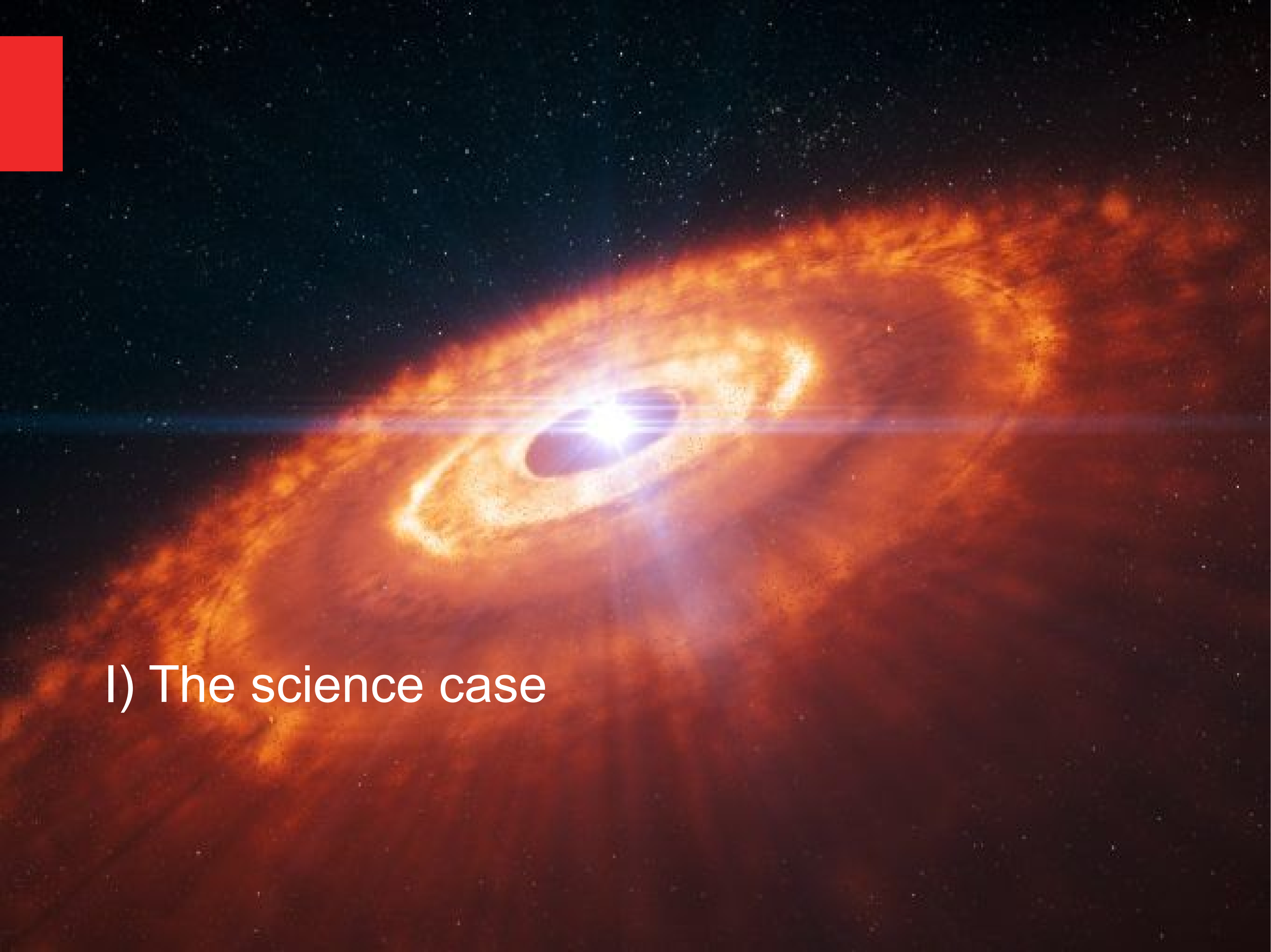
Radiation Hydrodynamics with Flux Limited Diffusion in FARGOCA

E. Lega, A. Miniussi

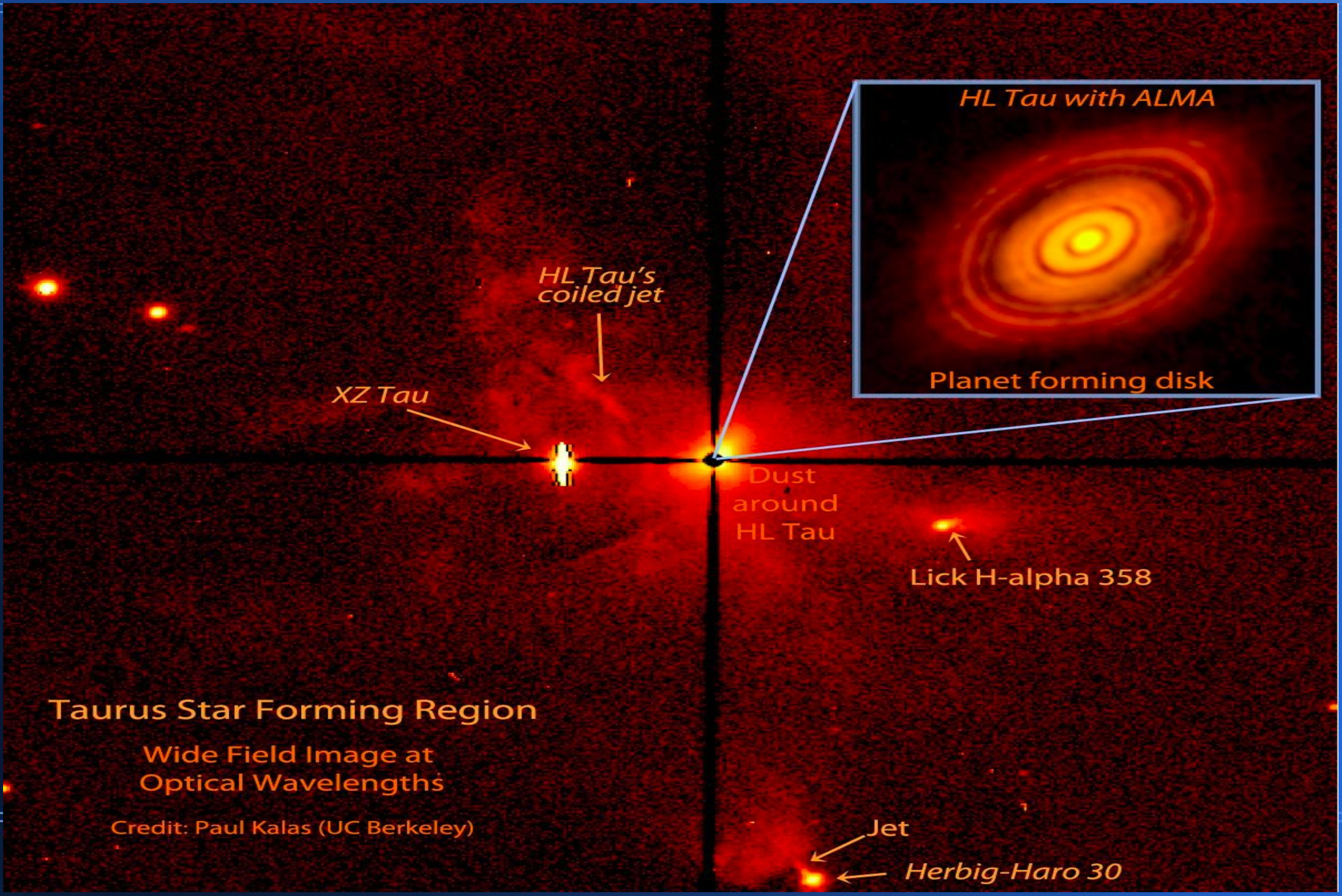
Journée sur la thématique du transfer radiatif
29 mai 2009

LAGRANGE



I) The science case

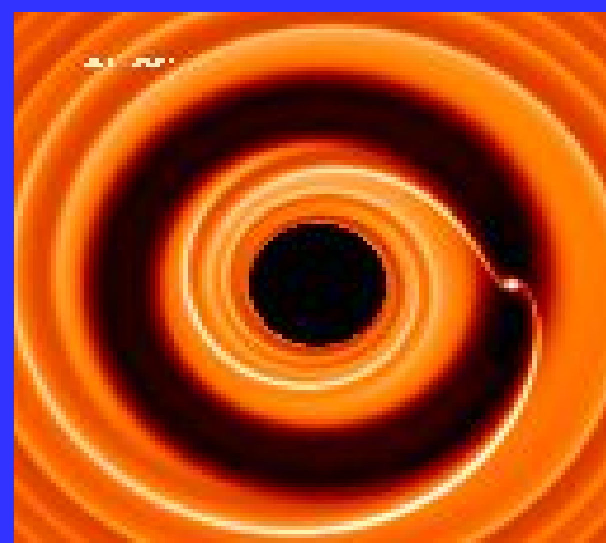
The study of protoplanetary discs and planet formation and evolution



Many Hydro-codes developed for this purpose:

FARGO : Fast Advection in Rotating Gaseous Objects
(Masset F., 2000)

2D grid-based code designed for the study of the interaction between protoplanetary locally ISOTHERMAL disc and newly forming planets

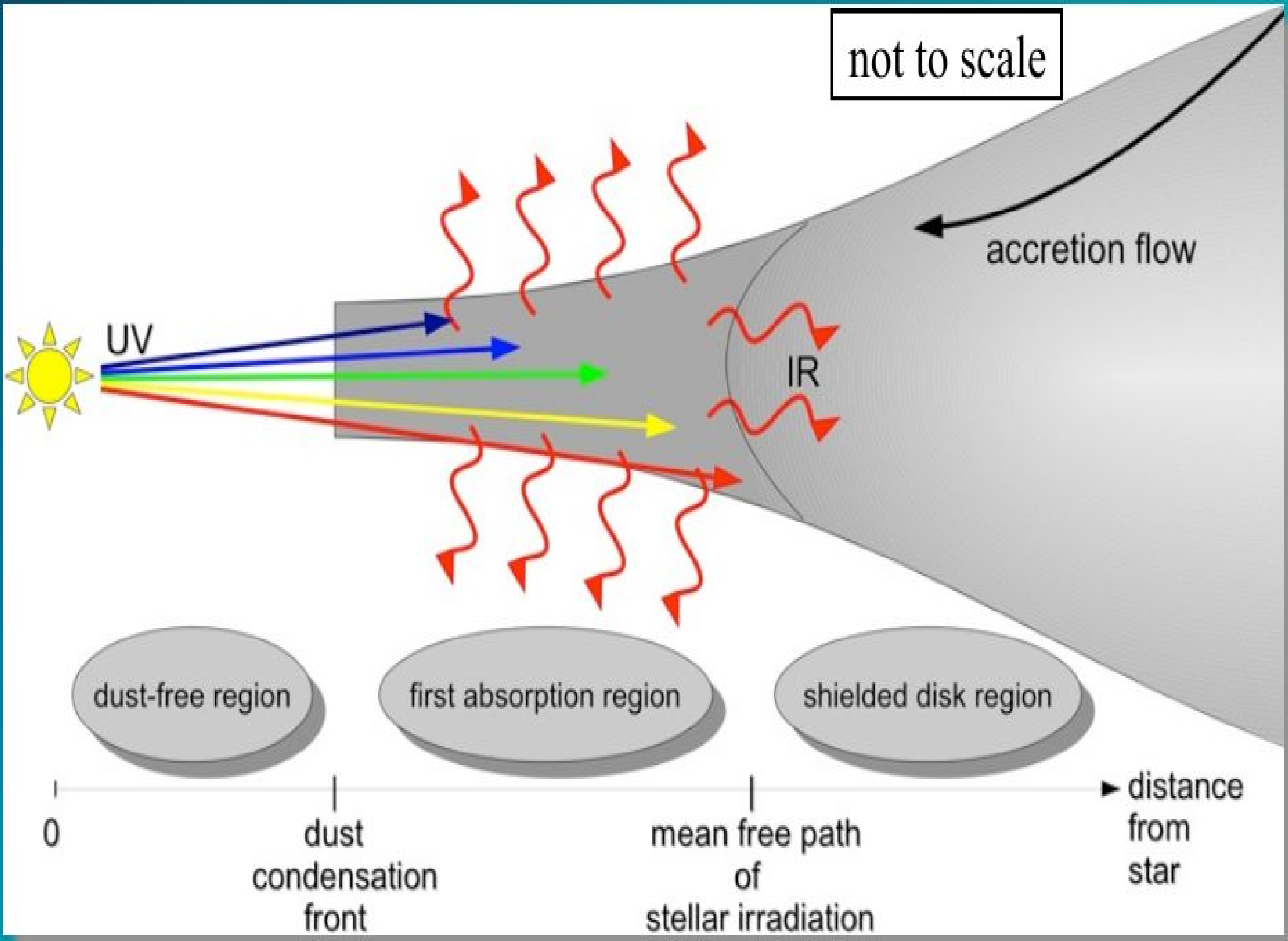


Gas density field

ALMA data of HItau

LAGRANGE

- Gray *Flux-Limited Diffusion* (FLD) for *thermal dust (re-)emission*

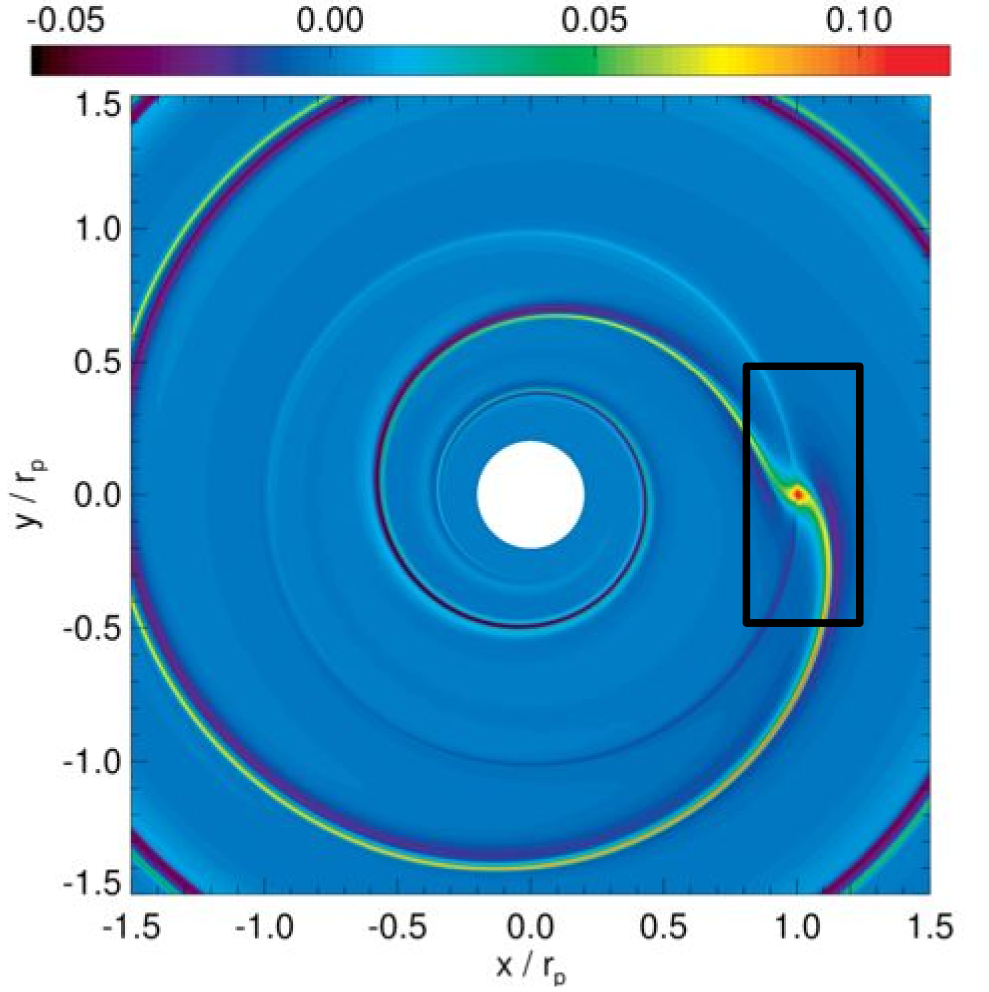


The problem of inward migration:

Planetary cores
FORM and MIGRATE
in protoplanetary discs.

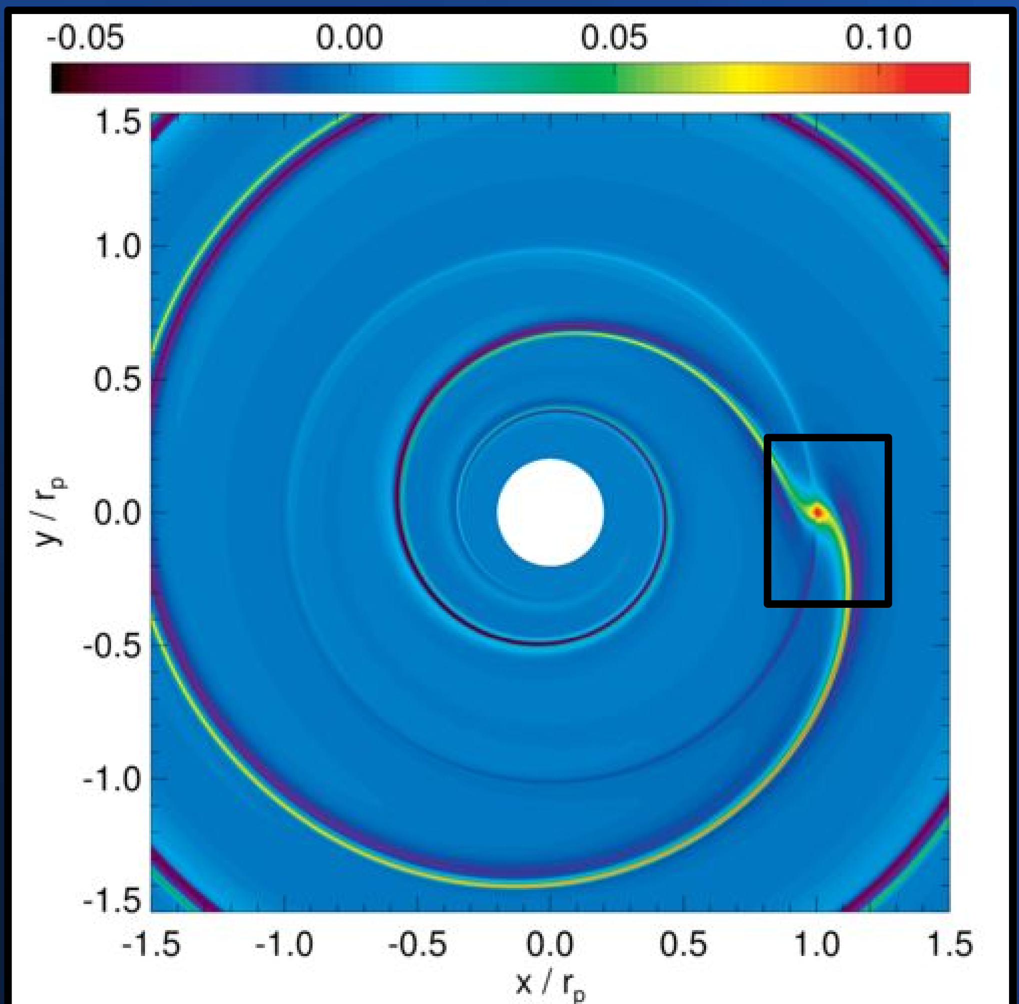
ISOTHERMAL DISCS:
too fast migration towards the central star for $M_p > 1\ M_{earth}$.

NON ISOTHERMAL EFFECTS
can change the magnitude and direction of migration in the inner part of the disc.

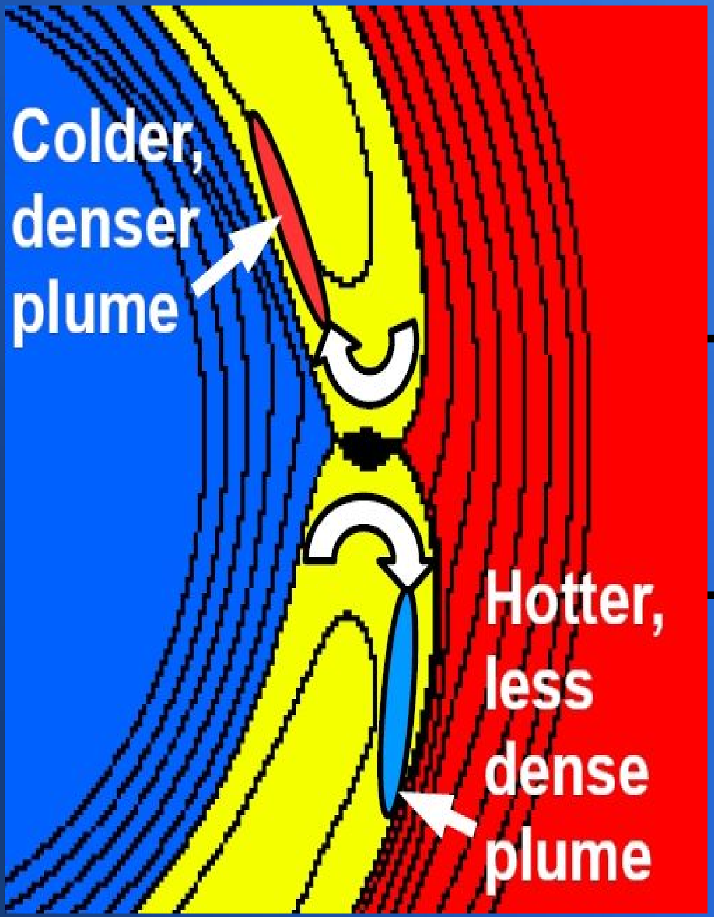


Perturbations of the gas density caused by a planet.

Can we have outward migration ?



Perturbations of the gas density caused by a planet.

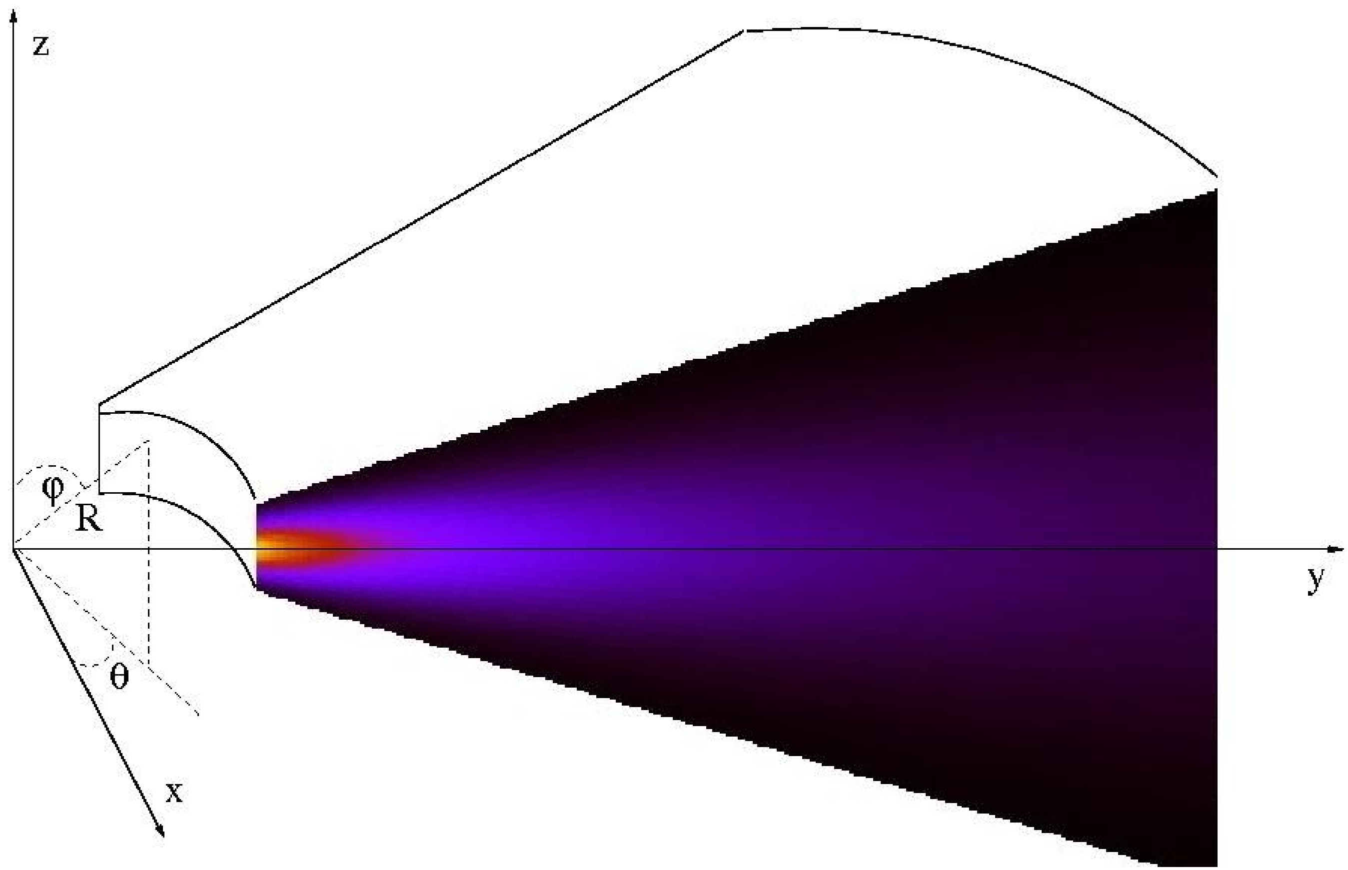


II) The fargOCA hydro code

The FARGOCA code: fargo with Colatitude Added at Observatoire Côte d'Azur

The FARGO code extended to 3D

with the additional introduction of energy equation to model thermal effects



The FARGOCA code in a slide :

1)The protoplanetary disc is treated as a three dimensional non self-gravitating gas whose motion is described by the Navier-Stokes equations.

2) The hydro equations in spherical coordinates are solved using finite difference with explicit multi-step procedure

3) Choice of the time step : CFL condition + FARGO algorithm

TALK TODAY

4) The energy equation -----

5) The code is parallelised using hybrid combination of MPI between the nodes and of OpenMP on shared-memory multi-core processors.

6) High resolution is achieved using a nonuniform grid geometry

Radiation-hydrodynamics
Andrea's questions:

a) input et output (e.g., il s'agit d'un code post-processing? statique ou dynamique? il y a des approximation sur certaines variables des modeles? pourquoi?). Certains approches sont inclus dans un code hydro, d'autres en post-processing…

→>> CODE DYNAMIQUE HYDRO

b) geometrie (1D versus 3D?)

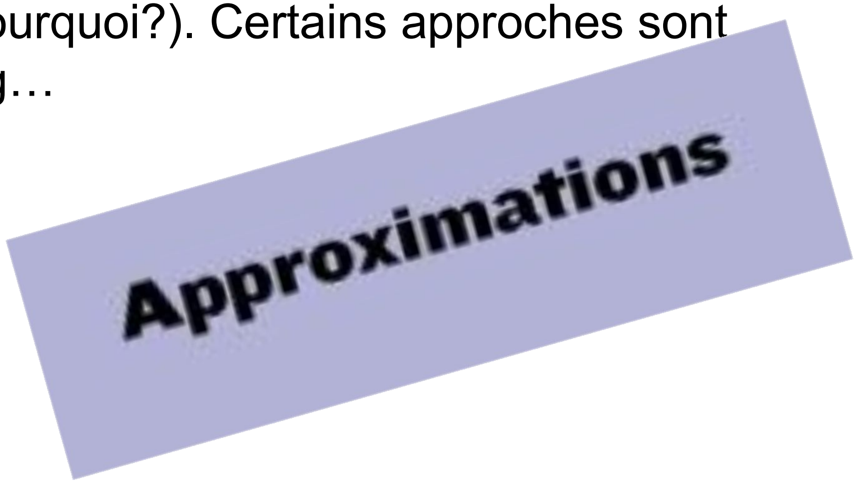
→>>> 3D

c) equilibre thermodynamique local ou pas?

d) opacites, besoin et limitations

e) propriete de particules

f) couplage gaz/poussière



h) quelques idees sur les solvers utilises et peut etre sur les moyens necessaires pour faire les calculs (parallel/serial?),

i) le magnetisme, approche et besoin. → pas de MHD (dans l'état actuel du code)

l) open source? acces a travers une collaboration? support pour l'utilisation? experience d'utilisation?

Radiation-hydrodynamics Approximations:

Radiative transfer equation

- Radiative transfer equation

Emission

$$\left(\frac{1}{c}\frac{\partial}{\partial t} + \mathbf{n}\cdot\nabla\right) I(\mathbf{x}, t; \mathbf{n}, \nu) = \eta(\mathbf{x}, t; \mathbf{n}, \nu) - \chi(\mathbf{x}, t; \mathbf{n}, \nu) I(\mathbf{x}, t; \mathbf{n}, \nu)$$

Specific intensity

Absorption

- Assuming TE (and neglecting scattering), thermal emission/absorption terms are

$$\eta_{\text{th}}(\mathbf{x}, t; \mathbf{n}, \nu) = \kappa(\mathbf{x}, t; \mathbf{n}, \nu) B(\mathbf{x}, t; \mathbf{n}, \nu)$$

$$\chi(\mathbf{x}, t; \mathbf{n}, \nu) = \kappa(\mathbf{x}, t; \mathbf{n}, \nu) I(\mathbf{x}, t; \mathbf{n}, \nu)$$

Radiation-hydrodynamics Approximations: consider the moments of the intensity

- Moments of the specific intensity
  - Energy $$E_\nu(\mathbf{x},t) = \frac{1}{c}\int I(\mathbf{x},t;\mathbf{n},\nu)d\Omega$$
  - Flux $$\mathbf{F}_\nu(\mathbf{x},t) = \int \mathbf{n}I(\mathbf{x},t;\mathbf{n},\nu)d\Omega$$
  - Pressure $$\mathbb{P}_\nu(\mathbf{x},t) = \frac{1}{c}\int \mathbf{n}\times\mathbf{n}I(\mathbf{x},t;\mathbf{n},\nu)d\Omega$$
    $$\mathrm{Tr}(\mathbb{P}_\nu) = E_\nu$$

Moments of the RT equation

- Radiative transfer equation

$$\left(\frac{1}{c}\frac{\partial}{\partial t}+\mathbf{n}\cdot\nabla\right)I(\mathbf{x},t;\mathbf{n},\nu)=\eta(\mathbf{x},t;\mathbf{n},\nu)-\chi(\mathbf{x},t;\mathbf{n},\nu)I(\mathbf{x},t;\mathbf{n},\nu)$$

TOO HEAVY for multidimensional dynamical calculations

- Zeroth-moment

$$\int d\Omega\times \qquad \frac{\partial E_\nu}{\partial t}+\nabla\cdot\mathbf{F}_\nu=\kappa_\nu(4\pi B_\nu-cE_\nu)$$

- First-moment

$$\int \mathbf{n}d\Omega\times \qquad \frac{1}{c}\frac{\partial \mathbf{F}_\nu}{\partial t}+c\nabla\cdot\mathbb{P}_\nu=-\kappa_\nu\mathbf{F}_\nu$$

Moments models

- System of two equations, three variables => need a closure relation

$$\begin{cases} \dfrac{\partial E_\nu}{\partial t} + \nabla \cdot \mathbf{F}_\nu = \kappa_\nu (4\pi B_\nu - cE_\nu) \\ \dfrac{1}{c}\dfrac{\partial \mathbf{F}_\nu}{\partial t} + c\nabla \cdot \mathbb{P}_\nu = -\kappa_\nu \mathbf{F}_\nu \end{cases}$$

- Flux-Limited Diffusion (FLD)
  - Optically thick medium <=> diffusion approximation. Radiation field is isotropic $\mathbb{P}_\nu = \frac{1}{3}\mathbb{I}E_\nu$ and radiative flux is stationary.

$$\mathbf{F}_\nu = -\frac{c\lambda}{\kappa_\nu}\nabla E_\nu$$

Flux Limiter

λ =1/3 Optically thick

λ $=K_\nu E_\nu / \nabla E_\nu$ Optically thin

Grey Flux Limited Diffusion

- Integration of all radiative quantities over frequency $E_\mathrm{r} = \int E_\nu d\nu$

$$\frac{\partial E_\mathrm{r}}{\partial t} - \nabla \cdot \frac{c\lambda}{\kappa_\mathrm{R}} \nabla E_\mathrm{r} = \kappa_\mathrm{P}(a_\mathrm{r} T^4 - cE_\mathrm{r})$$

- Planck mean opacity $$\kappa_\mathrm{P} = \frac{\int \kappa_\nu B_\nu(T) d\nu}{\int B_\nu d\nu}$$

- Rosseland mean opacity $$\frac{1}{\kappa_\mathrm{R}} = \frac{\int \frac{1}{\kappa_\nu} \frac{\partial B_\nu(T)}{\partial T} d\nu}{\frac{\partial B_\nu(T)}{\partial T} d\nu}$$

Coupling Radiation with Hydrodynamics

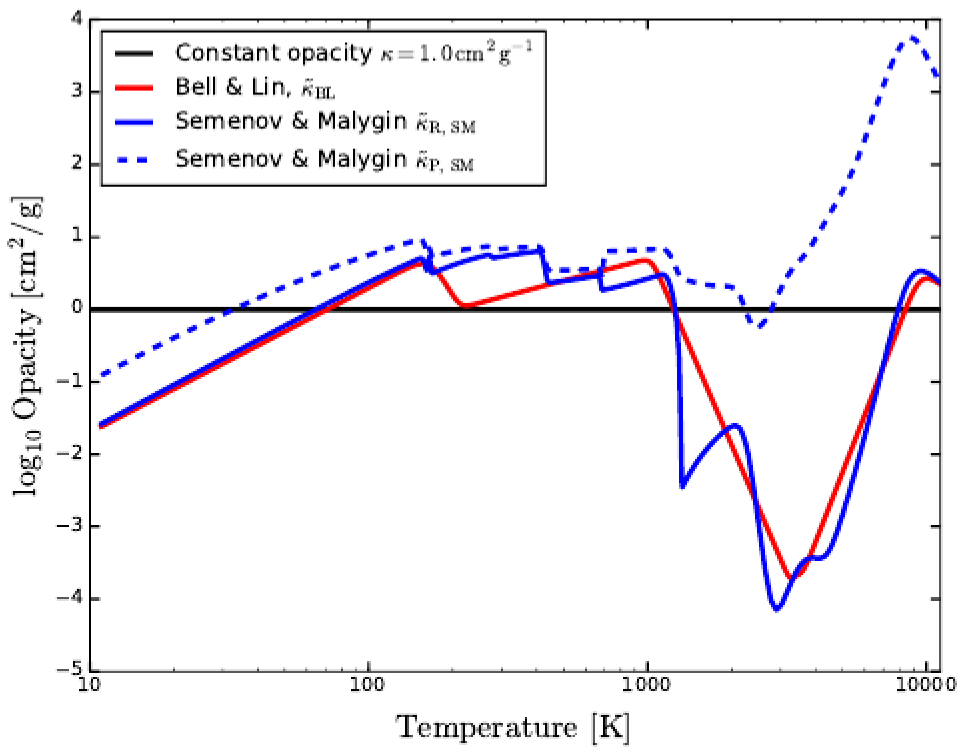
$$\begin{cases} \frac{\partial E_{\mathrm{r}}}{\partial t} - \nabla \cdot \frac{c\lambda}{\rho k_R} \nabla E_{\mathrm{r}} & = & \rho \kappa_p (a_r T^4 - c E_{\mathrm{r}}) \\ \frac{\partial E_{\mathrm{gas}}}{\partial t} + \nabla \cdot (E_{\mathrm{gas}} \vec{u}) & = & -P \nabla \cdot \vec{u} - \rho \kappa_p (a_r T^4 - c E_{\mathrm{r}}) + Q^+ + S \end{cases}$$

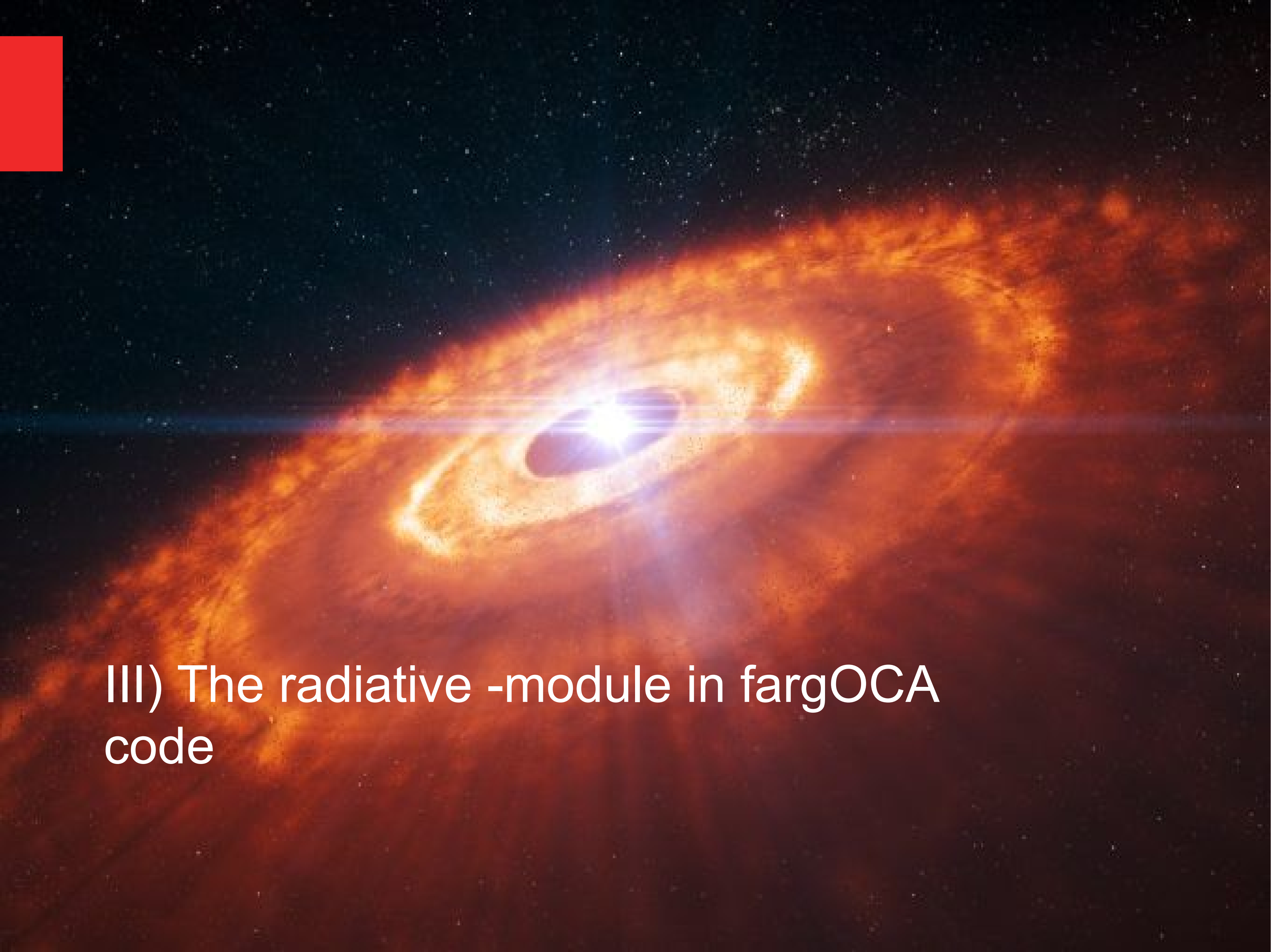
- $P\nabla \cdot \vec{v} \rightarrow$ compressional heating
- $Q^+ = (\mathbb{T}\nabla) \cdot \vec{v} \rightarrow$ viscous heating
- $S = F_\star e^{-\tau} \rho k_\star \rightarrow$ stellar heating

Opacity computations suited to protoplanetary discs:

The physical conditions typical of protostellar nebulae and protoplanetary discs around low-mass young stellar objects. Virtually everywhere within the medium dust grains are the main opacity source, as they absorb radiation much more efficiently compared to the gas and because the temperature in these regions is low enough to prevent their destruction. However, for hotter domains (T < 1500 K), where even the most stable dust materials cannot survive, it is necessary to take absorption and scattering due to gaseous species into account.

Different Opacity models depending on the various chemical constituents considered,





III) The radiative -module in fargOCA code

Solving heating equation: implicit scheme (solved with Successive-over Relaxation)

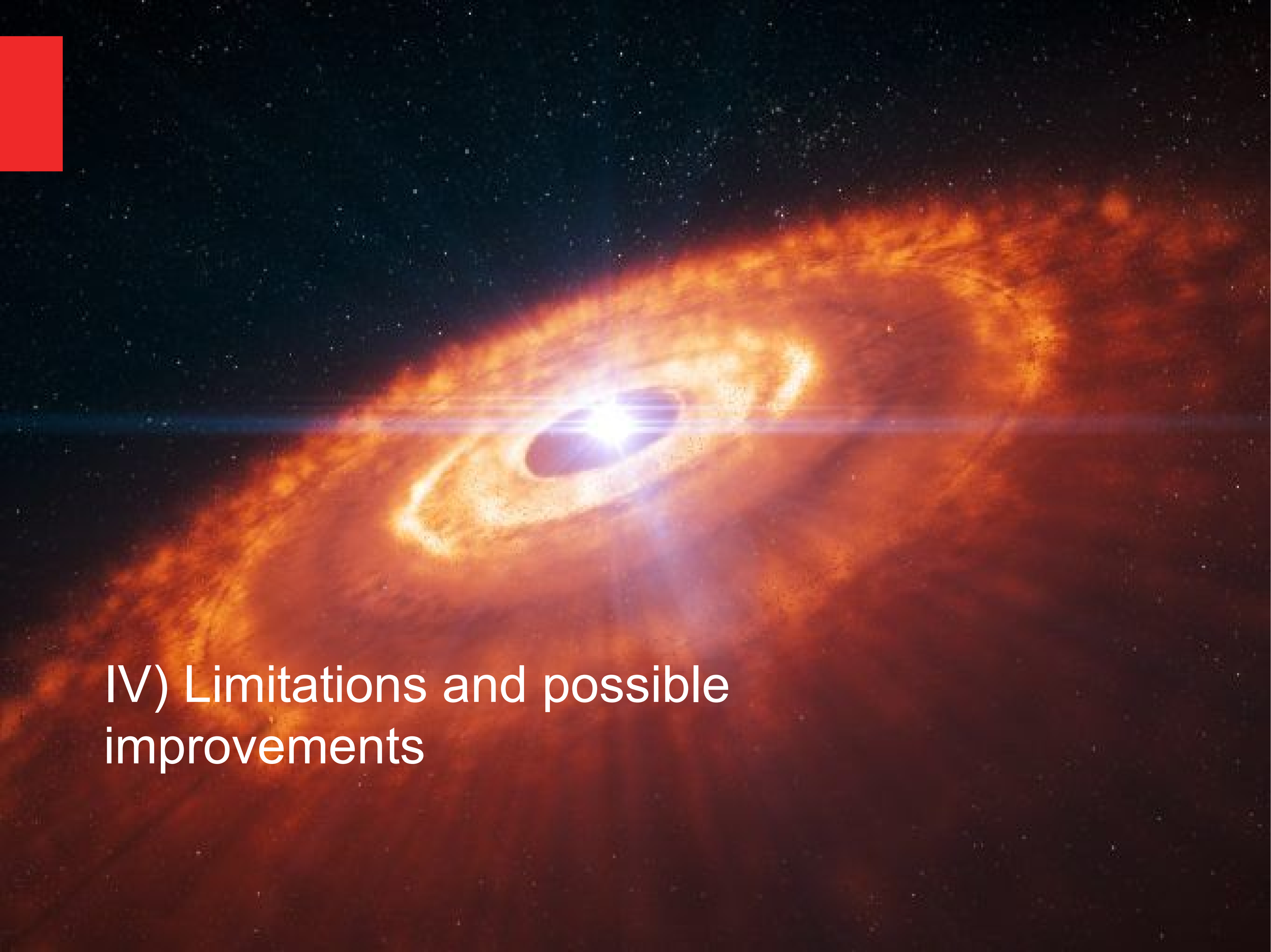
- Heat equation
$$\frac{\partial E_{\mathrm{r}}}{\partial t} = \nabla . K \nabla E_{\mathrm{r}}$$

- Implict discretization
$$\frac{E_{\mathrm{r},i}^{n+1} - E_{\mathrm{r},i}^{n}}{\Delta t} = K \frac{E_{\mathrm{r},i+1}^{n+1} - 2E_{\mathrm{r},i}^{n+1} + E_{\mathrm{r},i-1}^{n+1}}{\Delta x^2}$$

- Truncation error
$$TE = \frac{\Delta t}{2}\frac{\partial^2 E_{\mathrm{r}}}{\partial t^2} - K\frac{\Delta x^2}{12}\frac{\partial^4 E_{\mathrm{r}}}{\partial t^4} + 0(\Delta t^3, \Delta x^5)$$

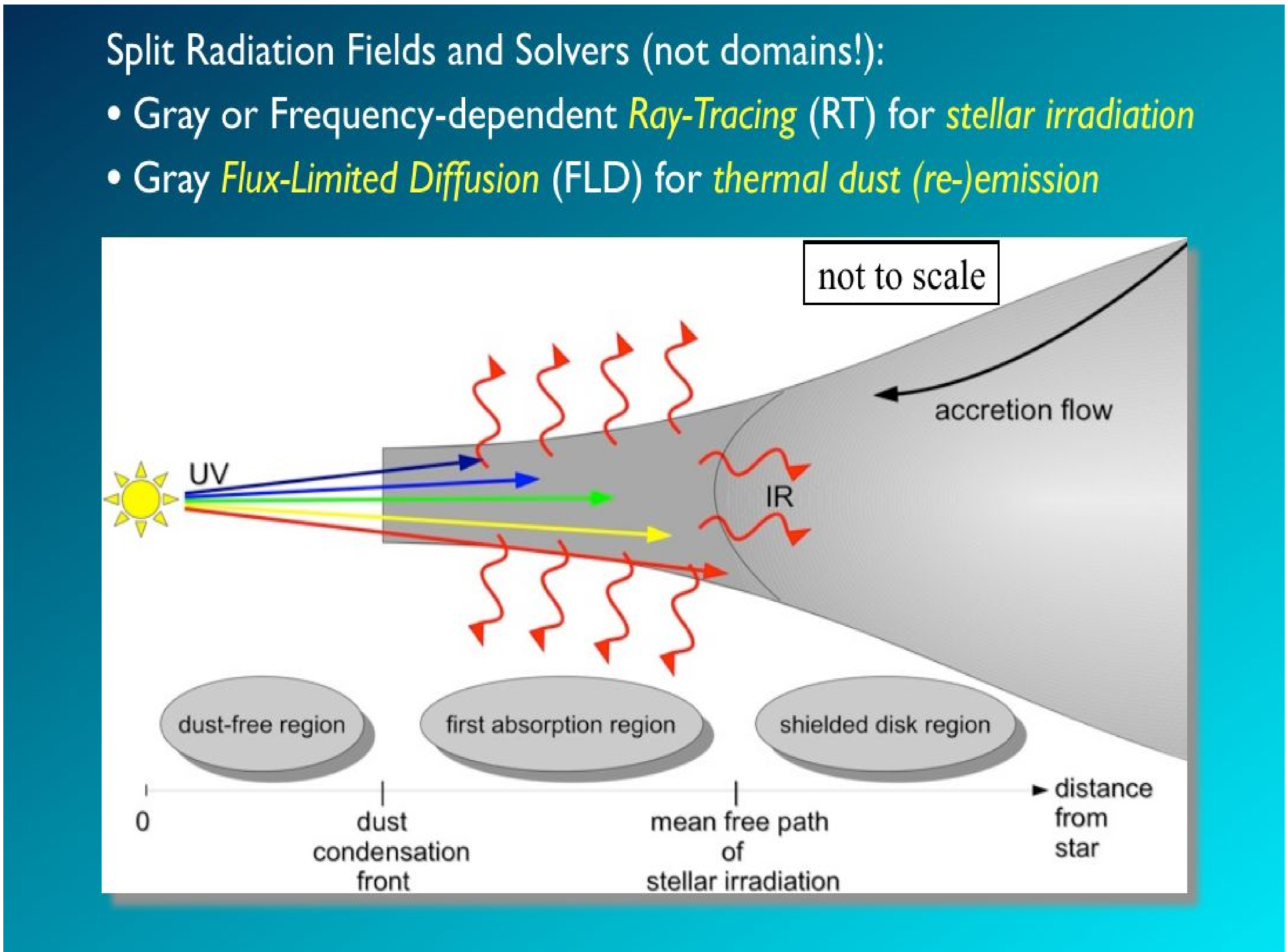
- Unconditionnaly stable

The SoR is a sequential method → parallelization



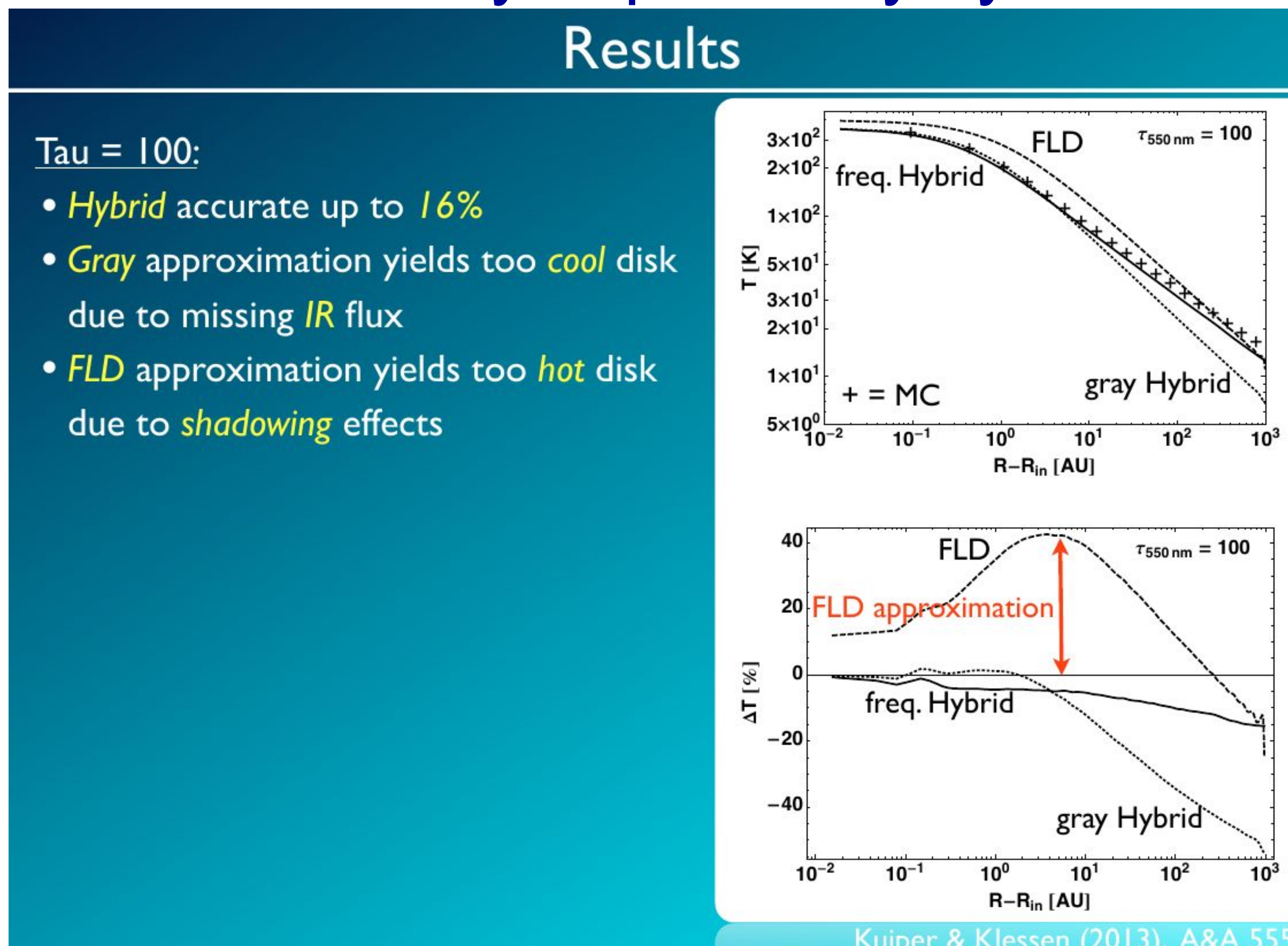
IV) Limitations and possible improvements

Is gray FLD reliable for irradiated circumstellar disks? (Kuiper and Klessen 2013)



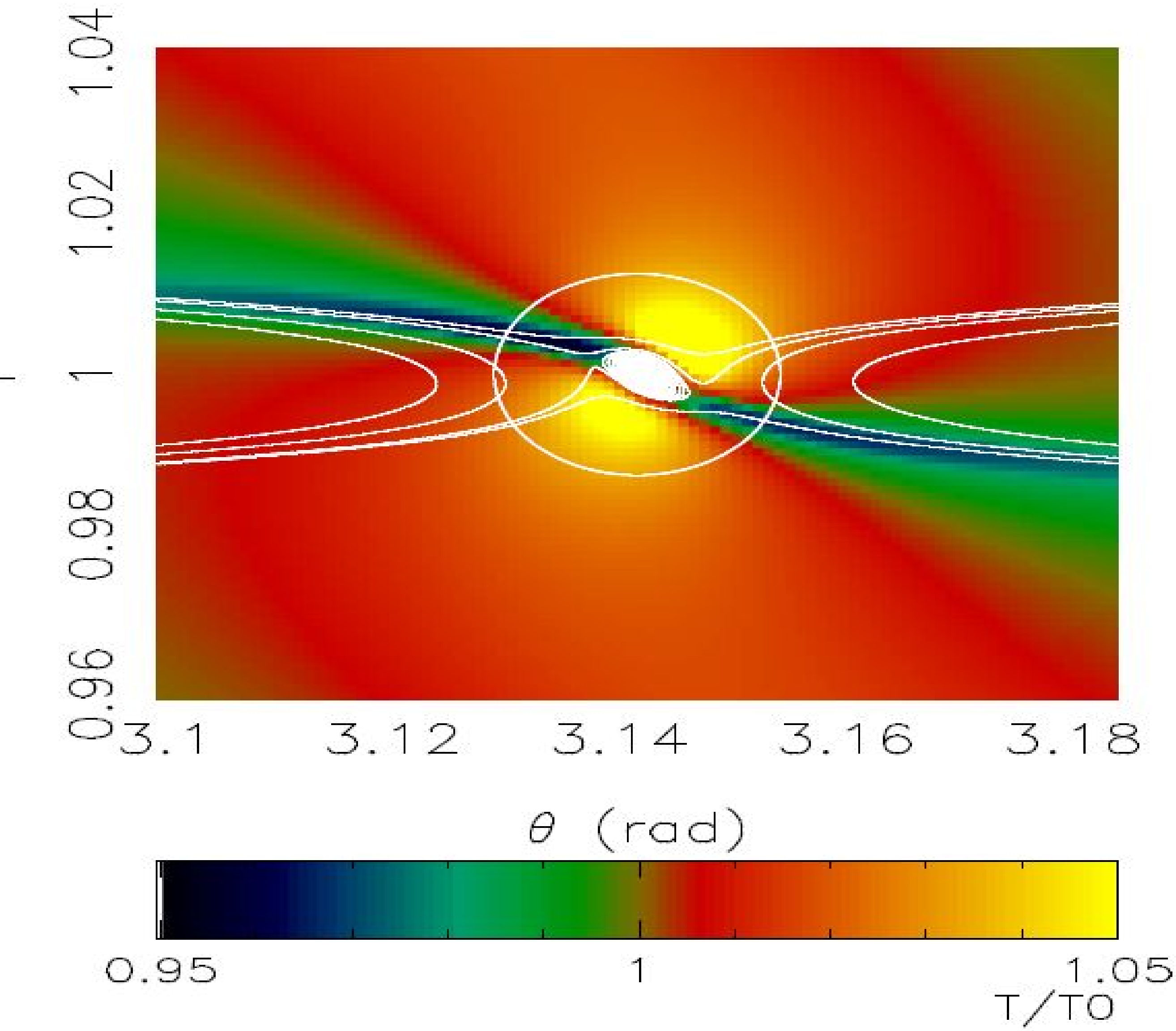
Is gray FLD reliable for irradiated circumstellar disks? (Kuiper and Klessen 2013)

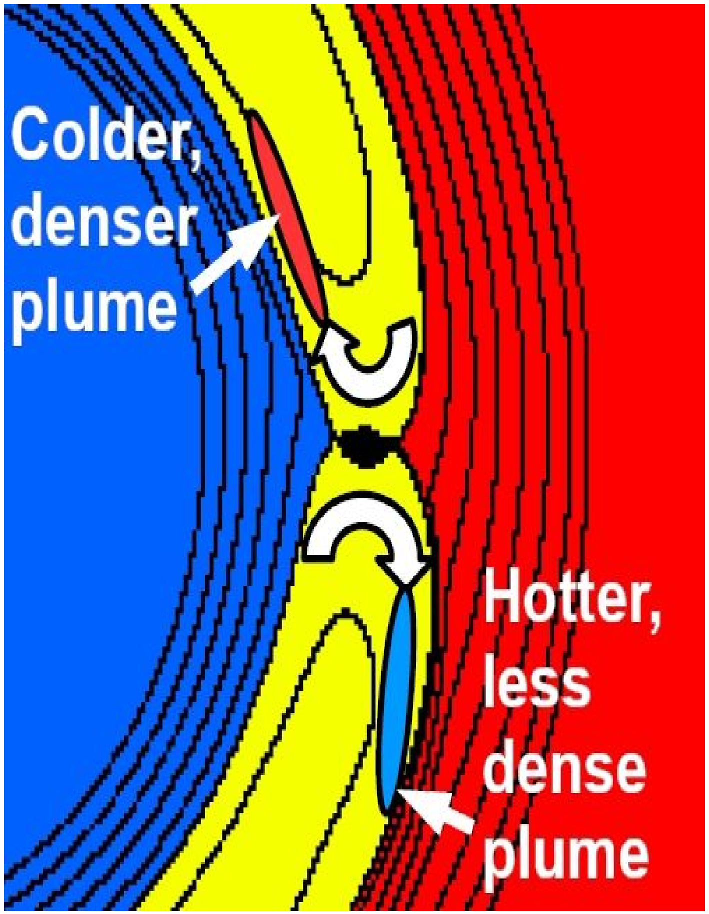
→ Important for disc structure
→ Long integration time for study of planetary dynamics



III) Some results on migration and disc structure

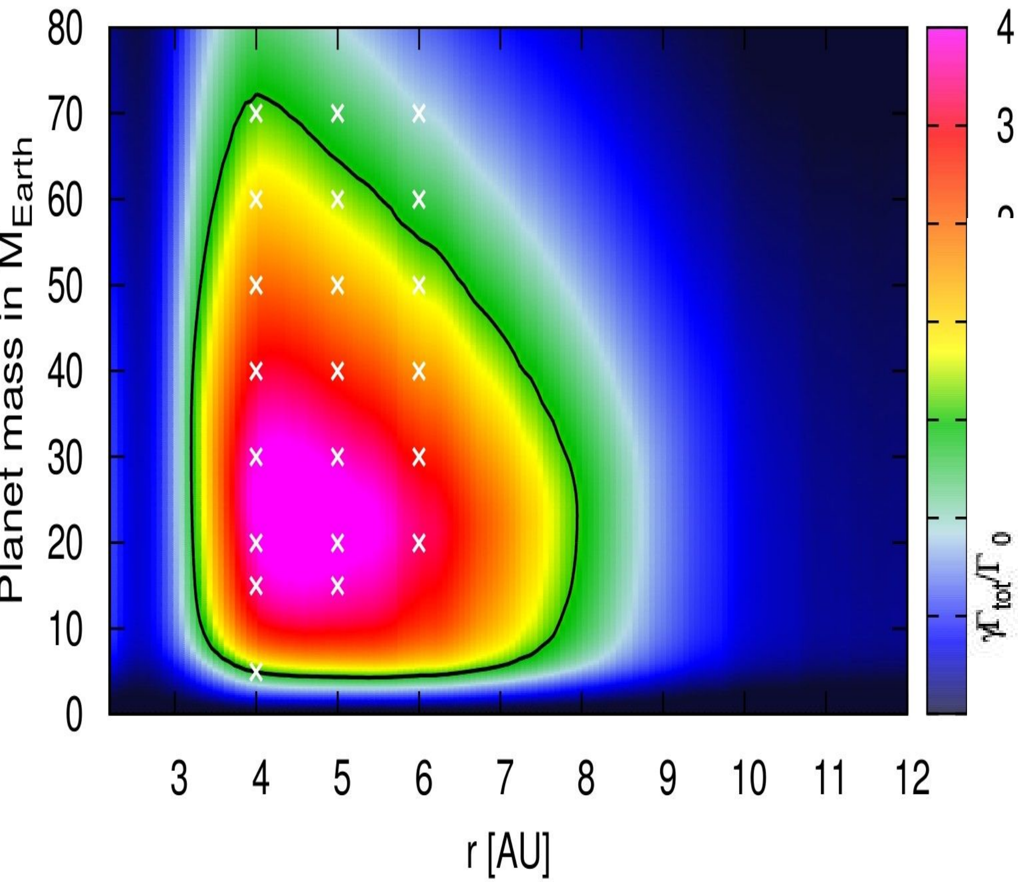
Results: new phenomena at play in the planetary migration process: 3D effects and thermal effects (Lega et al. MNRAS, 440, 2014)

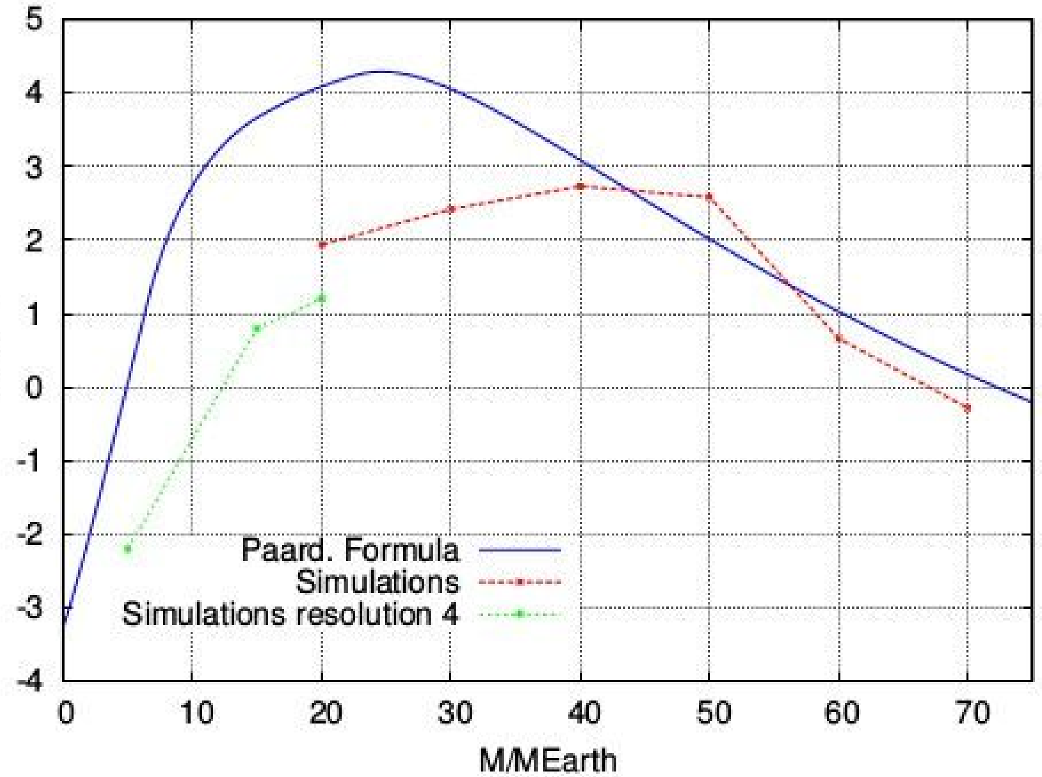




Results:

Check the validity of analytic formulae providing migration maps
Example : The case of a stellar irradiated accretion disc
Bitsch et al. 2013, Lega et al. MNRAS, 452, 2015)

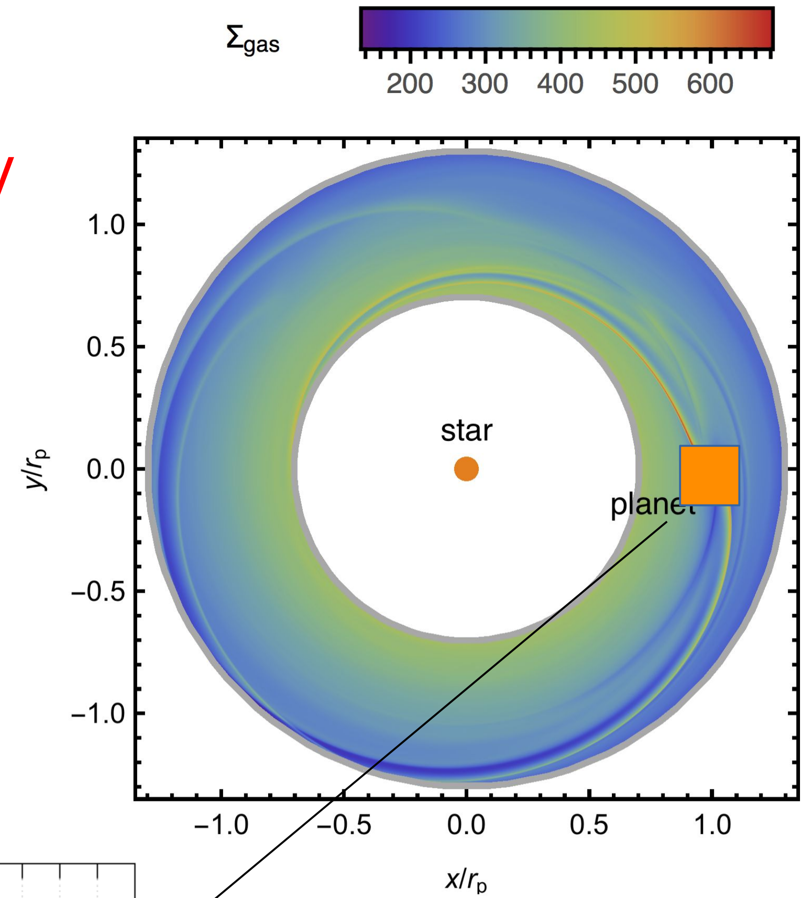


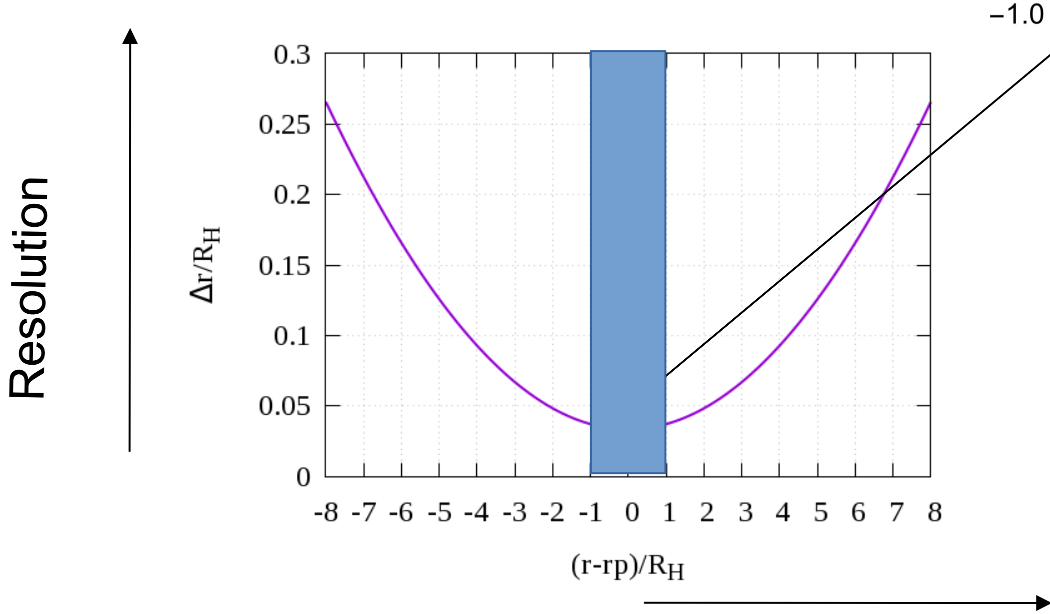


In order to study phenomena at play in the close vicinity of a planet :

Introdution of a nonuniform grid geometry

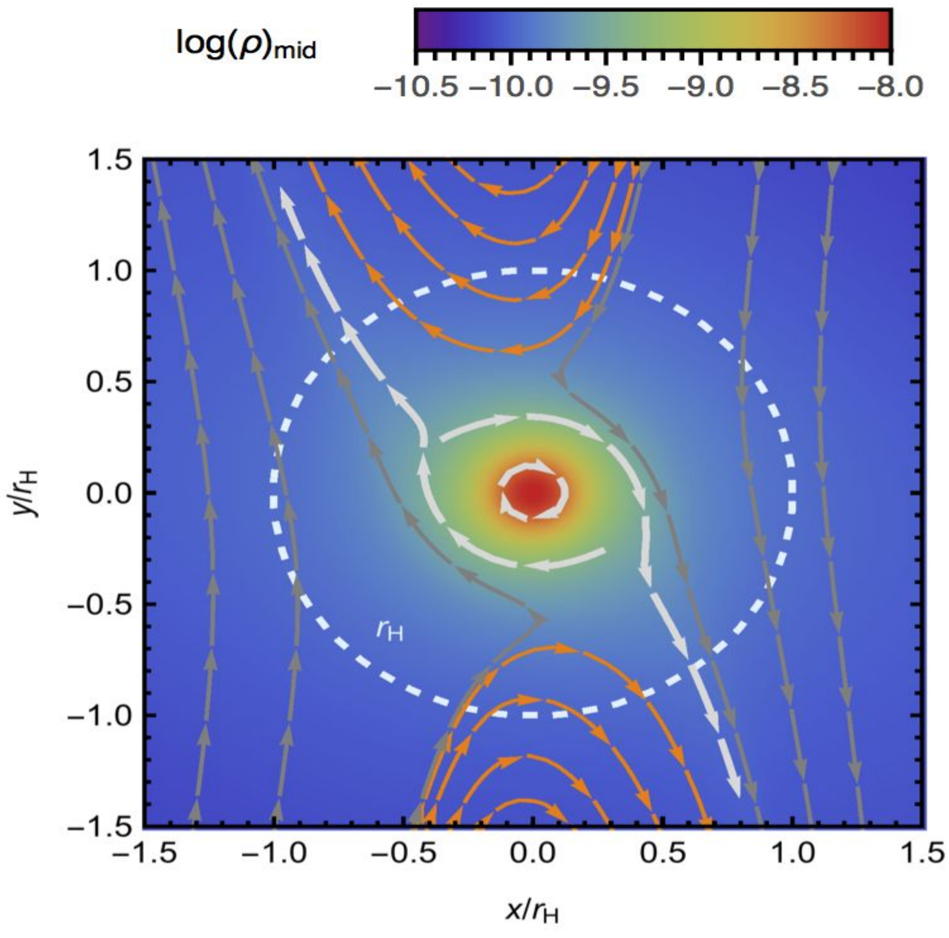
Resolution :
In order to acheive high resolution close to the planet -> use of a nonuniform grid geometry



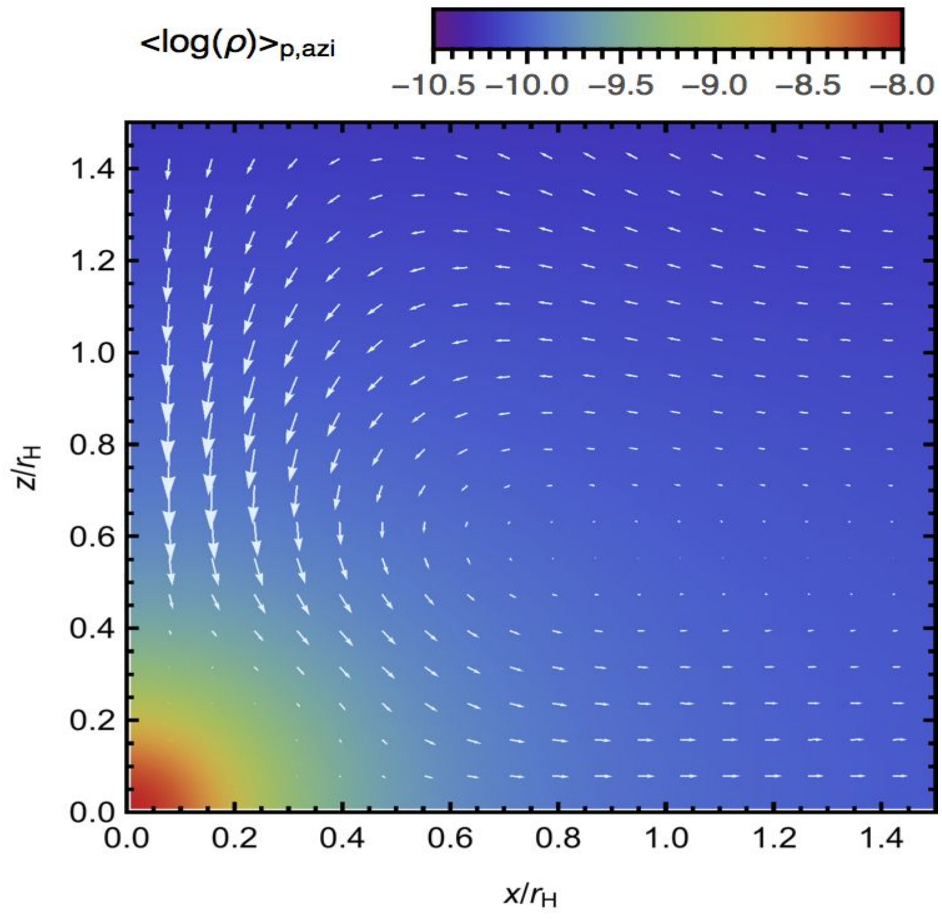


LAGRANGE

Results: Reduced gas accretion on super-Earths and ice giants (Lambrechts, M. Lega E. A&A 2017)



Midplane density field



Vertical view : azimuthally averaged density and gas flow

The main dust constituents include amorphous pyroxene ([Fe, Mg]SiO3), olivine ([Fe,Mg]2 SiO4), volatile and refractory organics (CHON material), amorphous water ice, troilite (FeS), and iron1. As in HS, we vary relative iron content in the silicates considering “iron-rich” (IRS) silicates with Fe/(Fe+Mg) = 0.4, “normal” silicates (NRM) with Fe/(Fe+Mg) = 0.3, and “iron-poor” (IPS) silicates with Fe/(Fe+Mg) = 0. However, the absolute amount of metallic iron in all models is kept constant, which leads to the absence of solid iron in first case and enhanced mass fraction of Fe in third case. Such a variety of silicate models allows us to study the influence of iron distribution within the grain constituents on the extinction properties of dust. Another reason is that the exact mineralogical composition of the silicates in the protostellar clouds and protoplanetary discs is poorly constrained and can be different for various environments. Compared to HS, we re-estimated the absolute abundances of the silicates, iron, and troilite by chemical equilibrium calculations. The mass fractions of all dust constituents and their densities are quoted in Table 1 (see also Table 2 in PHB). Note the difference between the iron mass fractions in the different dust models.

The sublimation temperatures of the grain constituents are adopted from PHB (see Table 3 therein). We suppose that destruction of dust materials occurs in a narrow range of temperatures (z 10 –30 K). Given that the evaporation of the silicates and iron happens at approximately the same conditions, we do not distinguish between their evaporation temperatures and assume that they evaporate in one temperature range, DT z 100 K. Thus, we account for six principal temperature regions:

T < 155 K - all dust constituents are present;
z 165 K < T < 270 K - no ice;
z 280 K < T < 410 K - no ice and volatile organics;
z 440 K < T < 675 K - silicates, iron, and troilite are present;
z 685 K < T < 1500 K - silicates and iron are present;
T < 1500 K - gas-dominated opacities;