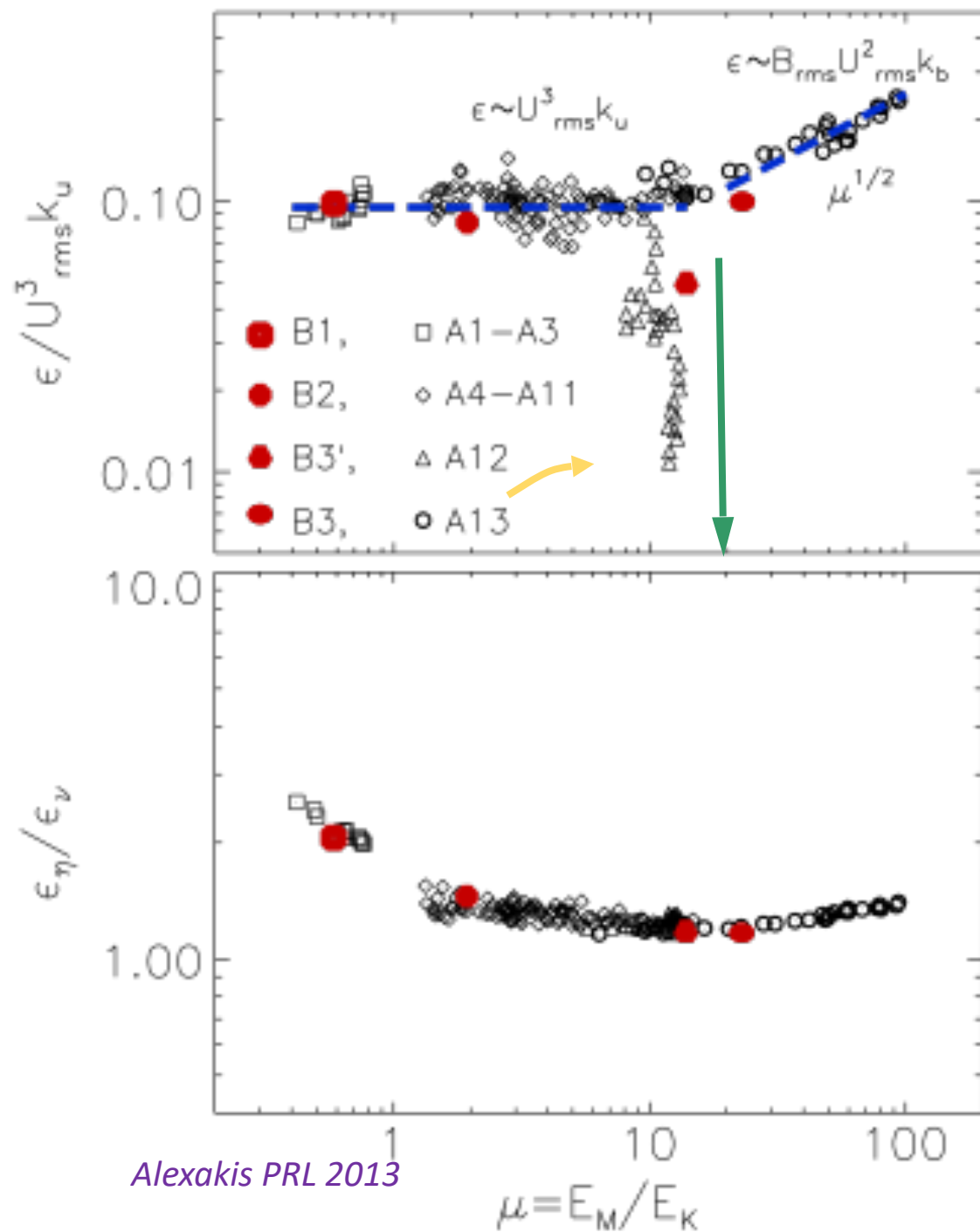
On the normalized dissipation parameter  $C_\epsilon$  in decaying turbulence



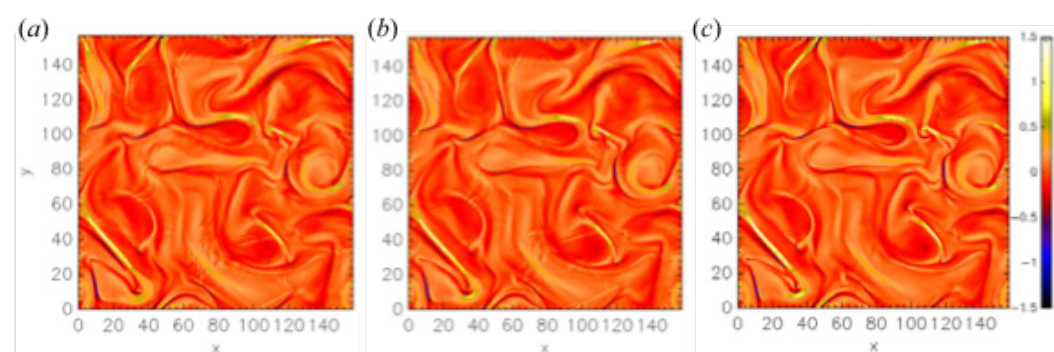
# Dissipation in MHD

DNS up to  $1024^3$  points

Change of regime for dominant magnetic energy and for fully helical forcing, with  
\*more\* dissipation





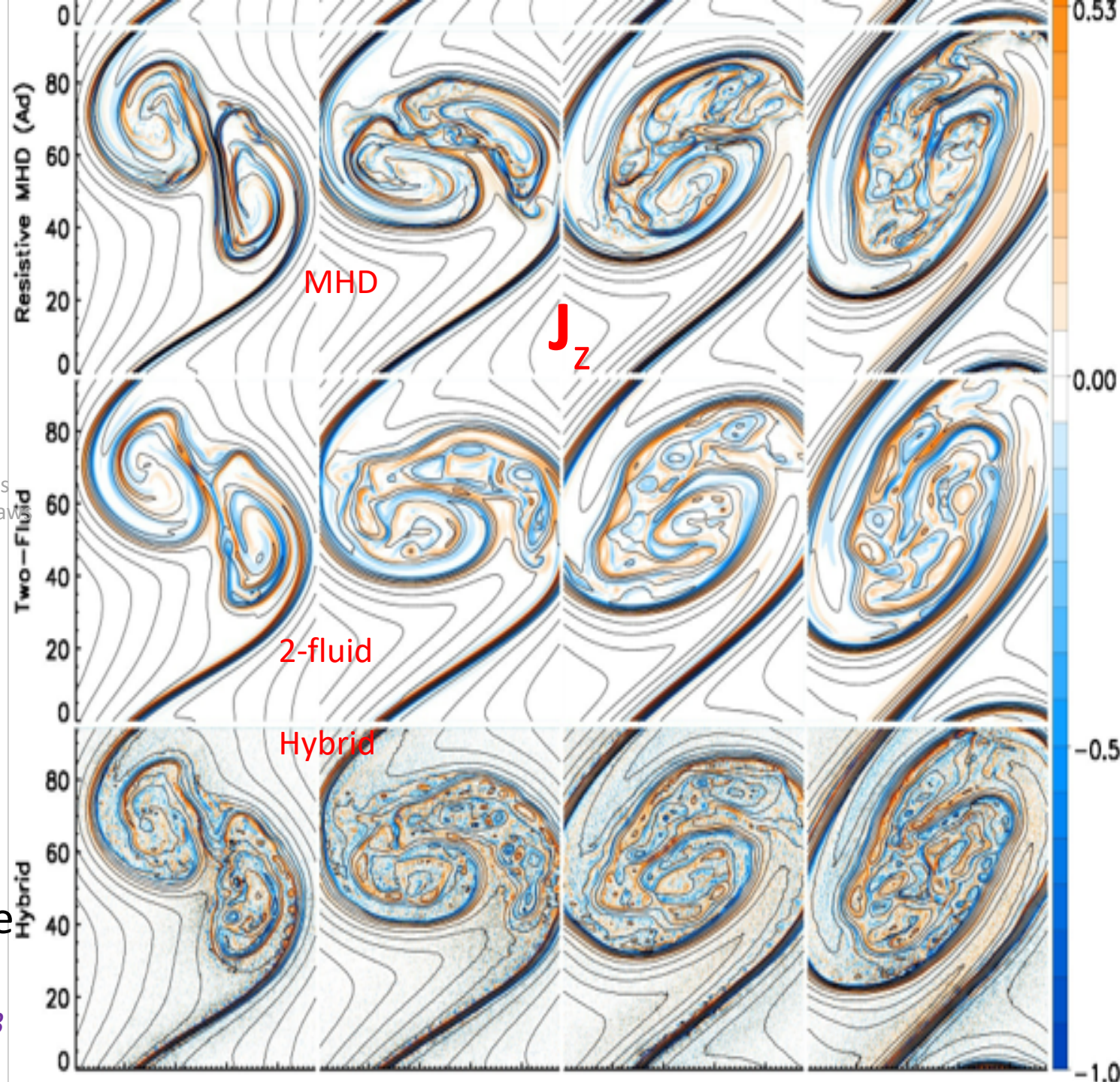


*Vasconez+21*  $J_z$  H-MHD    Landau fluid    Hybrid Vlasov Maxwell

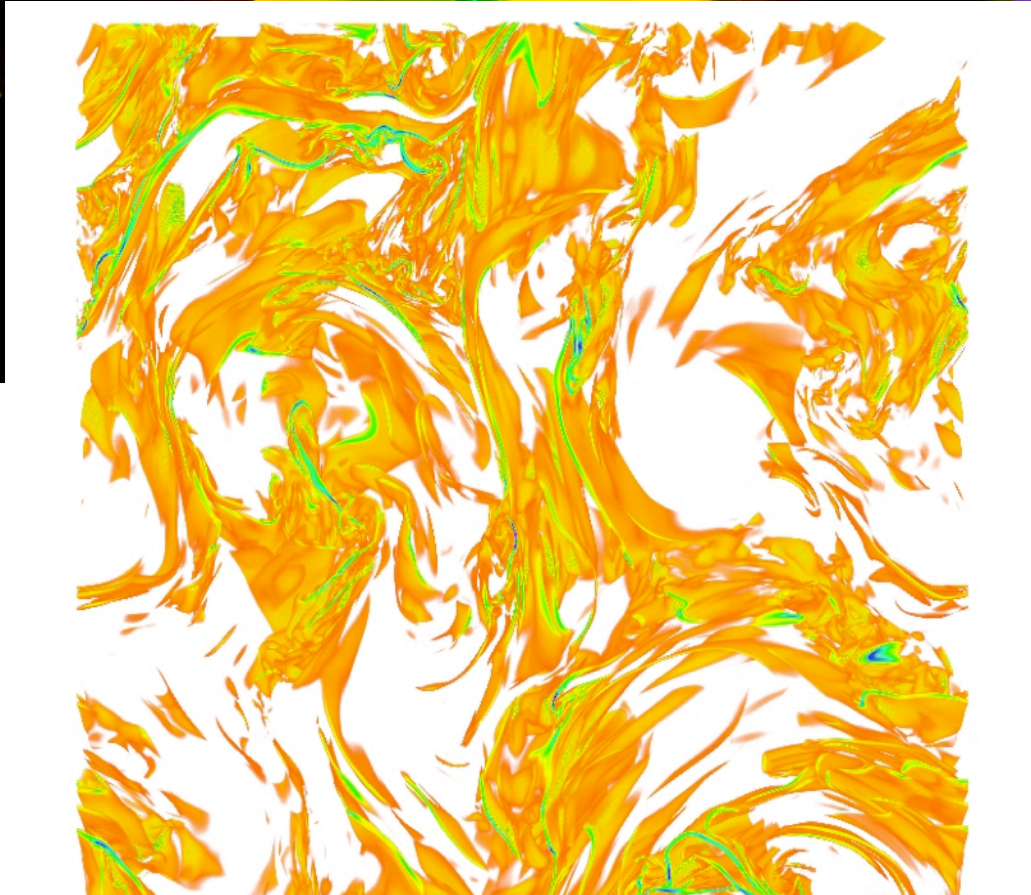
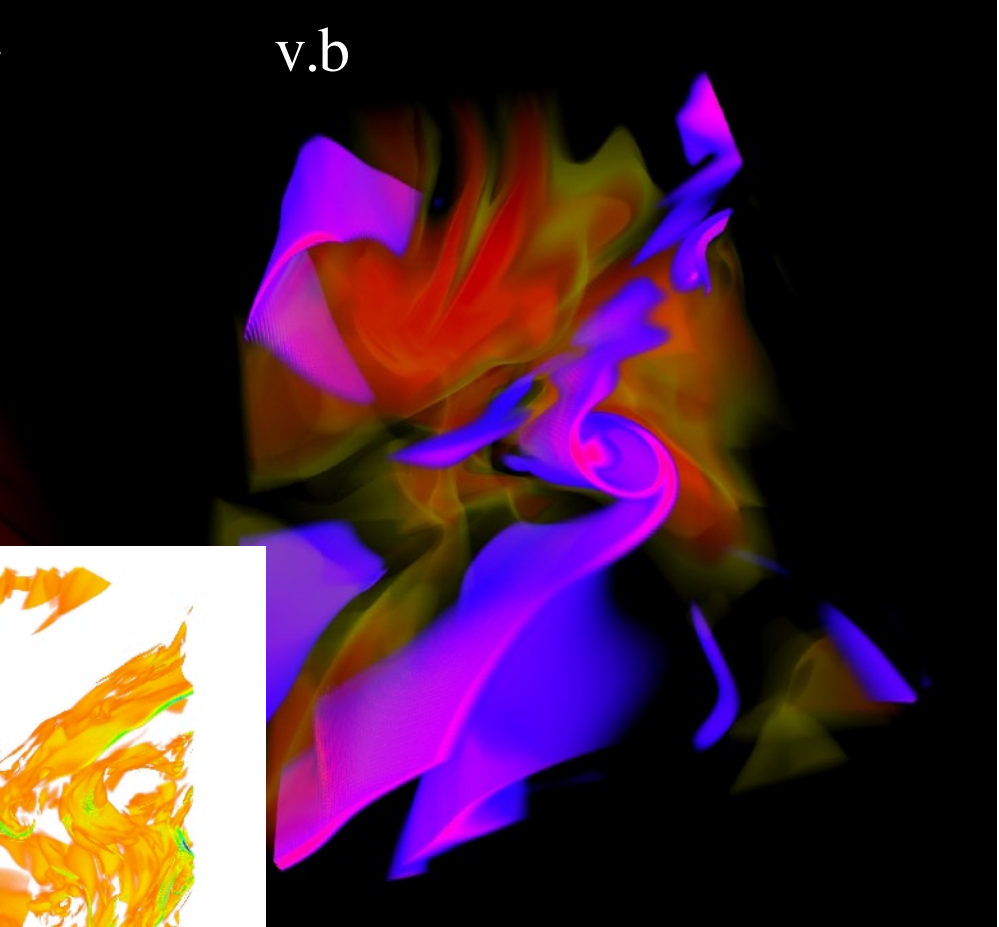
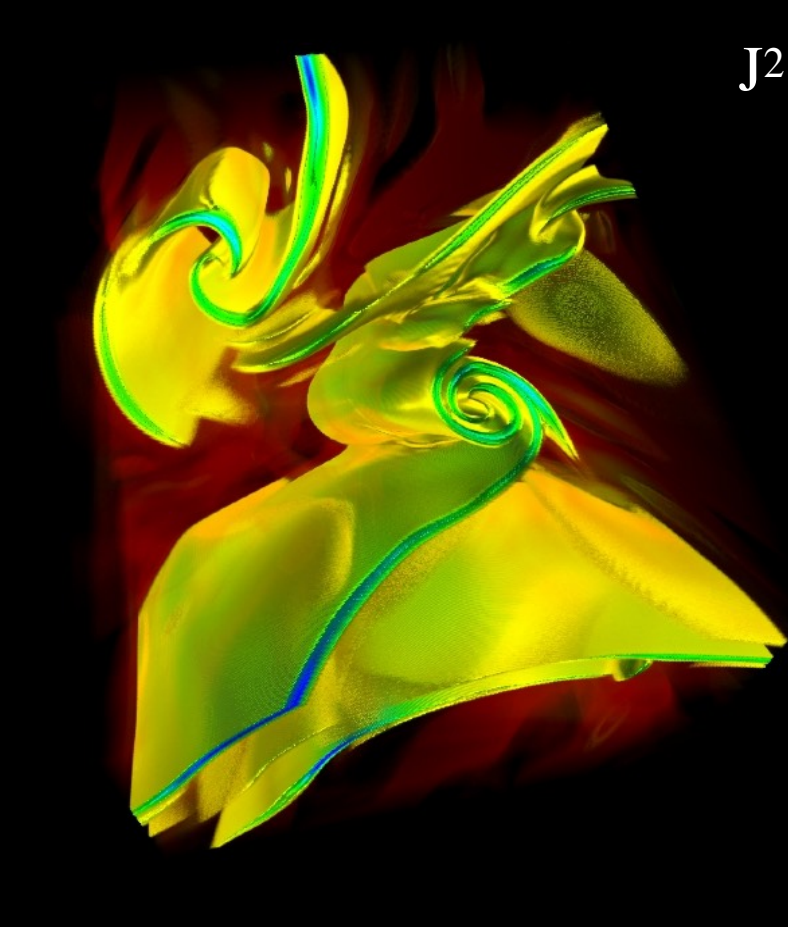
Landau fluid:  $\mathbf{V}$ ,  $P_{\perp}$ ,  $P_{\parallel}$ ,  $\mathbf{Q}$ ,  $\mathbf{B}$ , heat flux  $\mathbf{Q}_{\perp}$  &  $\mathbf{Q}_{\parallel}$ . Hybrid Vlasov Maxwell: ions are PIC,  $e$  are isothermal, quasi-neutrality; Two-fluid: ion-electron conservation laws

- Aspect ratio:  $L_0 / l_d$
- Non-locality of interactions
- Intermittency, non-Gaussian vorticity and current PDFs, *as well as sometimes  $v$  &  $b$*

Vortex filaments in superfluid turbulence  
Current sheets and filaments in MHD+  
*Internal small-scale structures* *Henri+2013*







Global current at early times

And at later times →

*Mininni+ 2006*



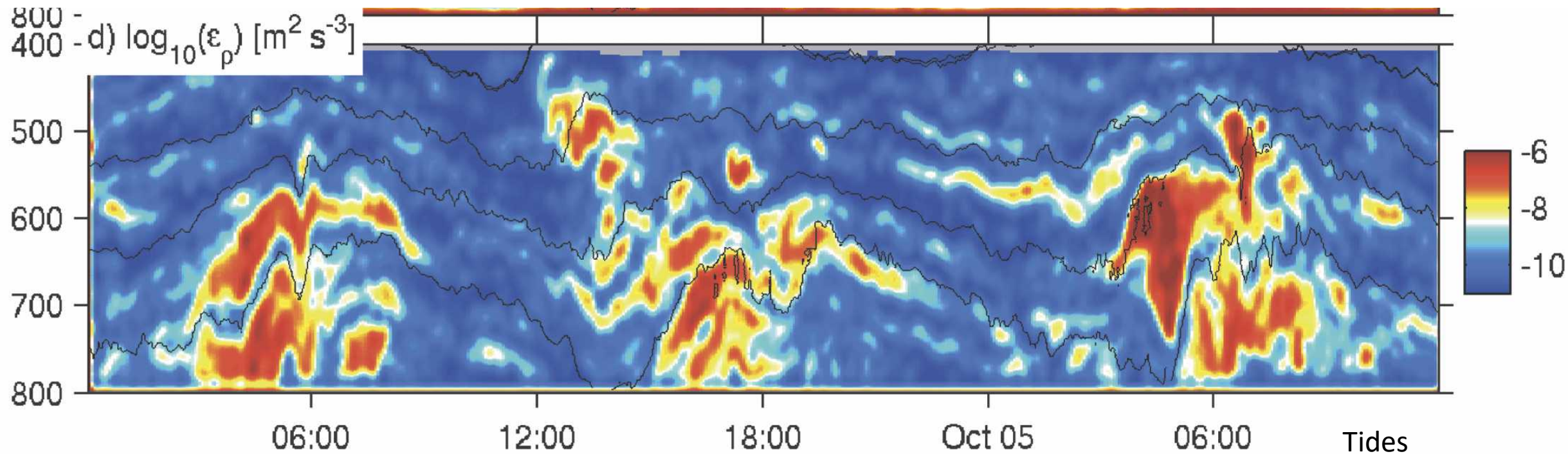


# Ocean, measured dissipation rate $\varepsilon$ in the Hawaiian ridge *(Klymak+2008)*

$$U=0.1\text{ms}^{-1}, \text{Re} \sim 10^8, L_0 \sim 10^3\text{m}, \langle \varepsilon_v \rangle \sim 10^{-10} \rightarrow \tau_{\text{NL}}=L_0/U \sim 2.8 \text{ hr}$$

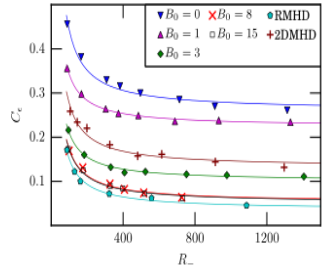
$$\text{Brunt-V frequency } N=0.001\text{s}^{-1} \rightarrow \text{Fr} \sim 0.1$$

$$\text{Active sites: } \varepsilon_v \sim 10^{-6} \sim \varepsilon_D \sim U^3/L_0$$



# Dissipation efficiency in rotating stratified turbulence, $Ro, Fr < 1$

RIDDHI BANDYOPADHYAY *et al.*



1024<sup>3</sup> DNSs, Reynolds nb.,  $Re \sim 8000$   
 Froude number  $Fr = U/[LN]$ ,  
 Rossby nb.  $Ro = U/[Lf]$ , variable  $N/f > 2$   
*Pouquet+ JFM 844, 2018; PoF 31, 2019*

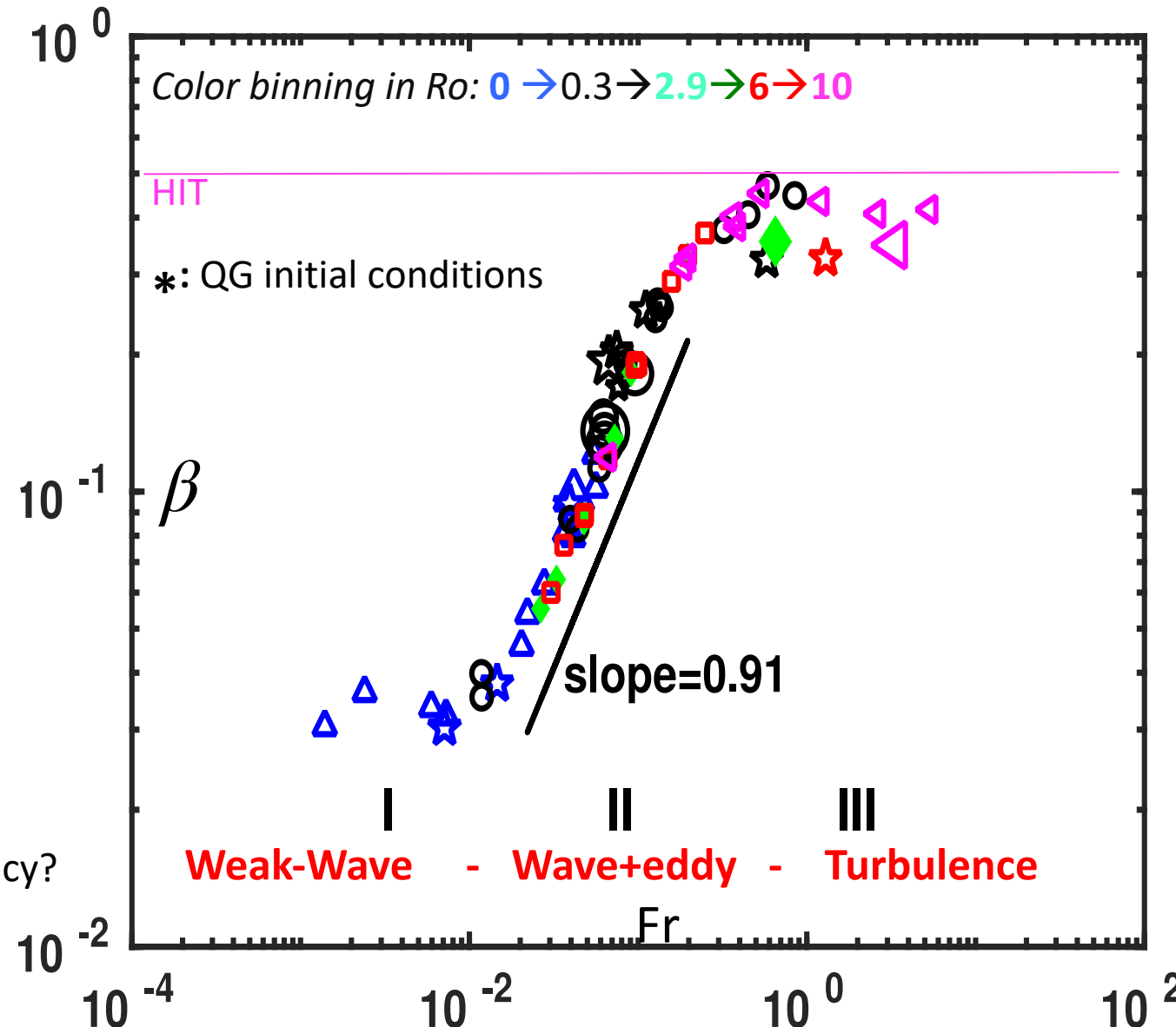
$$\beta \equiv \varepsilon_v / \varepsilon_D \sim Fr \quad \text{with} \quad \varepsilon_D = U^3/L$$

Weak (wave) turbulence **\*phenomenology\***

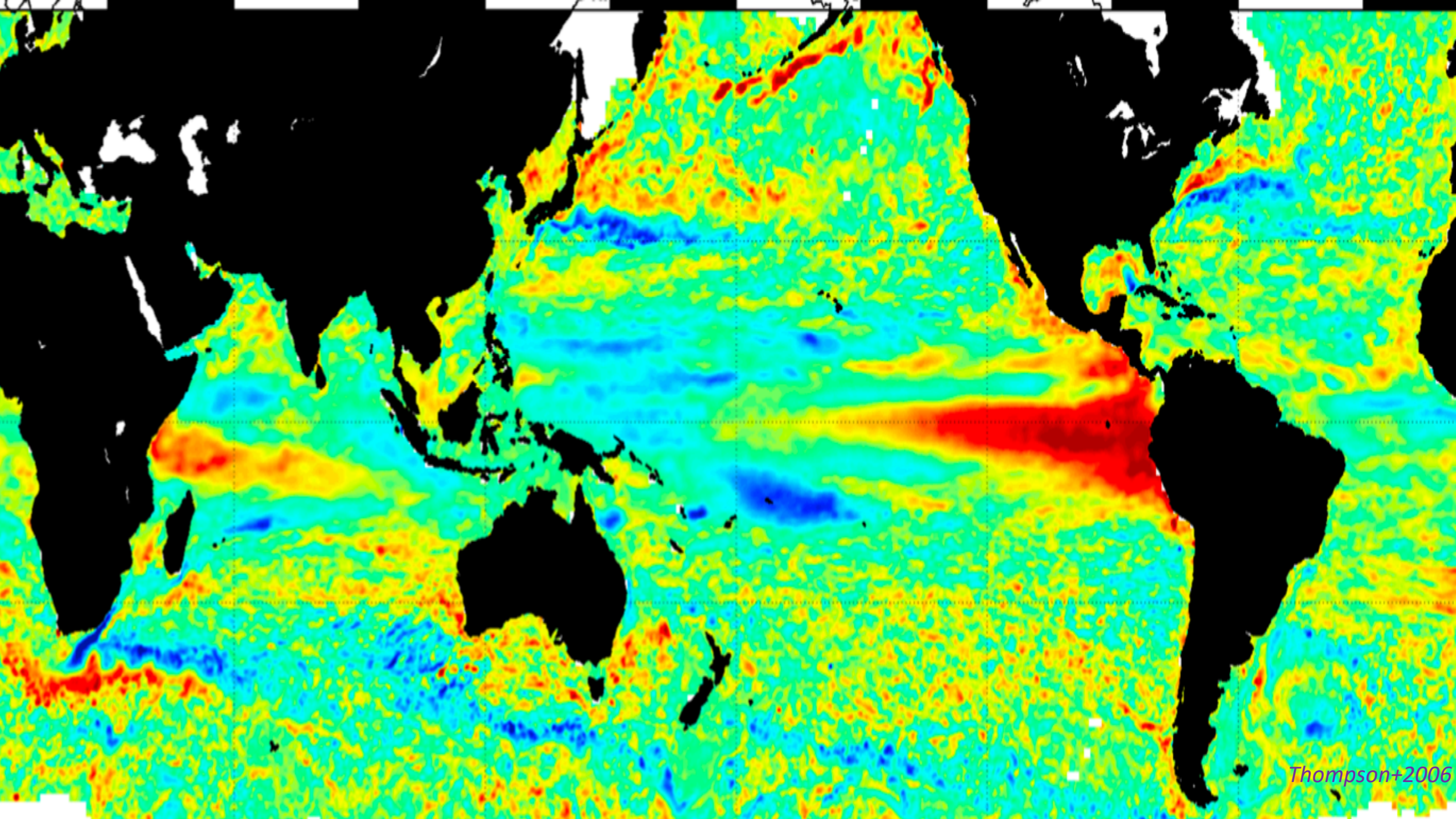
$$\tau_{\text{transfer}} = \tau_{\text{NL}} * [\tau_{\text{NL}} / \tau_{\text{wave}}] = \tau_{\text{NL}} / Fr$$

$$\beta = \varepsilon_v / \varepsilon_D = [U^2 / \tau_{\text{transfer}}] \cdot [L / U^3] \sim Fr$$

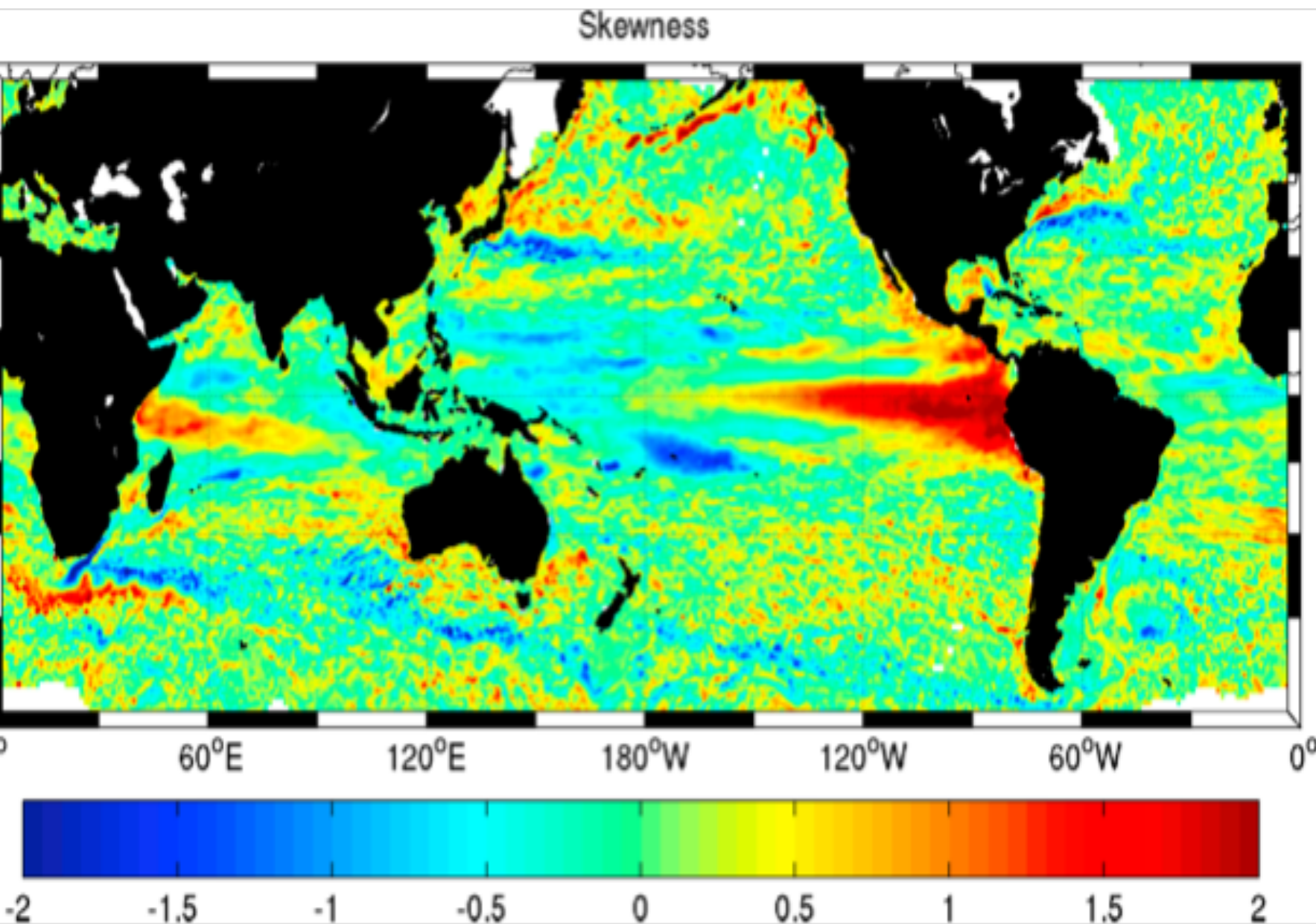
0.91 vs. 1 slope: Insufficient data? Anisotropy? Intermittency?  
 Rotation?  $Re$ ? Or is it a critical scaling?



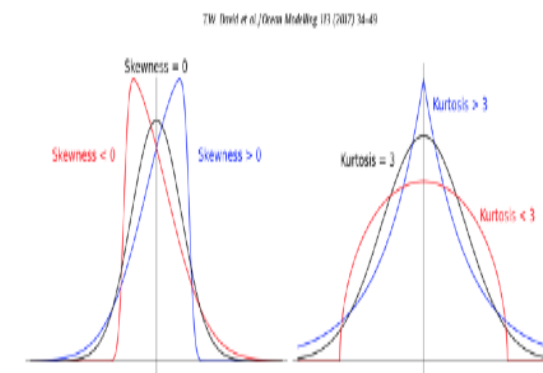


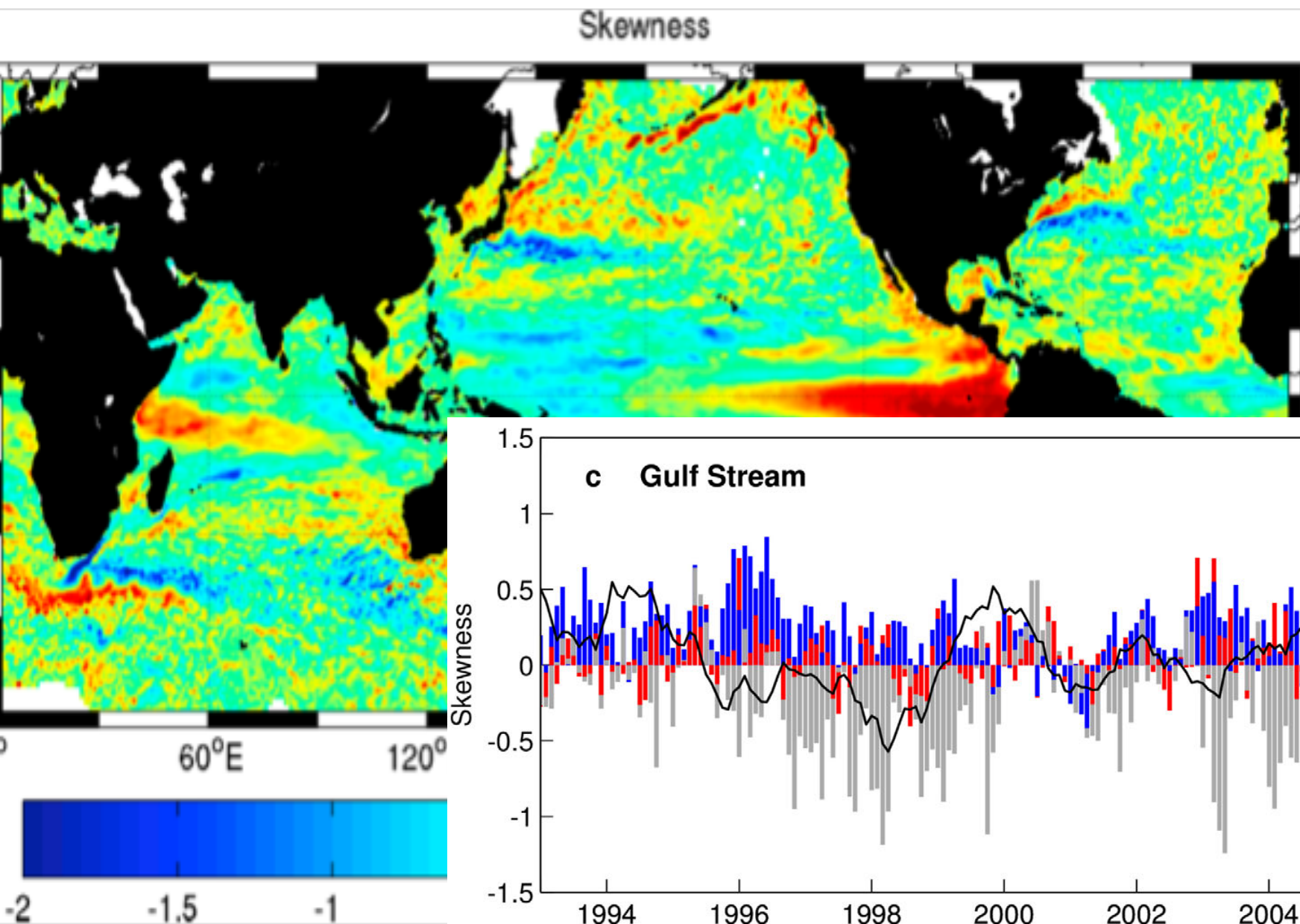




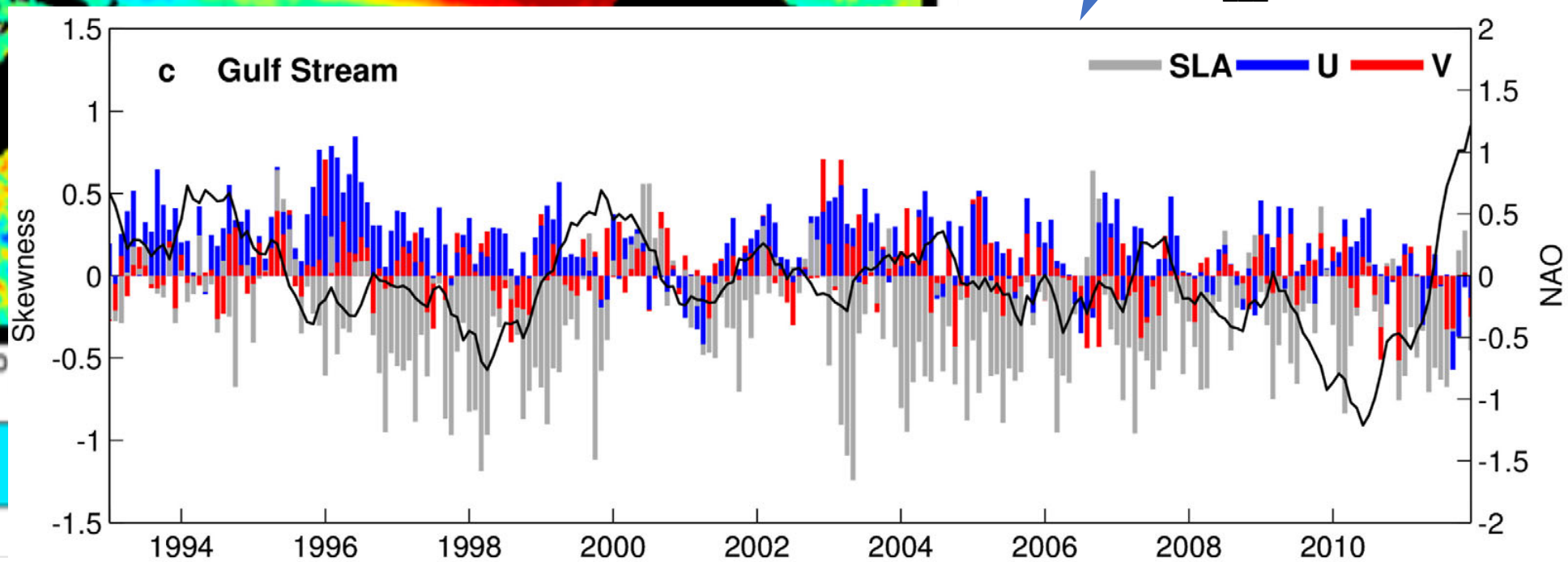


Weekly-gridded 7 years data  
Topex-Poseidon+  
1/3 degree res.  
Error in skewness  $\sim 0.2$   
El Niño 1997-98  
Meso-scale variability



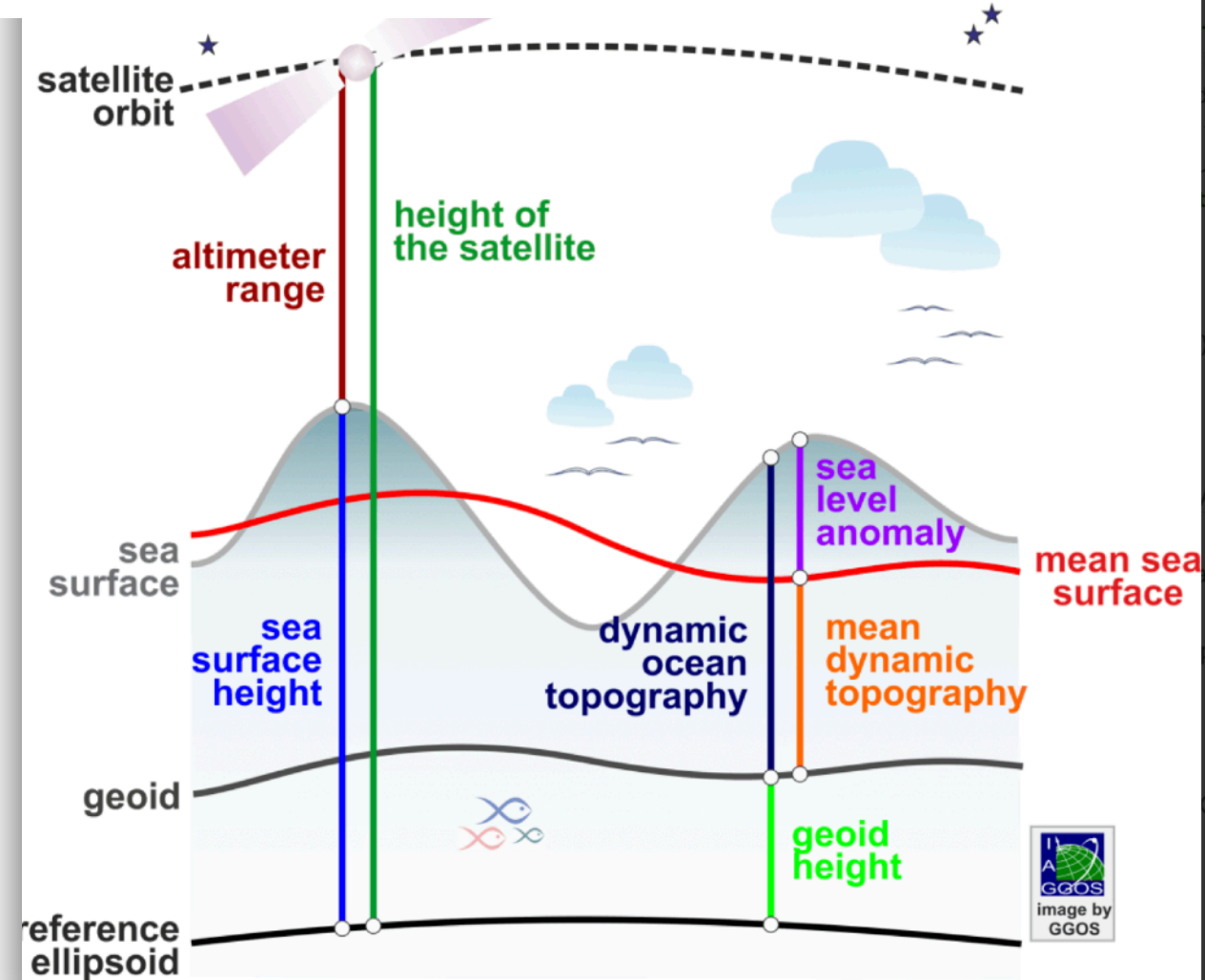
*Biri+2015*

Skewness for SLA  
(sea level anomaly) &  
horizontal velocities  
& \_\_\_ North-Atlantic Oscillation





**Satellite altimetry** systems are designed to map the sea surface. These systems **measure the satellite-to-sea surface** round-trip travel time of radar or light pulses to determine the height of the satellite (altimetric range) above the instantaneous sea surface. The difference between the satellite altitude above a reference surface and the altimetric range provides the sea surface height with respect to the same reference surface. The range from the satellite to the sea surface is **corrected for various components** of the atmospheric refraction and to mitigate effects caused by instrumental biases and sea state induced systematics. A number of corrections due to different geophysical effects are also taken into account. **Different products are distinguished:**

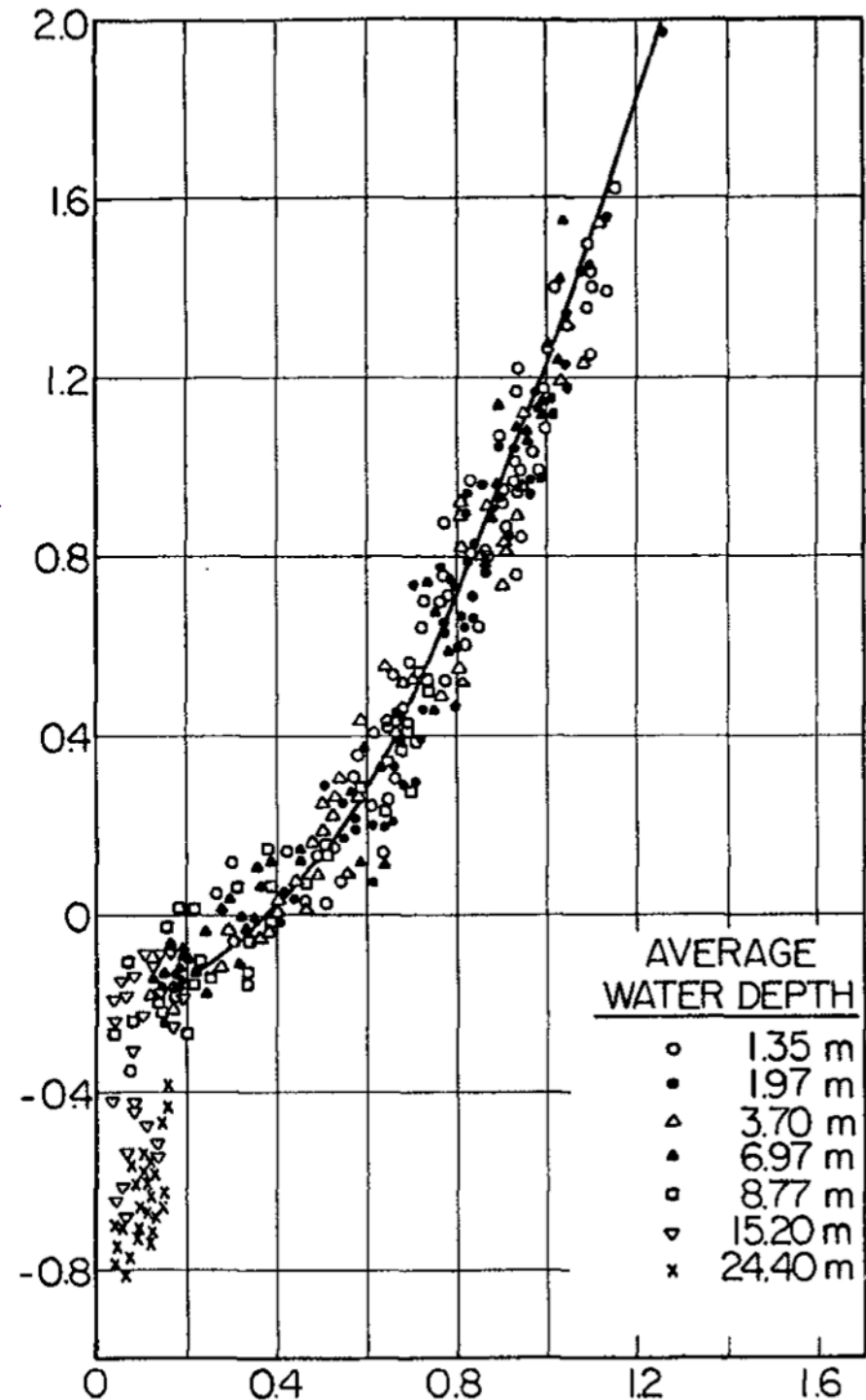


Breaking waves on a sloping beach

Longuet-Higgins 1963

$K(S)$  for sea surface deviation

Ochi 1984 →



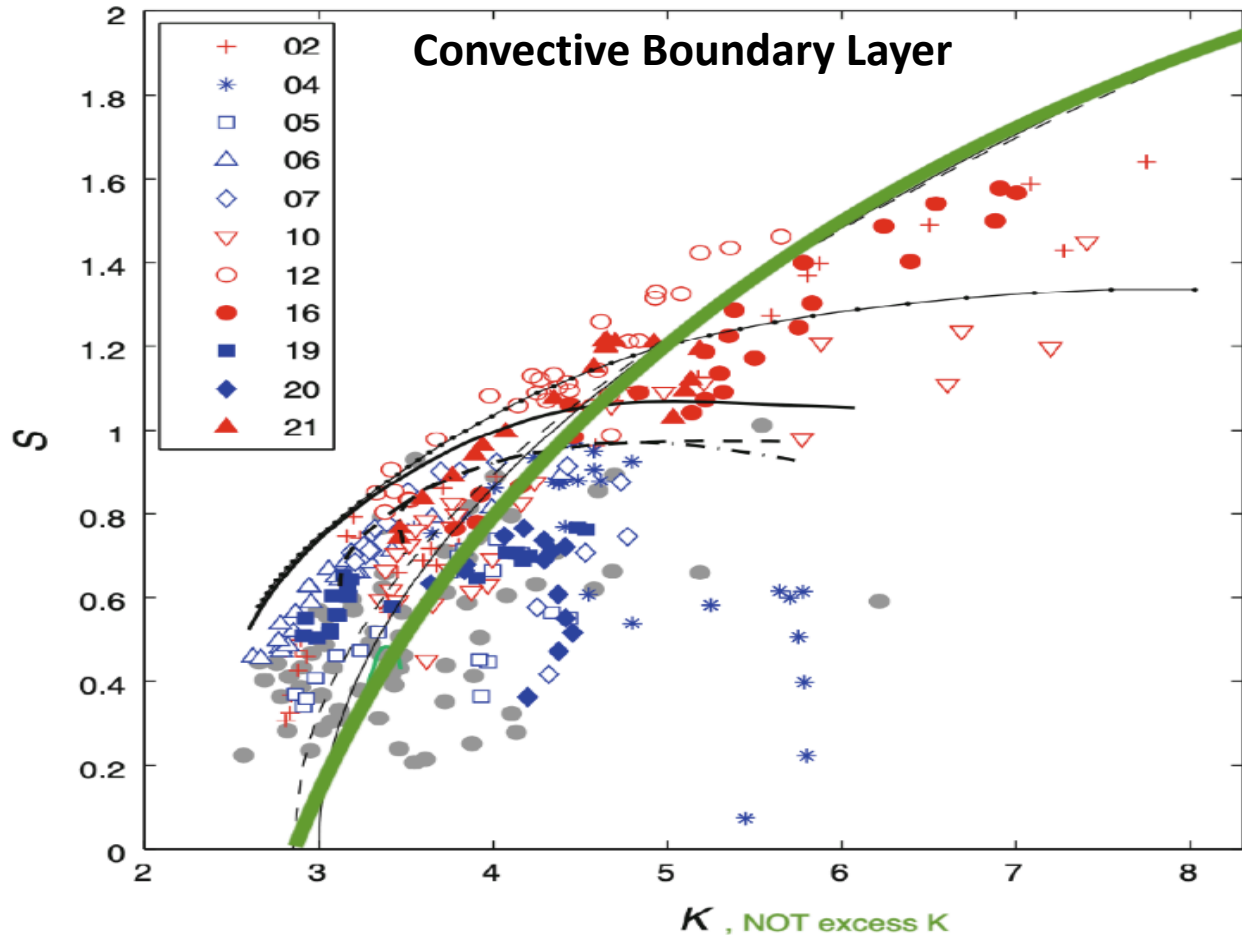


Lenschow+, 1994: PDF = G + αG<sup>2</sup>, G Gaussian.

Large α: PDF~ 3(S<sup>2</sup>/2 + 1), S<sub>max</sub>~ 2.83, K<sub>max</sub>~ 1

Convective Boundary Layer, S(K)

Lenschow+'12: Measurements and LES



Fit  
 $K - 3 = K_{\text{excess}}$   
 $\sim 1.4S^2 - 0.15$

Lines: LES  
Circles: Data  
(LIFT, AMMA)  
Lidar In Flat  
Terrain, 1996  
African  
Monsoon, 2006

*Lenschow+, 1994*:  $PDF = G + \alpha G^2$ ,  $G$  Gaussian.

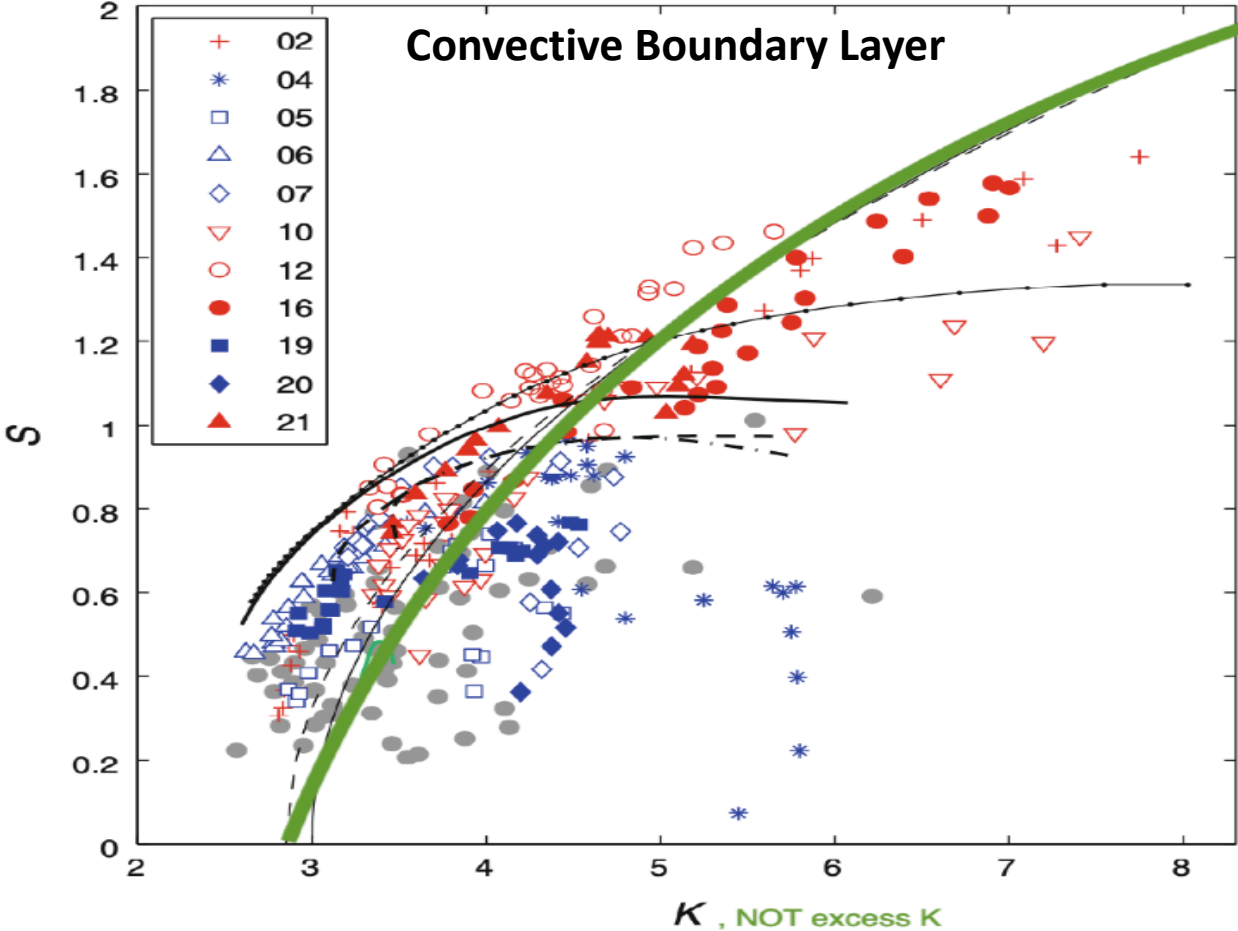
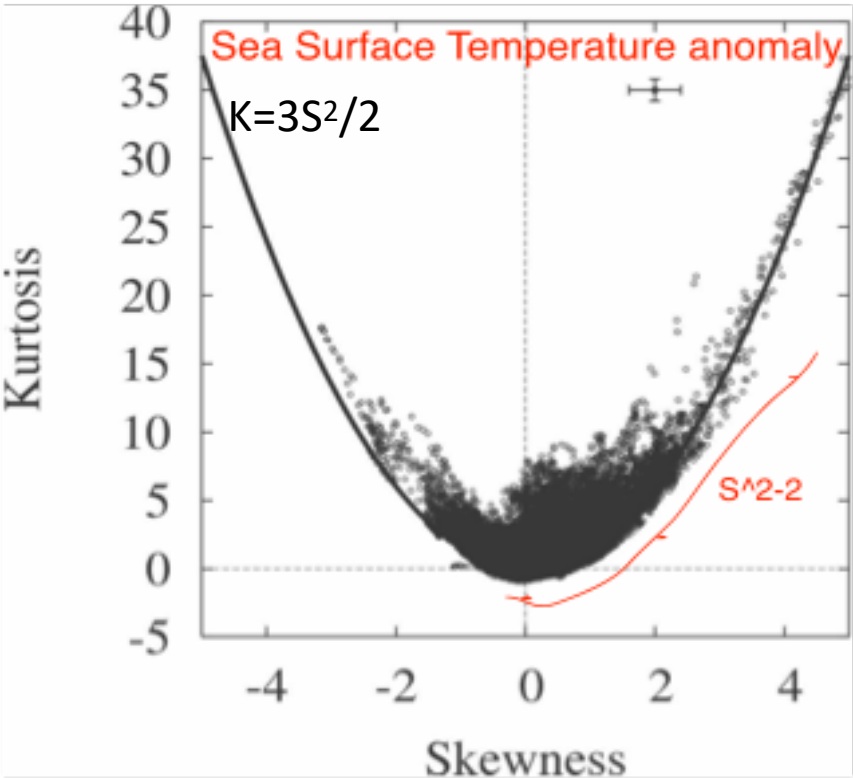
Large  $\alpha$ :  $PDF \sim 3(S^2/2 + 1)$ ,  $S_{max} \sim 2.83$ ,  $K_{max} \sim 1$

Convective Boundary Layer,  $S(K)$  →

*Lenschow+'12*: Measurements and LES

Sea-Surface Temperature anomaly

*Sura+2008*



Fit  
 $K - 3 = K_{excess}$   
 $\sim 1.4S^2 - 0.15$

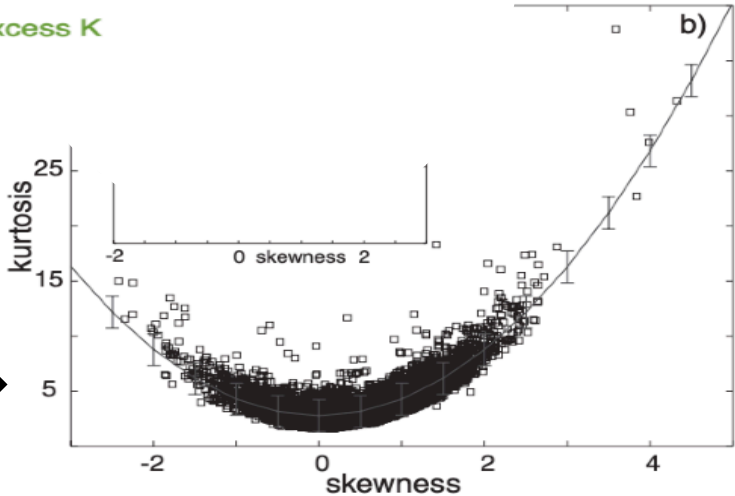
Lines: LES  
 Circles: Data (LIFT, AMMA)  
 Lidar In Flat Terrain, 1996  
 African Monsoon, 2006

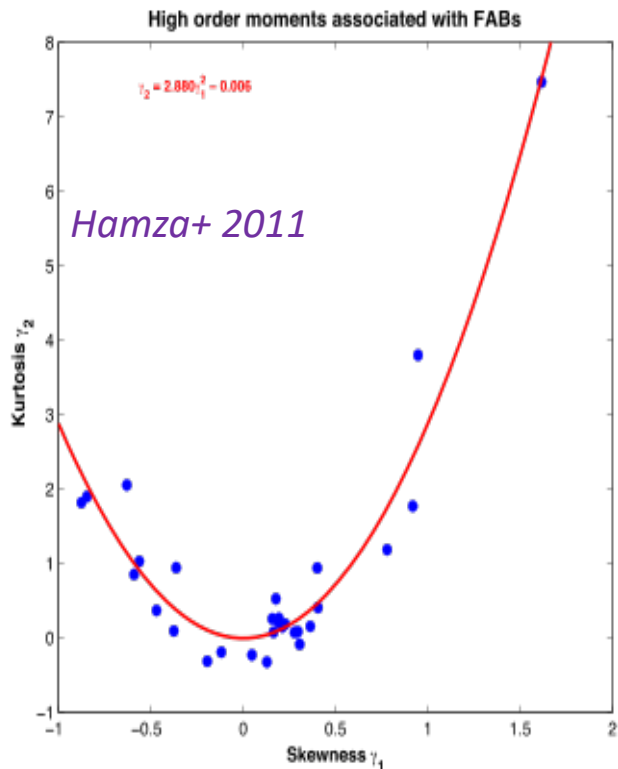
$K(S)$

Toroidal magnetized plasma

*Labit+2007*

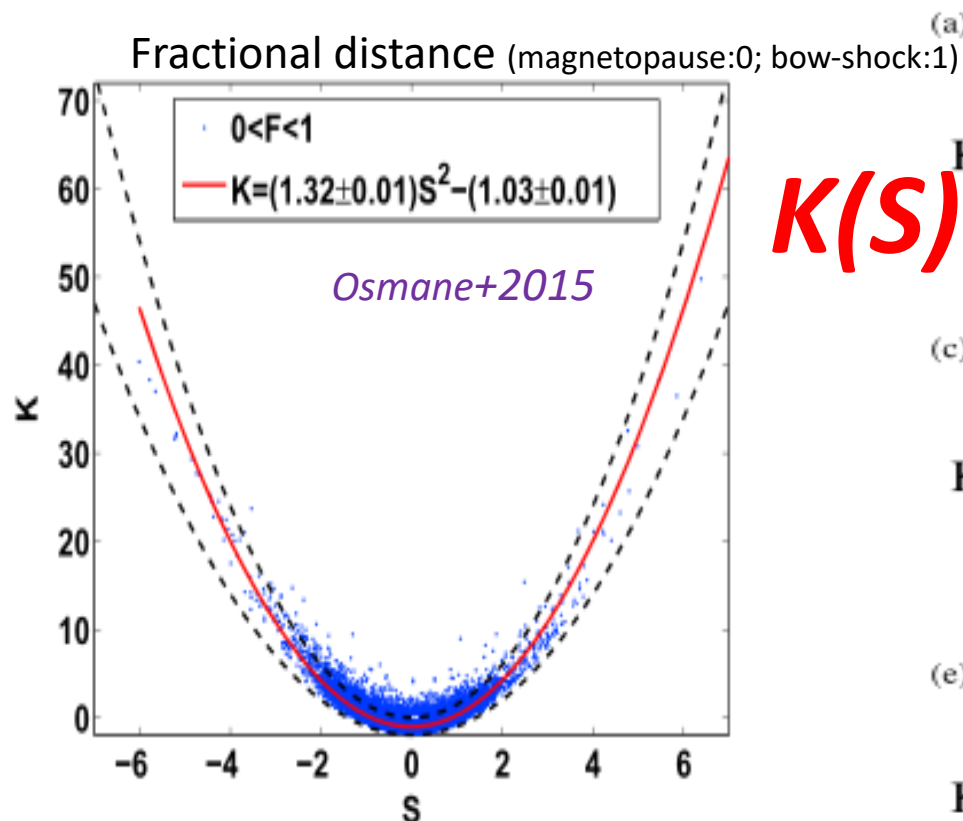
→





Hamza+ 2011

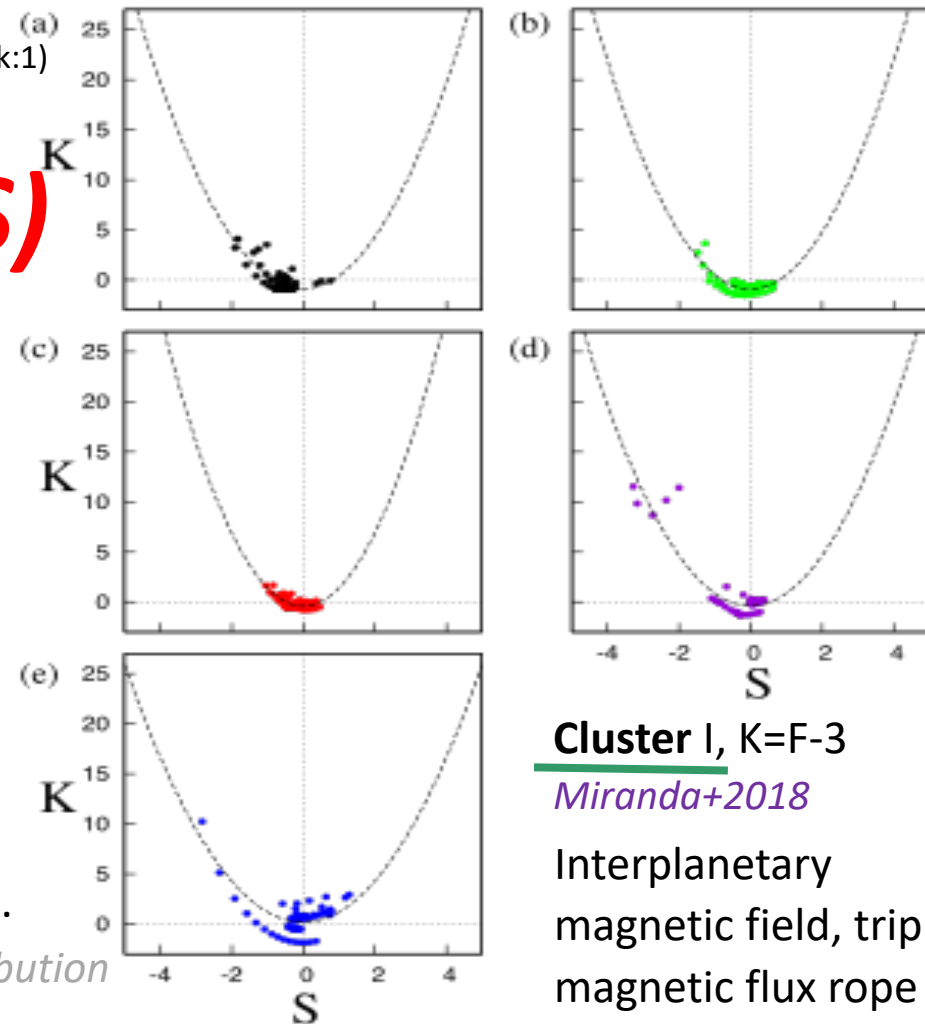
Ion distribution, **Cluster**  
Quasi-perp. bow shock



Osmane+2015

Earth's magnetosheath, **Themis**. 7 years,  
~ 50000 points, mirror modes fluctuations.

$K(S)$

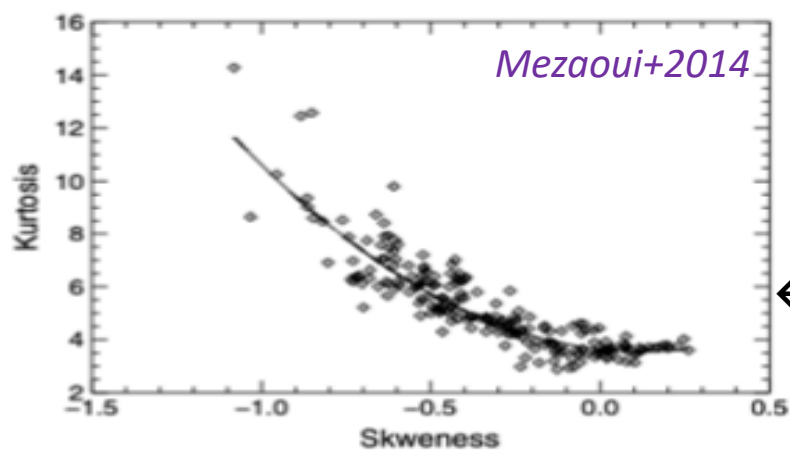


**Cluster I, K=F-3**

Miranda+2018

Interplanetary  
magnetic field, triple  
magnetic flux rope

Black dots: constraints for  $\beta$  distribution



High-latitude ionospheric turbulence  
Power fluctuations of electron density  
 $K \sim 5.5 S^2 + \dots$

Interval	$K = \alpha S^2 + \beta$
$R_1$	$K = 1.29S^2 - 0.86$
$I_{12}$	$K = 1.42S^2 - 0.92$
$R_2$	$K = 1.82S^2 - 0.42$
$I_{23}$	$K = 1.26S^2 - 0.40$
$R_3$	$K = 1.03S^2 + 0.17$

interior  
interface

*Maier+2017* : LITTLETHINGS dwarf irregular galaxies ISM

Characteristics of turbulence through  
3<sup>rd</sup> and 4<sup>th</sup>-order normalized moments of H I

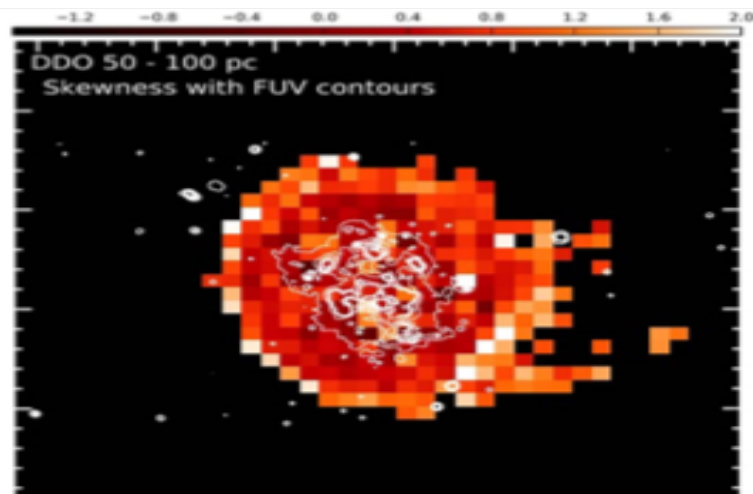
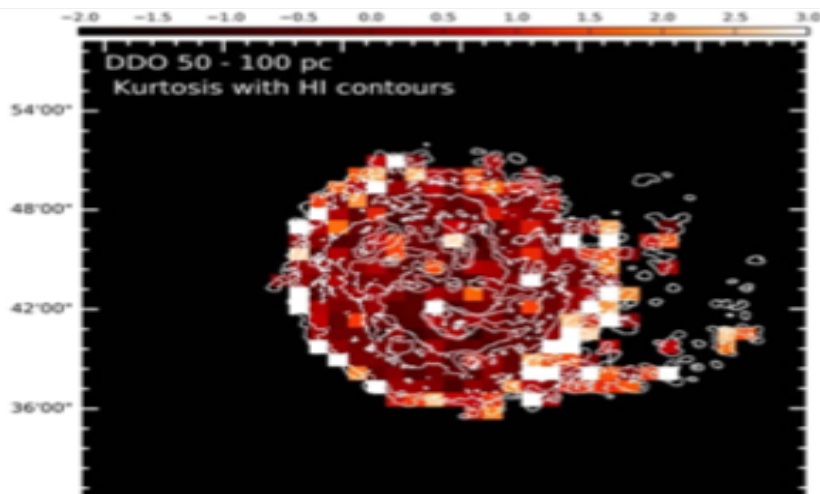
Local Irregulars That Trace Luminosity Extremes: The H I Nearby Galaxy Survey

Integrated HI column density, Very Large Array

AL JOURNAL, 153:163 (35pp), 2017 April

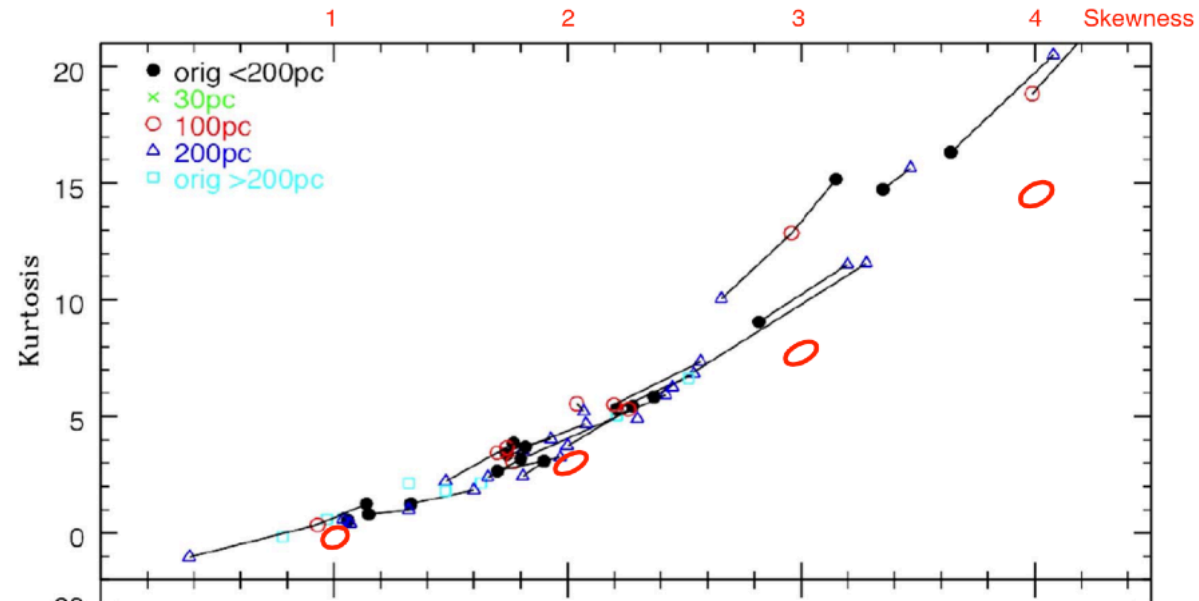
Kurtosis

Skewness

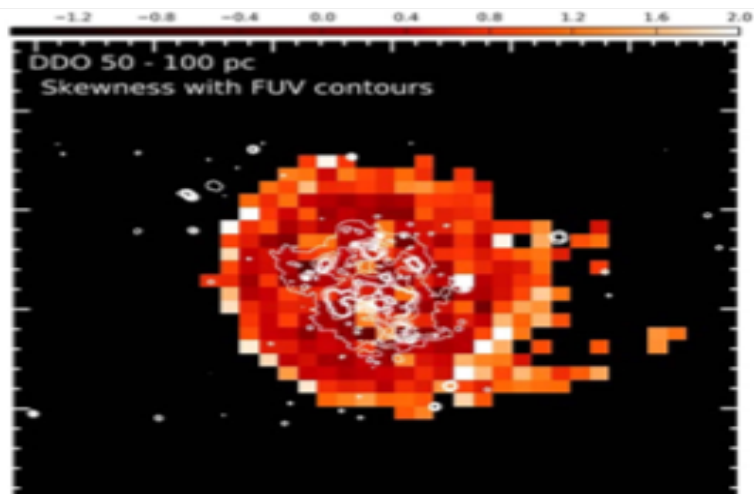
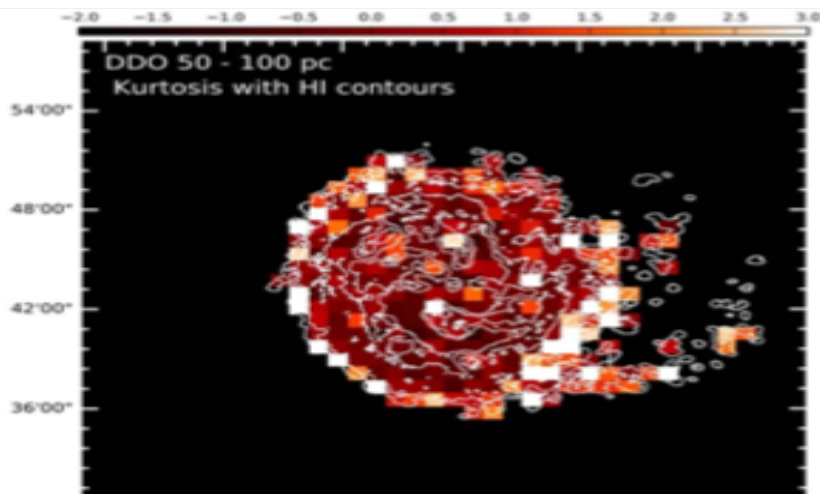


$$K = aS^2 + b \quad [Cauchy-Schwarz: K \geq S^2 - 3]$$

$$K \geq S^2 - 6/5 \quad (Klaassen+2000, \text{unimodal symmetric})$$

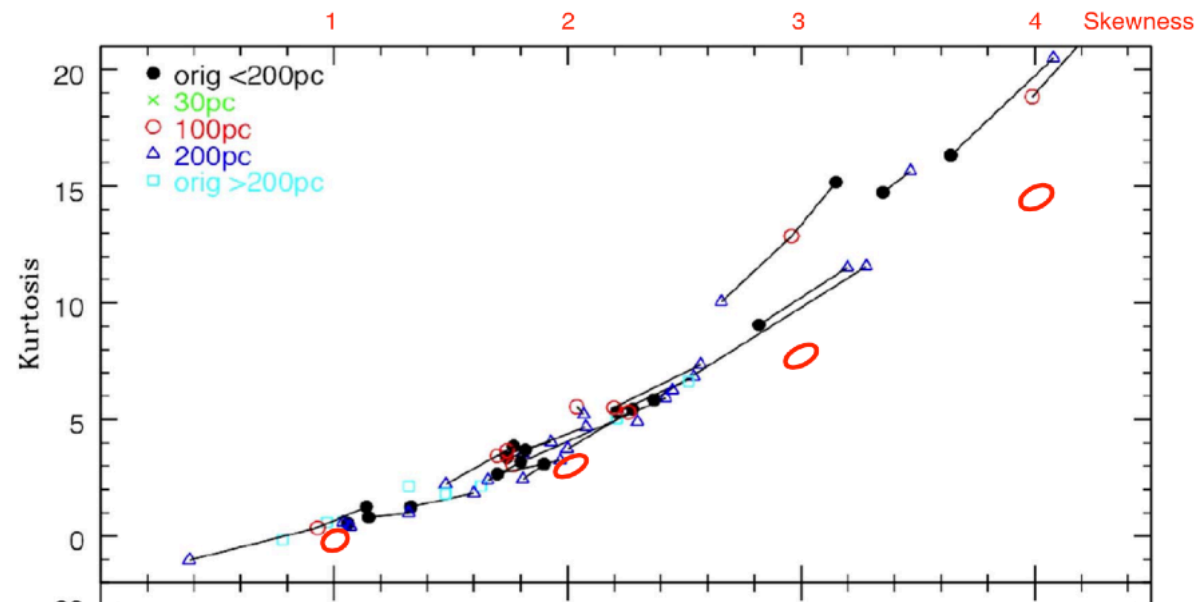


AL JOURNAL, 153:163 (35pp), 2017 April

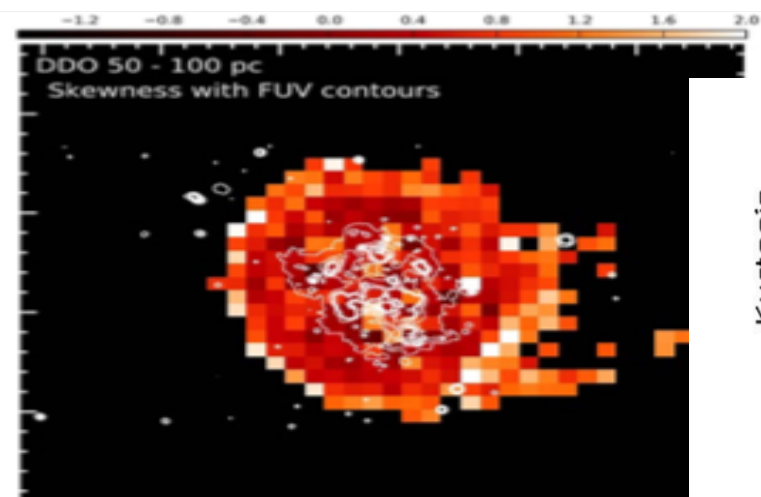
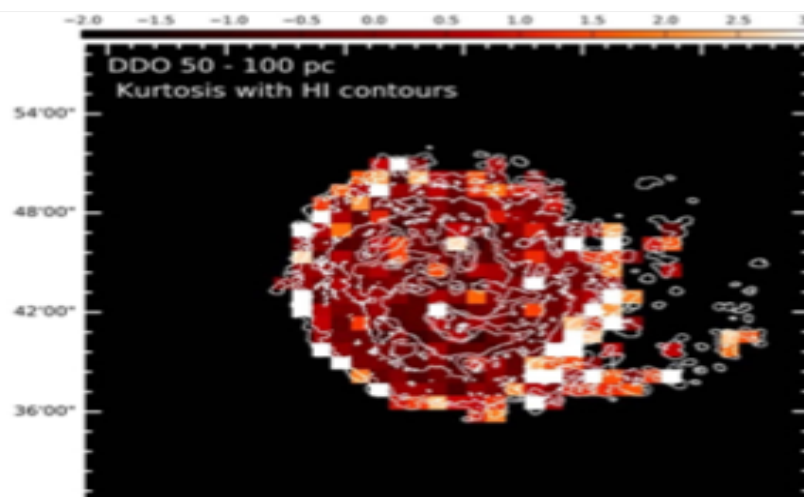


$$K = aS^2 + b \quad [\text{Cauchy-Schwarz: } K \geq S^2 - 3]$$

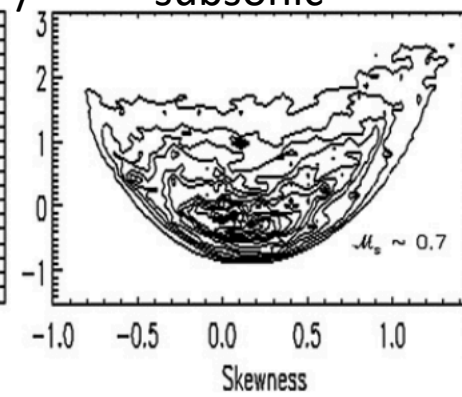
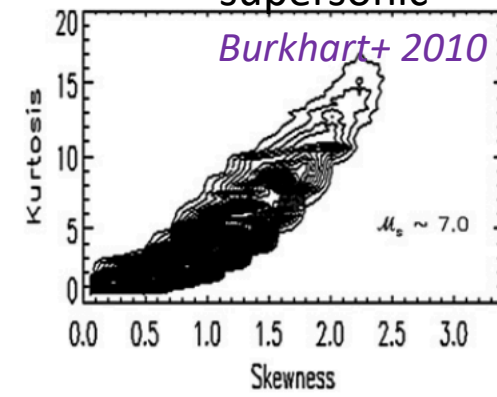
$$K \geq S^2 - 6/5 \quad (\text{Klaassen+2000, unimodal symmetric})$$



AL JOURNAL, 153:163 (35pp), 2017 April

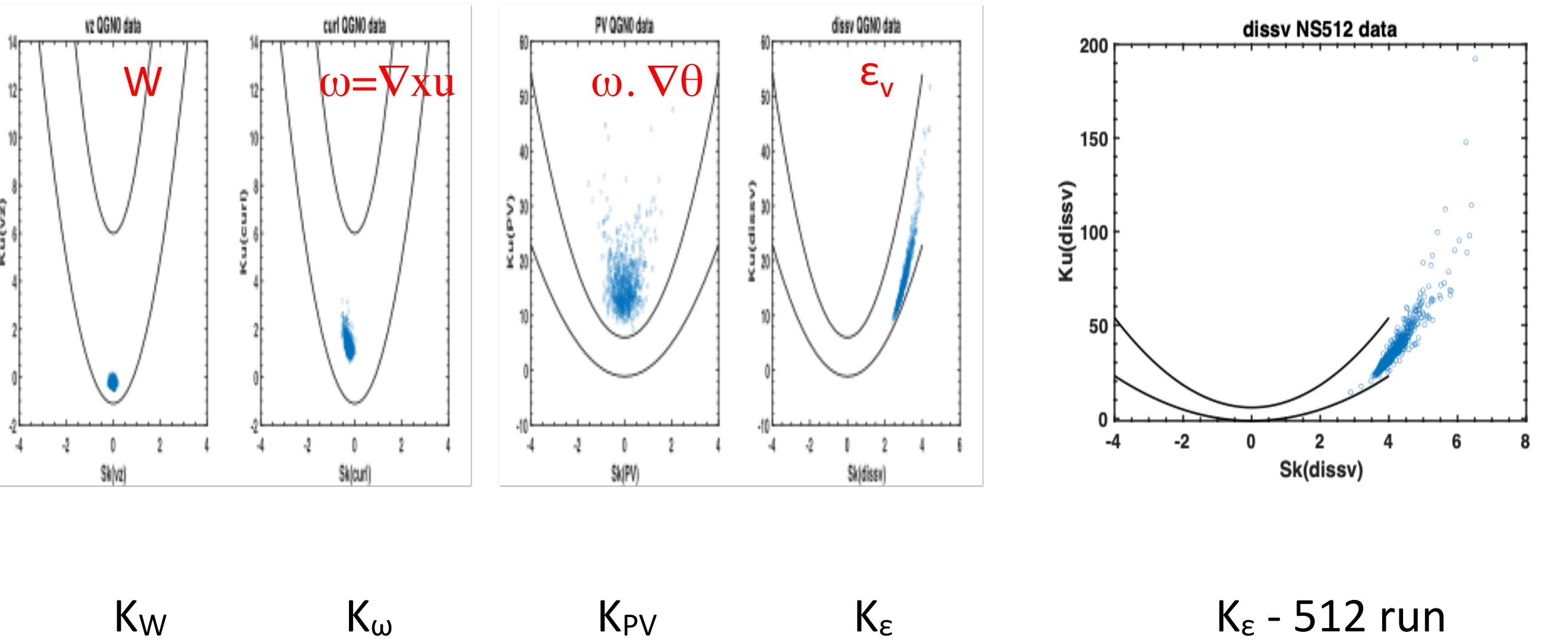


K(S) for HI, isothermal runs for SMC  
supersonic / subsonic





Navier-Stokes:  $128^3$ ,  $Re \sim 200$ ,  $R_\lambda \sim 28$ ,  $800\tau_{NL}$  &  $512^3$ ,  $Re \sim 800$ ,  $R_\lambda \sim 53$ ,  $80\tau_{NL}$



$$\begin{aligned}\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} &= -\nabla p - N\theta \mathbf{z}^* + 2\mathbf{u} \times \boldsymbol{\Omega} + \nu \nabla^2 \mathbf{u} \\ \partial_t \theta + (\mathbf{u} \cdot \nabla) \theta &= Nw + \kappa \nabla^2 \theta\end{aligned}$$

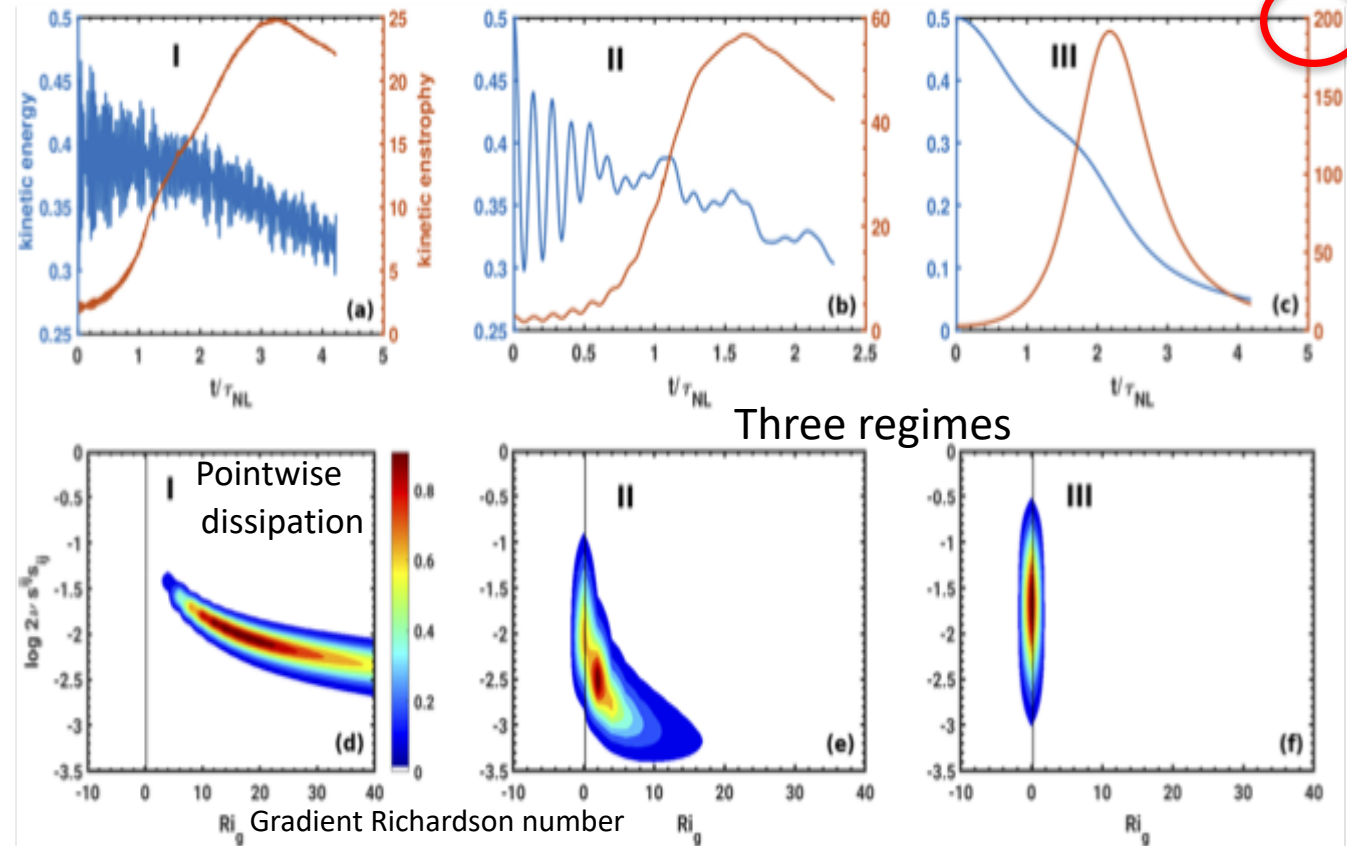
## Rotating stratified turbulence, no forcing

Kinetic energy and dissipation

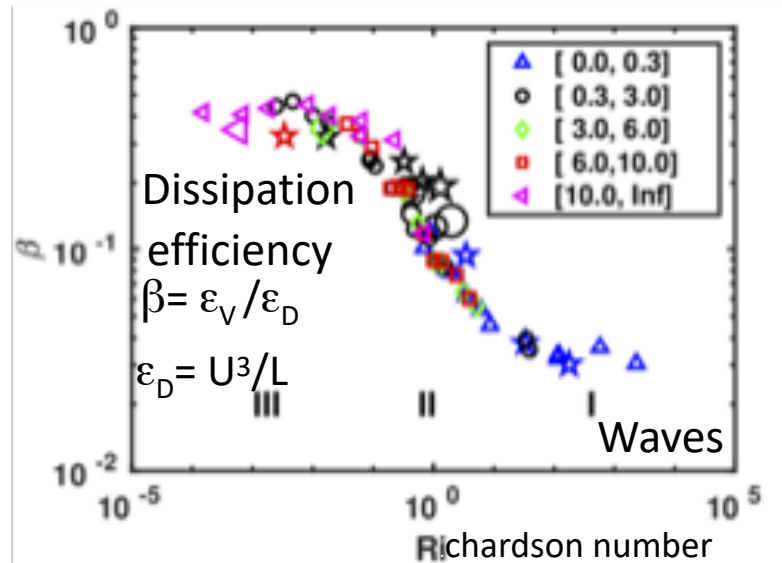
$N/f=31$ ,  $Re=14000$ ;  
 $Fr=0.007$ ,  $R_B=0.75$

$N/f=42$ ,  $Re=12000$ ;  
 $Fr=0.07$ ,  $R_B=65$

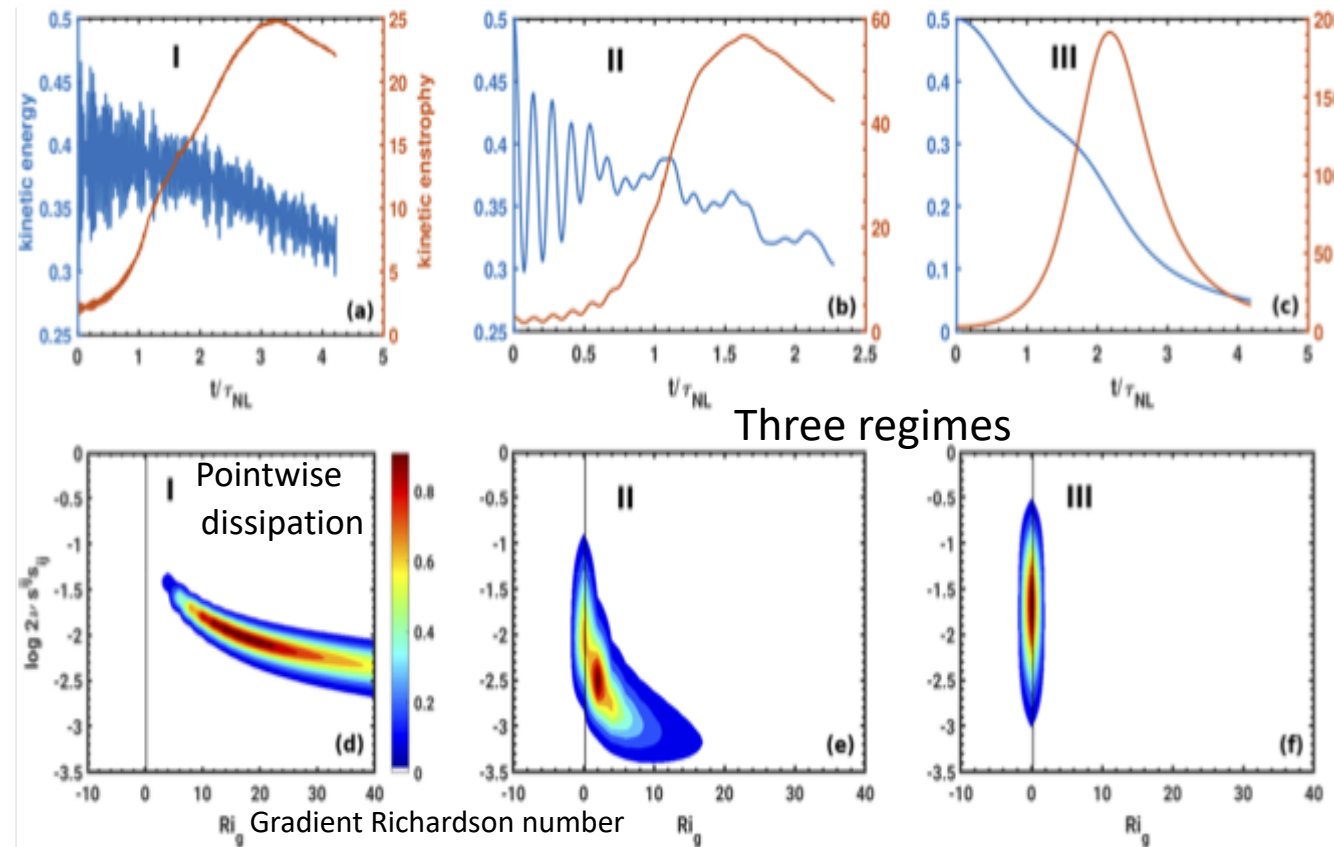
$N/f=2.5$ ,  $Re=4700$ ;  
 $Fr=0.89$ ,  $R_B=3760$

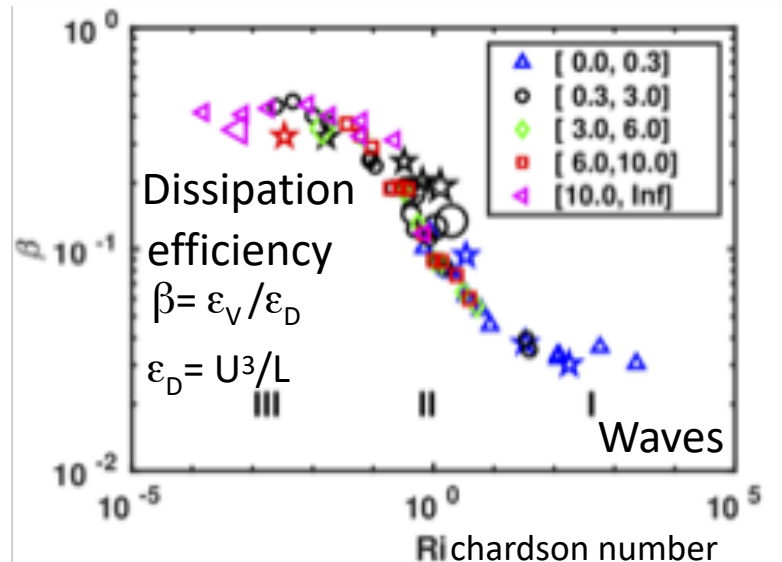




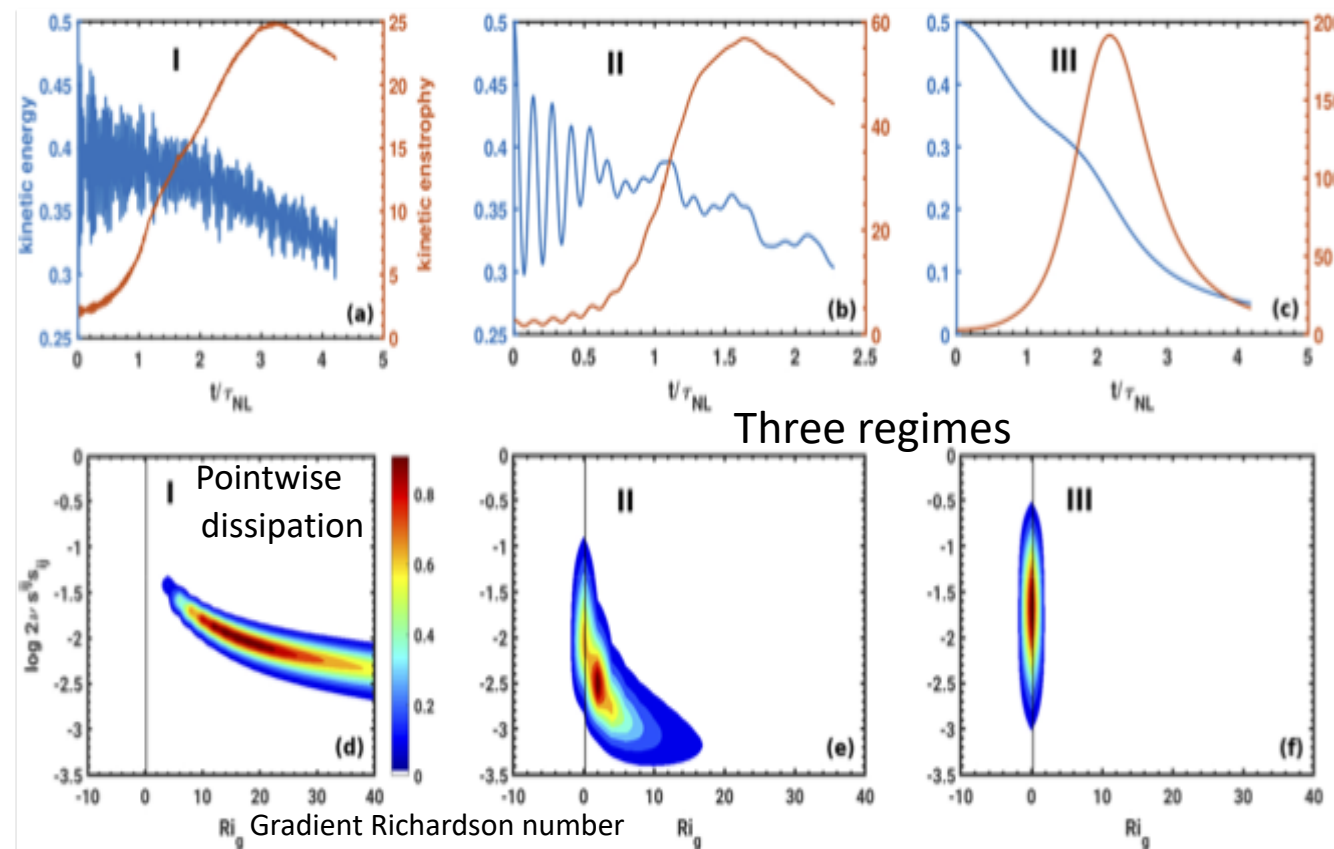
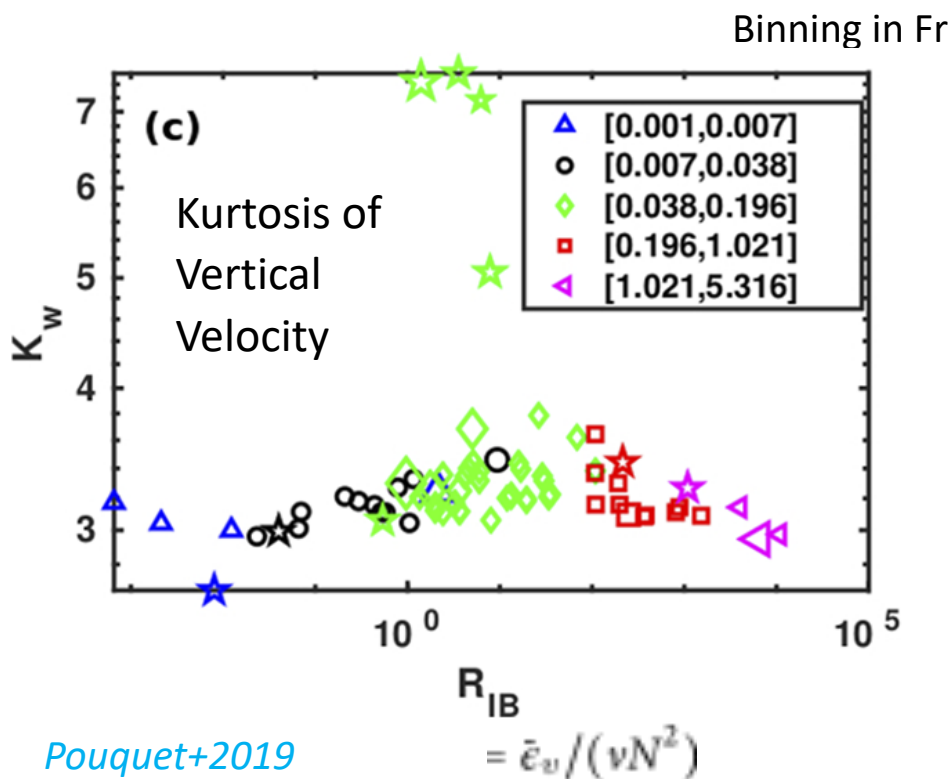


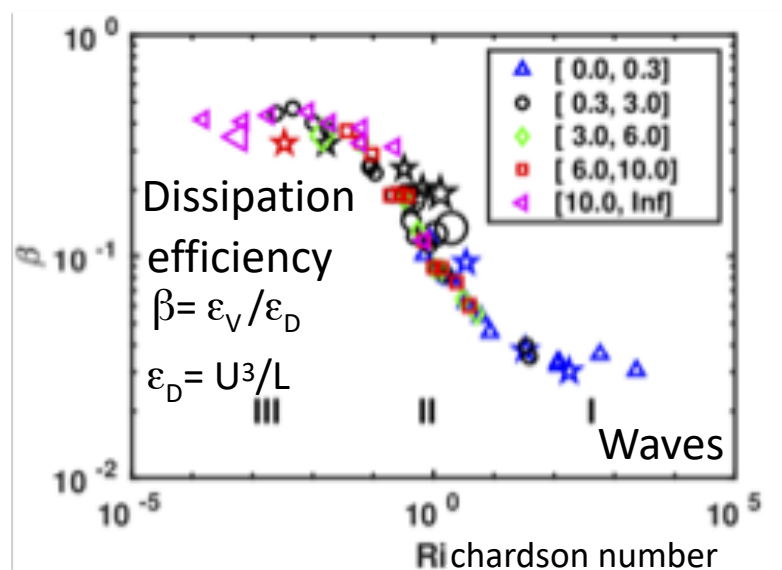
## Rotating stratified turbulence





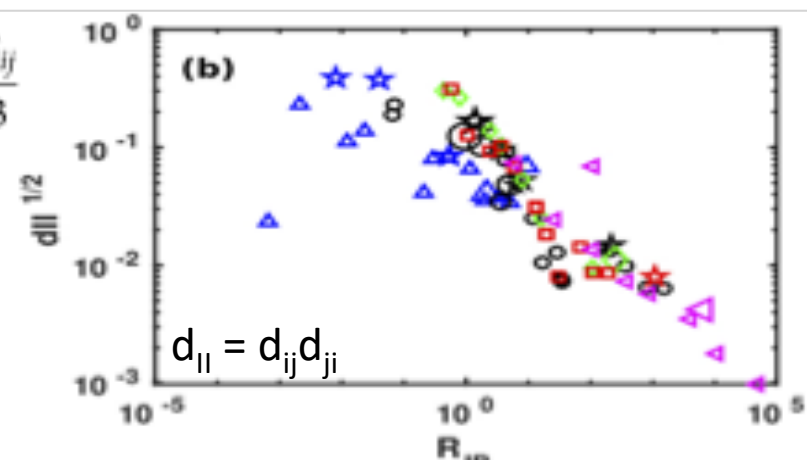
## Rotating stratified turbulence



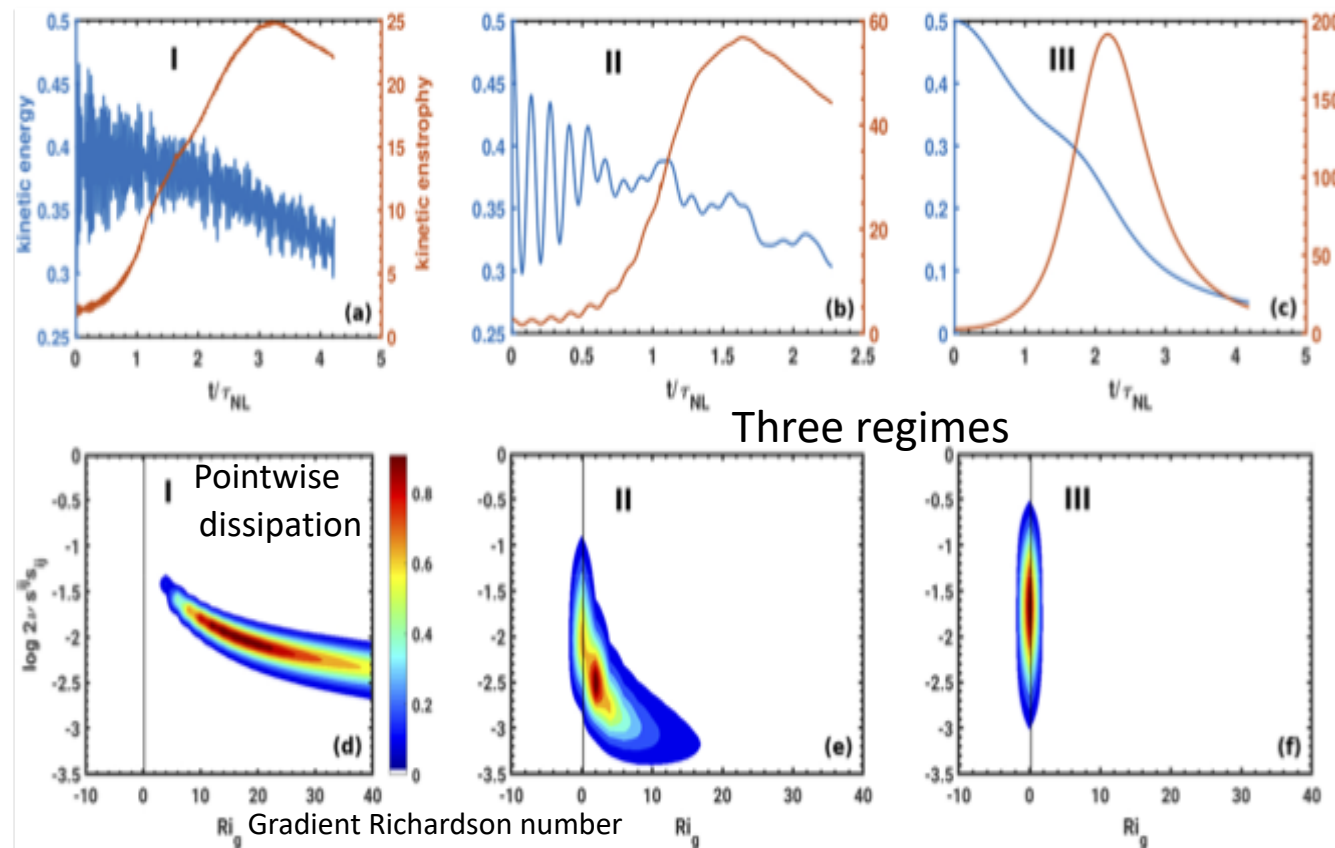
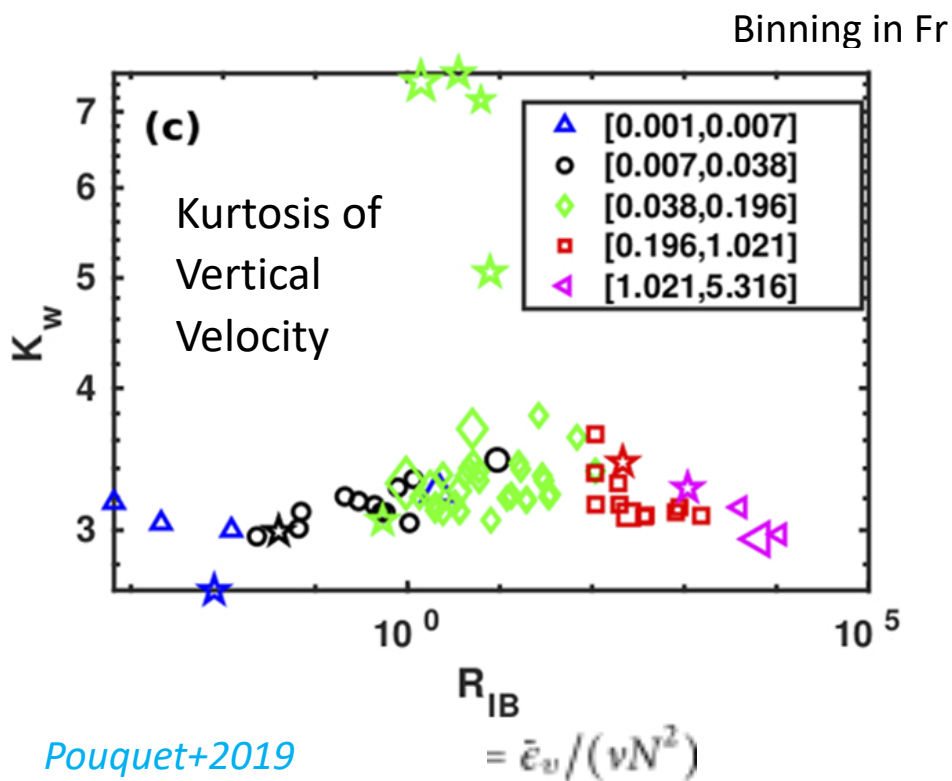


## Rotating stratified turbulence

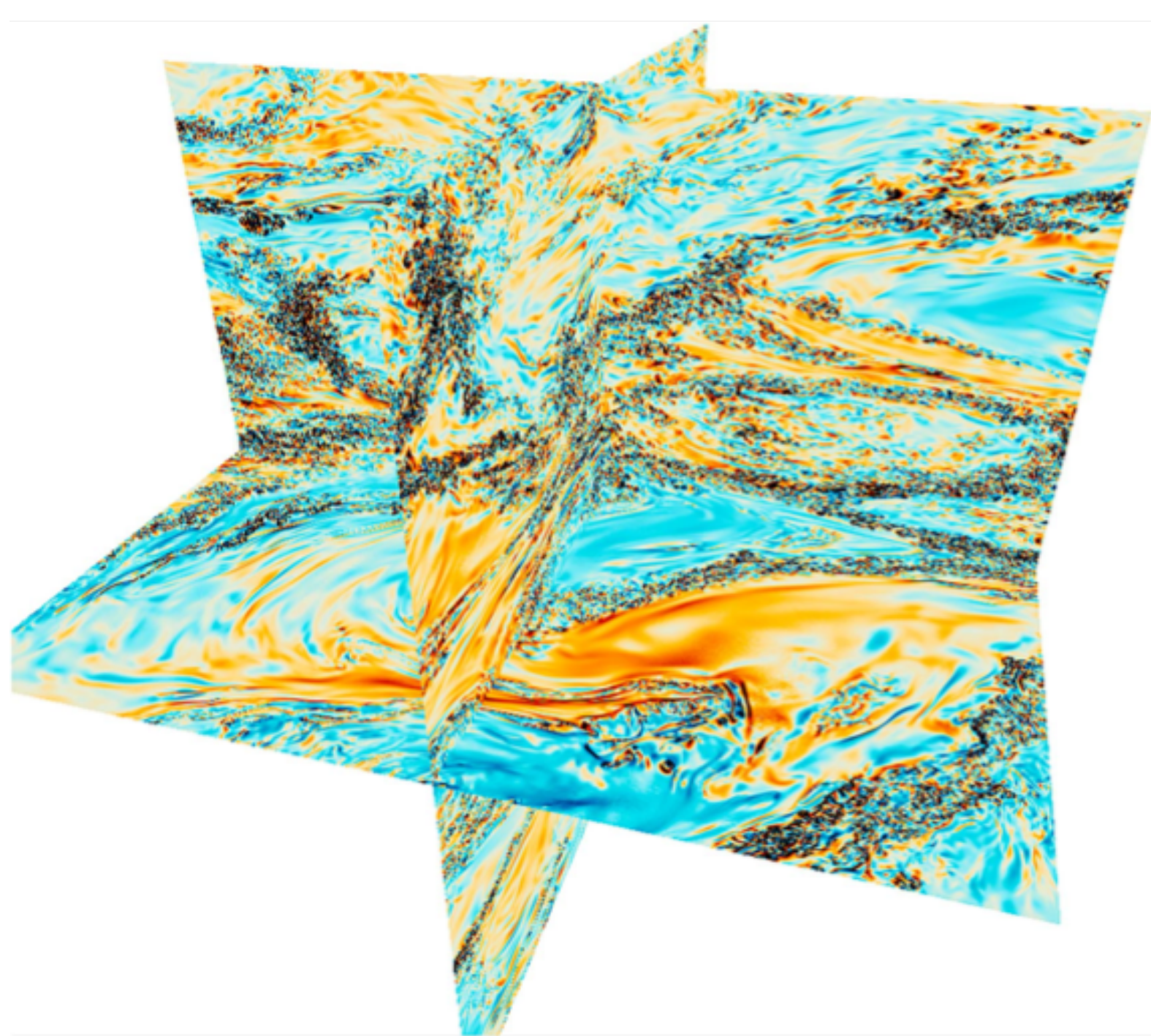
$$d_{ij} = \frac{\langle \partial_k u_i \partial_k u_j \rangle}{\langle \partial_k u_m \partial_k u_m \rangle} - \frac{\delta_{ij}}{3}$$



Slow return to isotropy







$4096^3$  ,  $N/f=5$ ,  $Re=55000$ ,  $R_b=32$

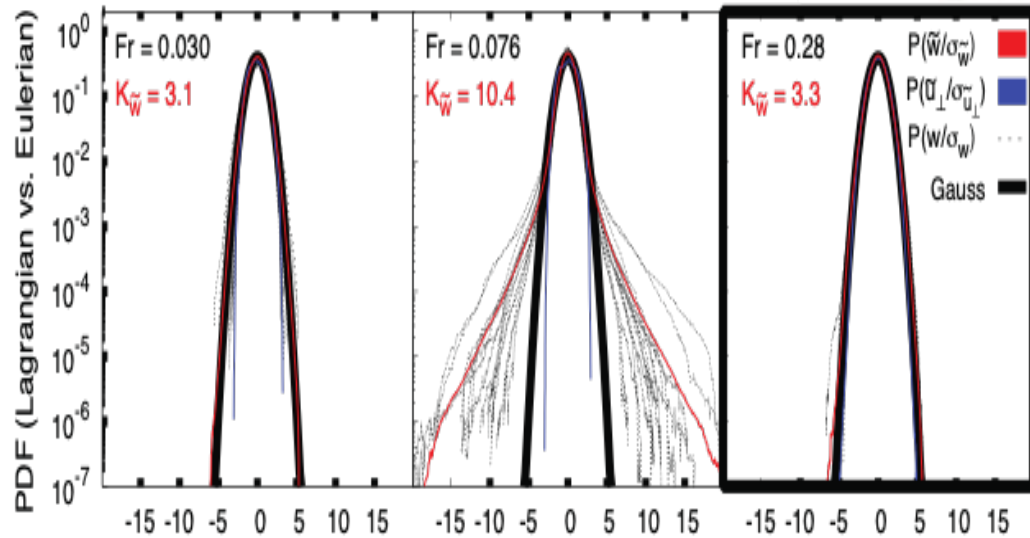
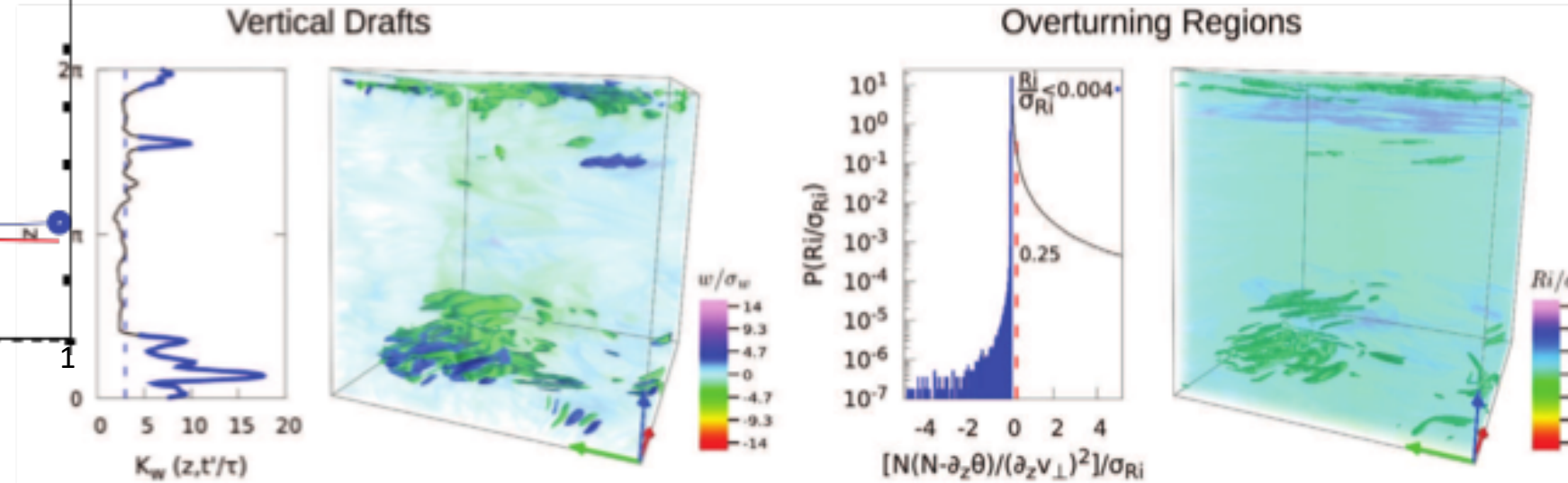
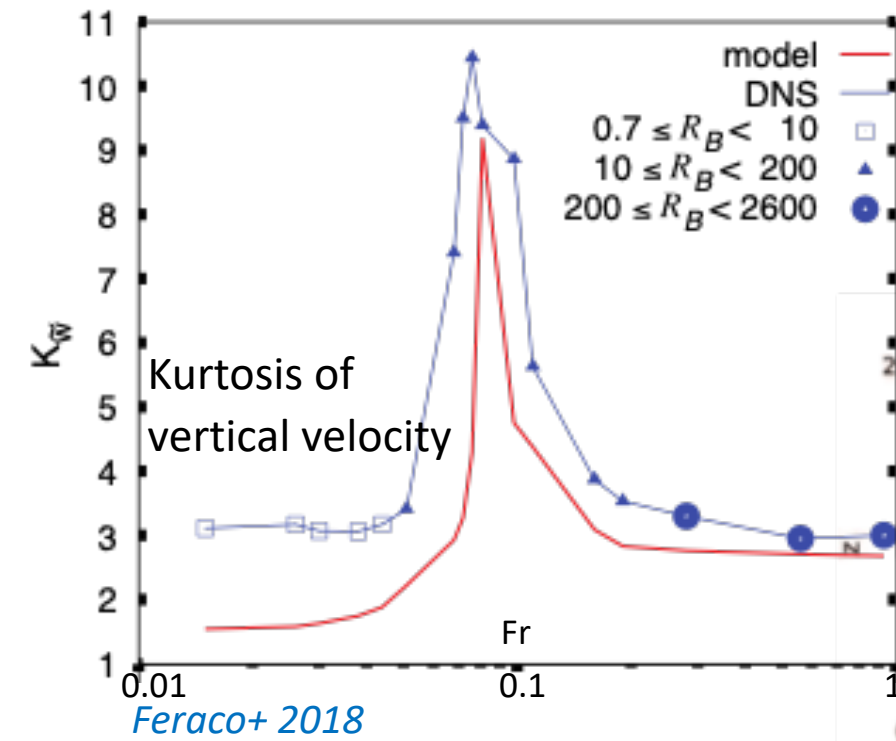
Unforced rotating stratified turbulence

Vorticity magnitude, 3D projection

Co-existence of large eddies  
(a signature of inverse transfer),  
and strong small-scale eddies

# Stratified turbulence

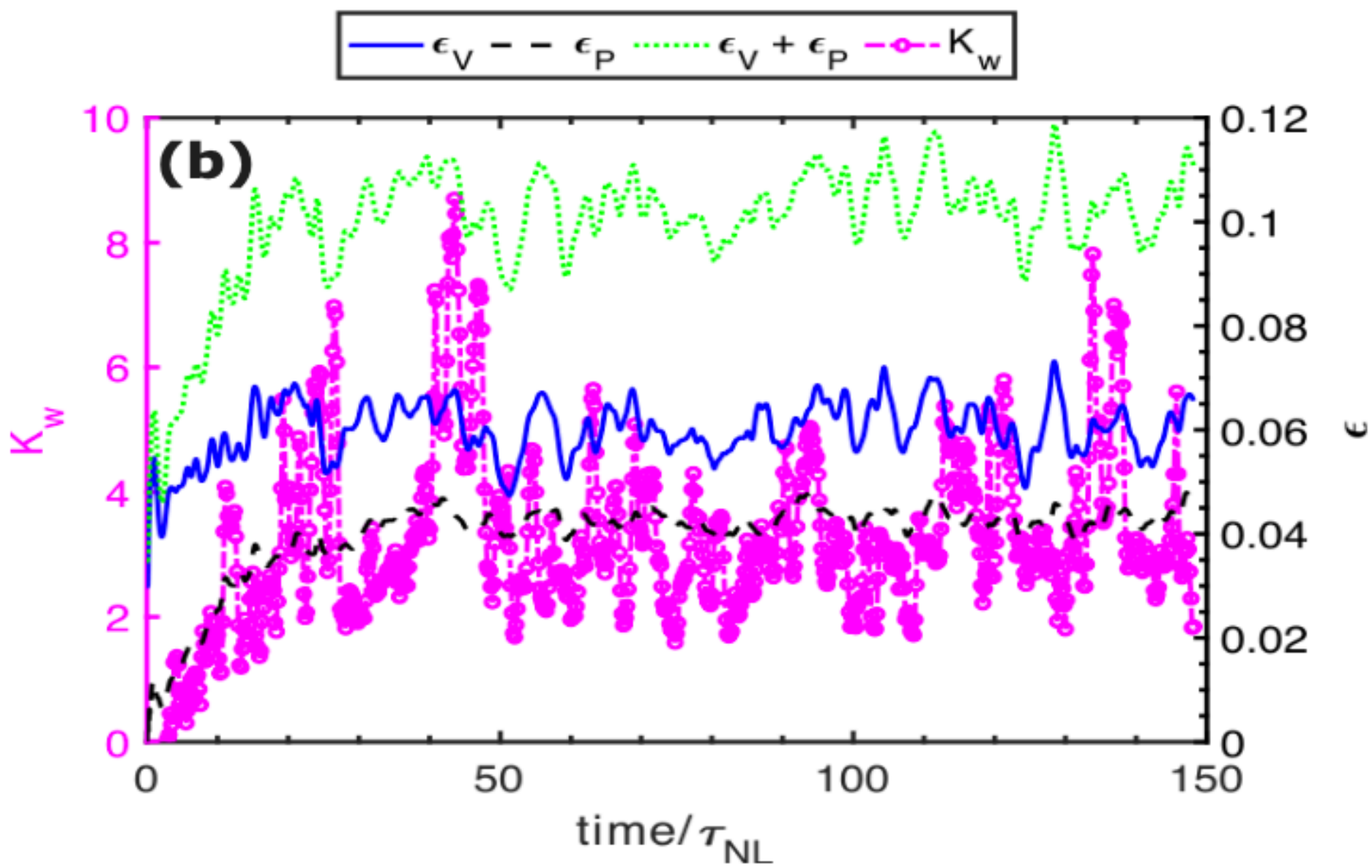
Efficiency of dissipation in volume @ the 50% level  
at different times



Marino+ PRF 2022

Overall kurtosis of vertical velocity at different times, forced flows





Stratified flows, no rotation

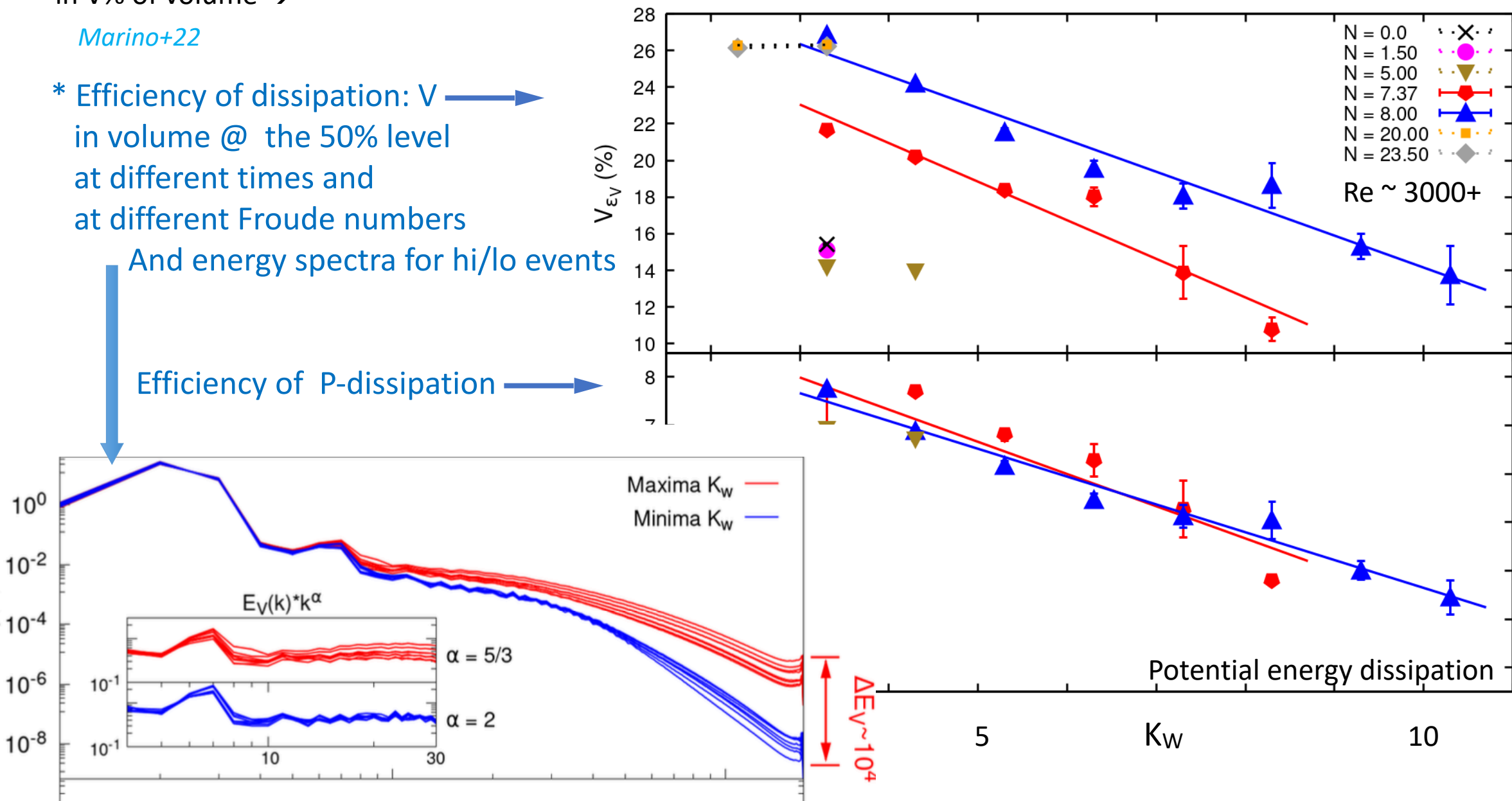
50% of dissipation

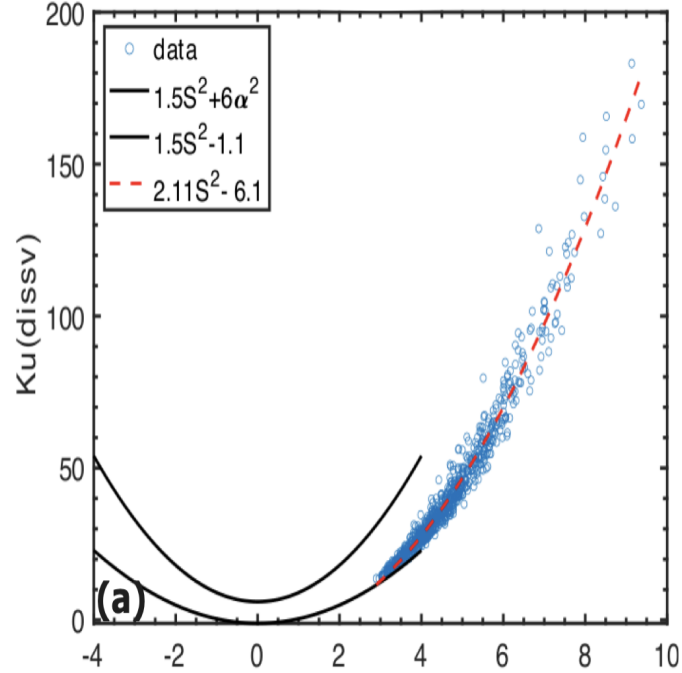
in  $V\%$  of volume  $\rightarrow$

*Marino+22*

\* Efficiency of dissipation:  $V$  in volume @ the 50% level at different times and at different Froude numbers  
And energy spectra for hi/lo events

Efficiency of P-dissipation  $\rightarrow$





## Rotating Stratified Turbulence

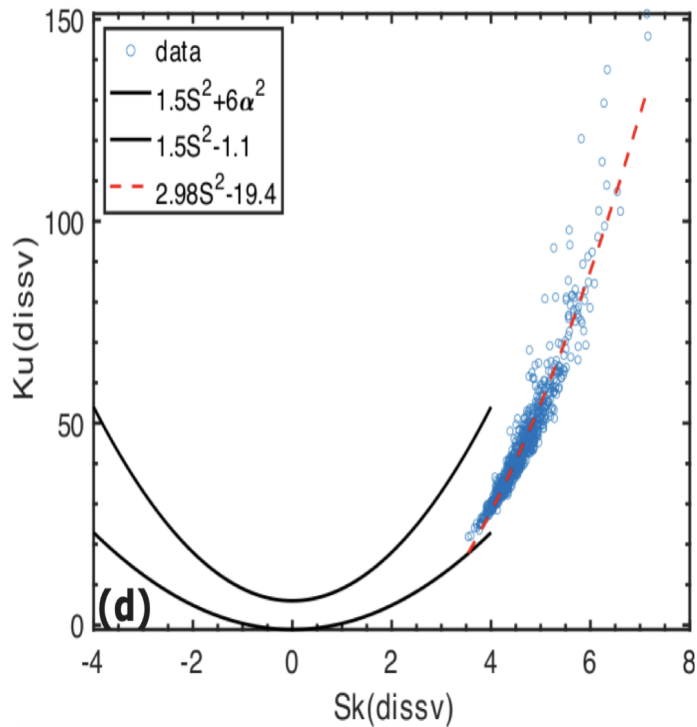
**K(S) for QG runs**

**Kinetic energy dissipation  $\epsilon_v$**

**Top:**  $Fr=0.07$ ,  $Re=631$

$Ri=3.1$ ,  $L_{Oz}/\eta_K=0.36$

$K(S) \sim 3.4S^2 + 16.5$

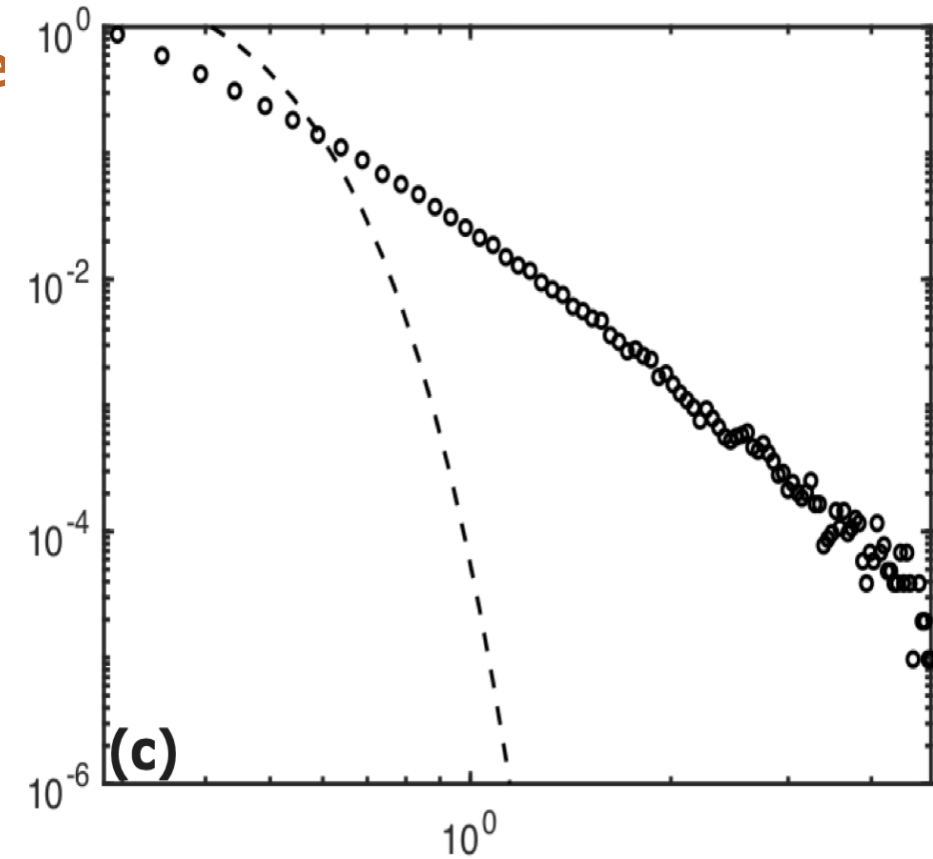


**Bottom:**  $Fr=0.36$ ,  $Re=694$

$Ri=0.6$ ,  $L_{Oz}/\eta_K=11.9$

$K(S) \sim 1.45S^2 + 6.2$

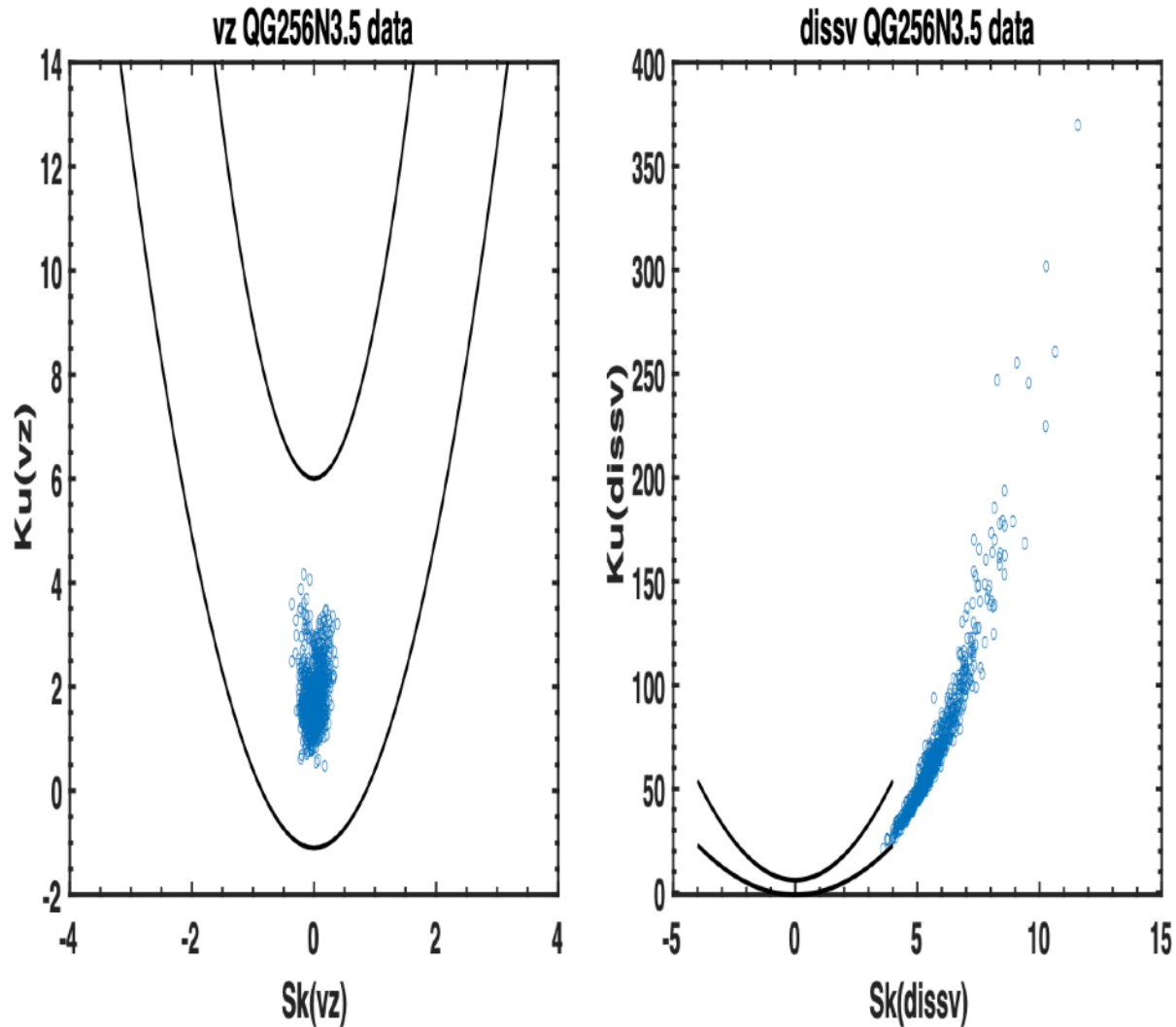
—> **Sharp transition** when isotropic  
K41 range begins to be present



**PDF of  $\epsilon_v$  at maximum of  $K_w$**

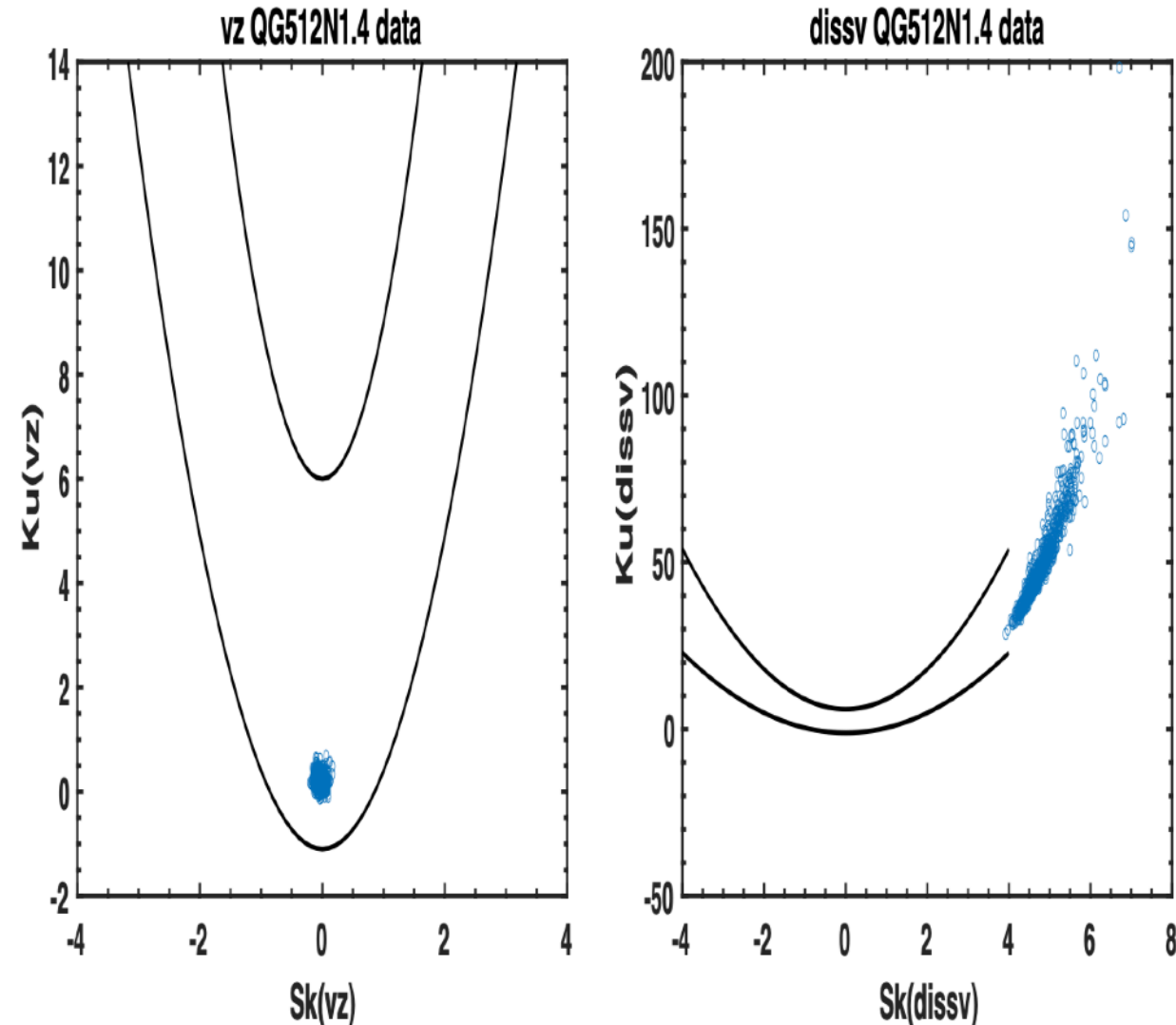
$Fr=.29$ ,  $Re=370$ ,  $Ri=.9$ ,  $L_{Oz}/\eta_K= 6.4$

Rotating stratified flows at different Froude numbers, with  $N/f=5, 80 T_{NL}$  :  $w$  &  $\varepsilon_v$

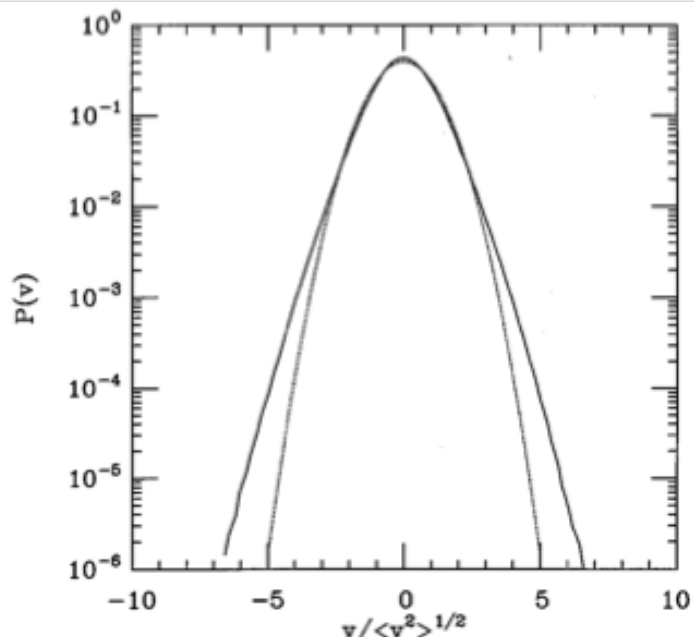


$Fr \sim 0.11$ ,  $Ro = 0.55$ ,  $Re = 942$ ,  $R_B = 11$ ,  $K \sim 4.4S^2$

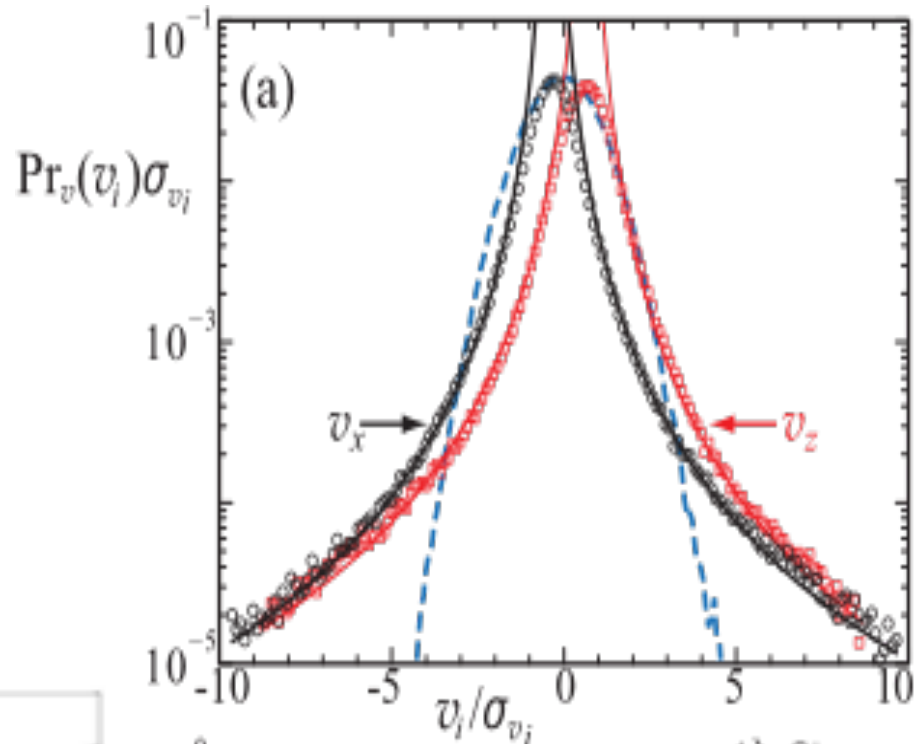
and



$Fr \sim 0.4$ ,  $Ro = 2.$ ,  $Re = 10^3$ ,  $R_B = 11$ ,  $K \sim 1.6S^2$

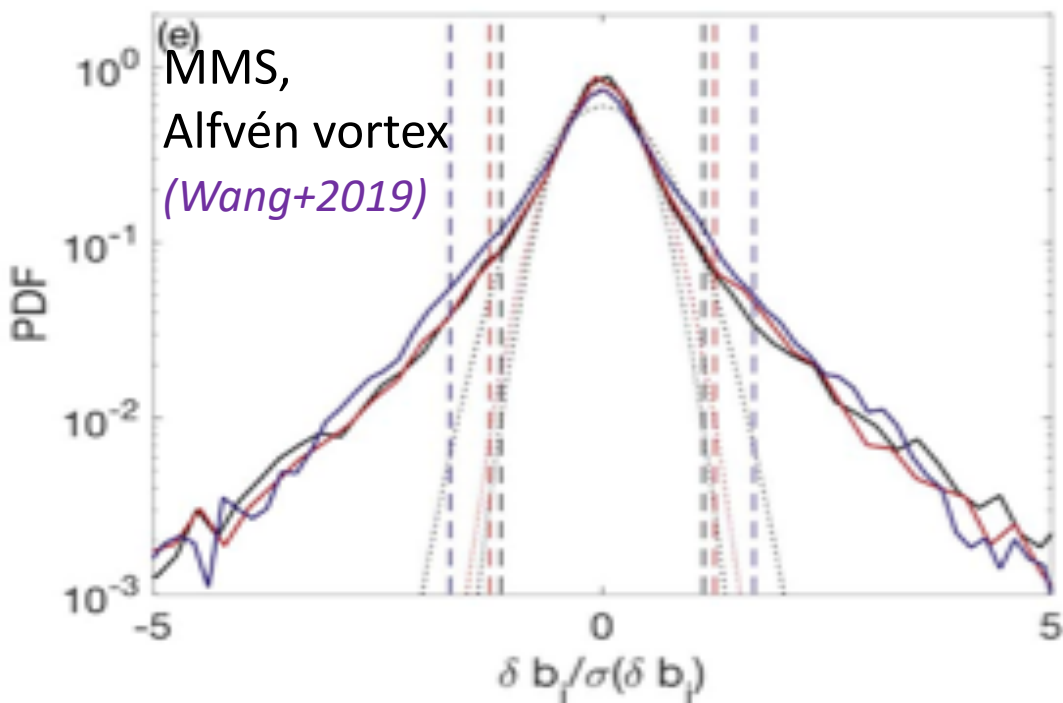


Shear flows  
*Pumir 1996*  
 DNS

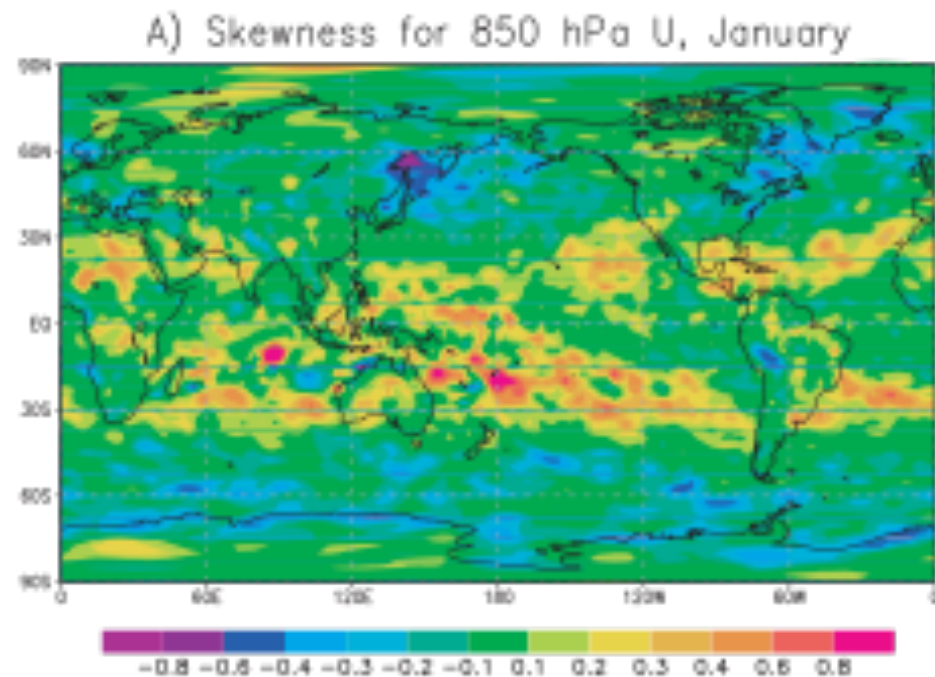


*Paoletti+2008*  
 Superfluid  
 turbulence

Skewness of temperature  
 ERA40 data. *Petoukhov+ 08*

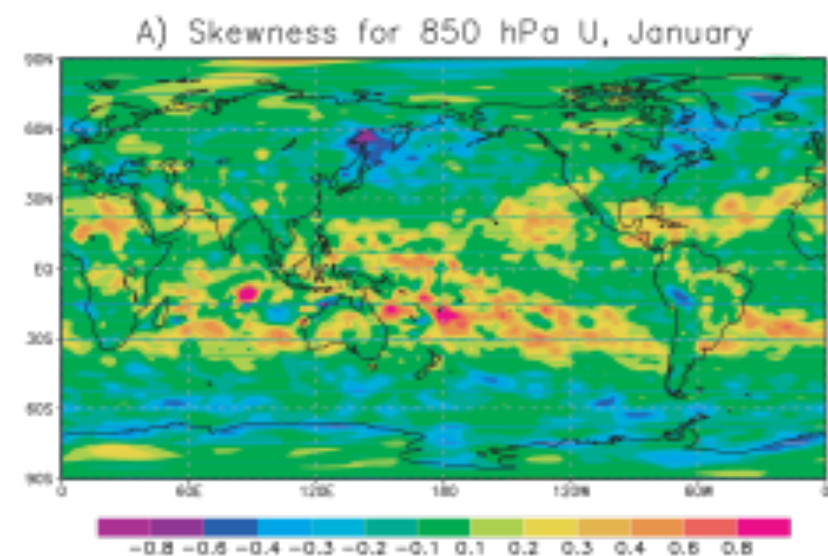
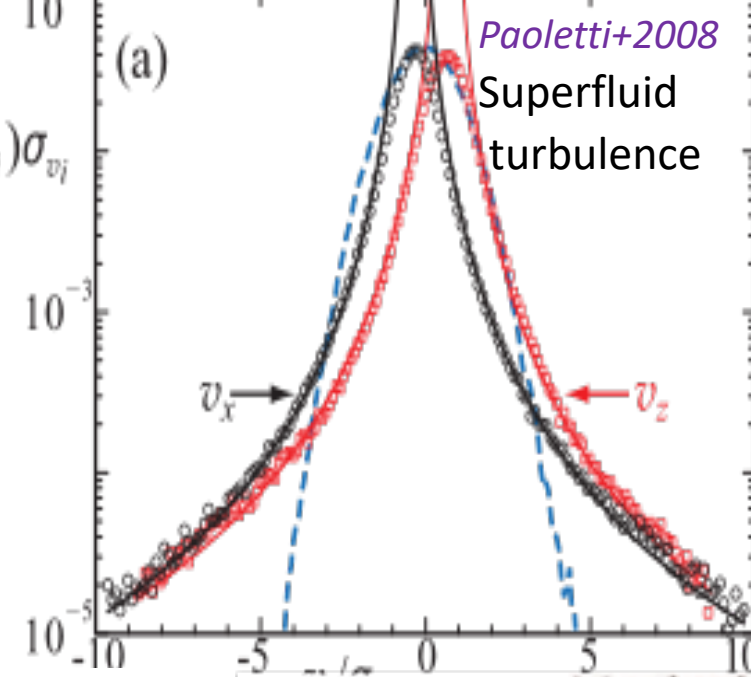
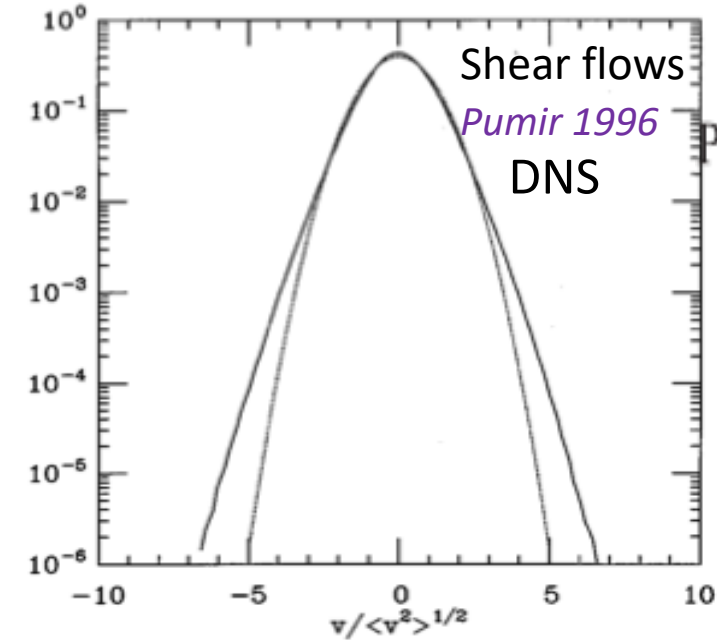


MMS,  
 Alfvén vortex  
*(Wang+2019)*

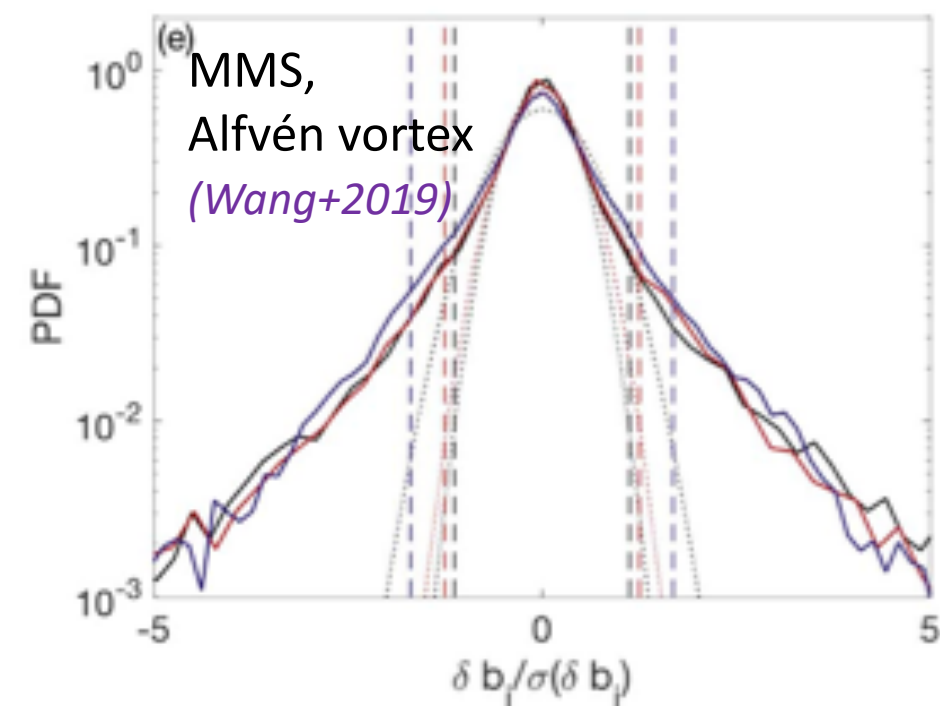


A) Skewness for 850 hPa U, January

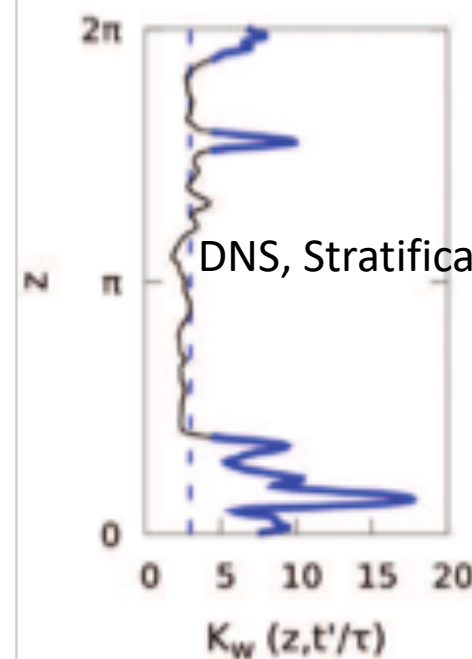




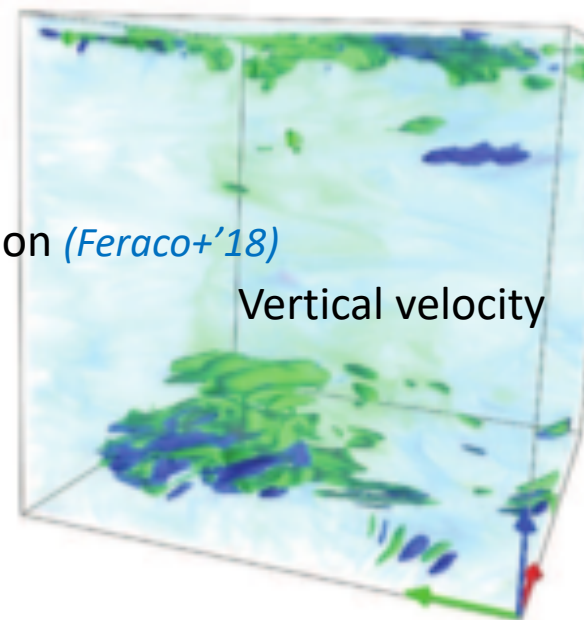
Skewness of temperature  
 ERA40 data. *Petoukhov+ 08*



Vertical Drafts



DNS, Stratification (*Feraco+'18*)



**Non-Gaussian  
 V, B, T**  
 (+ their gradients)

# Conclusions and perspectives

- \* Need for detailed explorations of dissipative structures in fluid, MHD and plasma turbulence (*e.g.*, sparseness scale as a function of dissipative scale)
- \* Dissipative structures not fully explored (*ongoing work in MHD*)
- \* Skewness  $S$  and excess kurtosis  $K$  as maps of nonlinear behavior
- \*  $K(S)$  laws in hydrodynamic turbulence, and (rotating) stratified turbulence
- \* Statistics: need for long-time integration, in excess of  $5000^+ \tau_{NL}$ ?
  - \* Scaling variation for different regimes? As a trace of what change in structures?
  - \* What intermittency do they correspond to (*ongoing work in MHD*)?
    - \* That of the dissipative range?
    - \* That corresponding to a critical shear instability
    - \* Or other: Langevin model, SOC, ...

# Thank you!

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