Stellar spectroscopy: physics and methods

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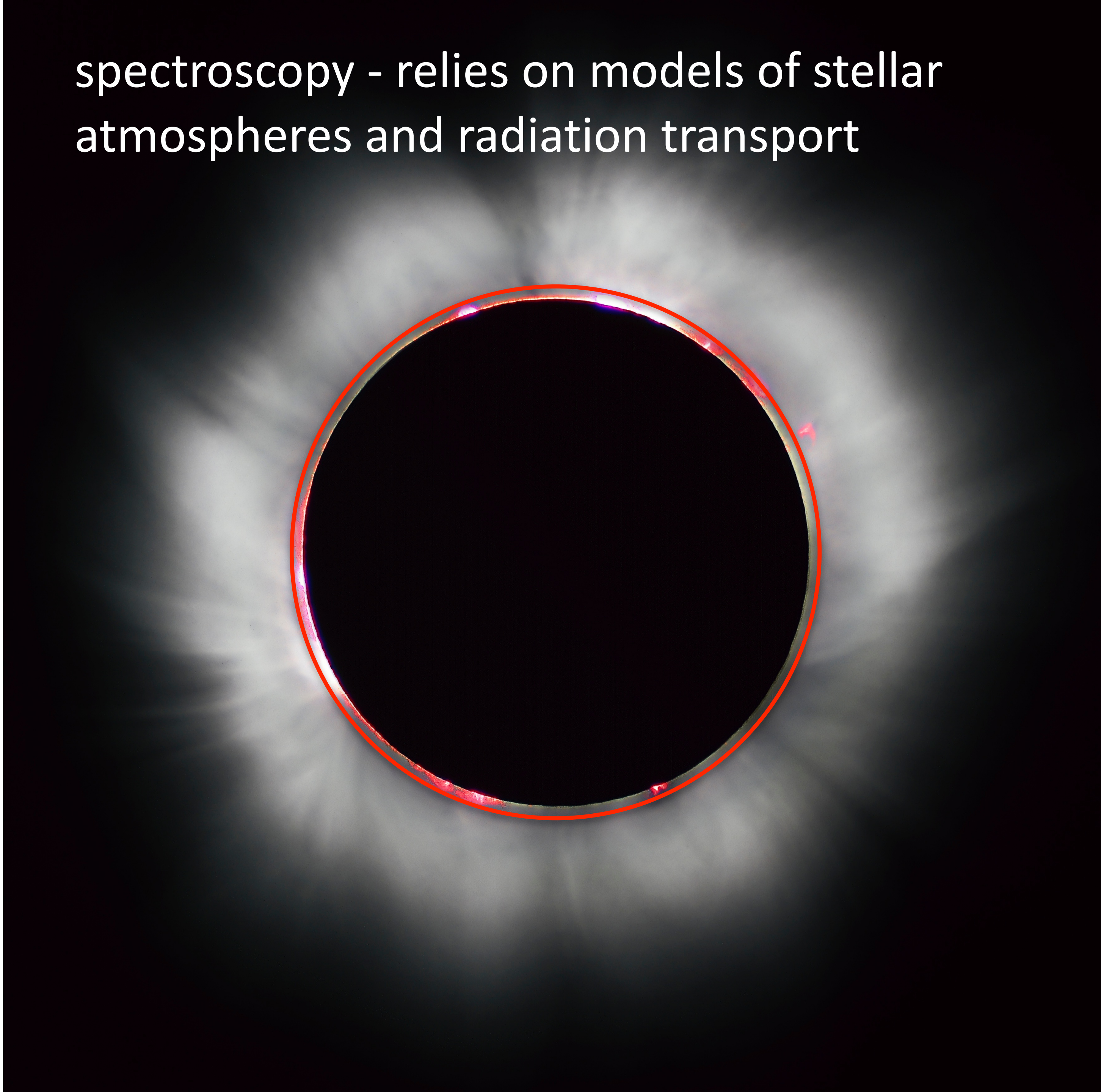
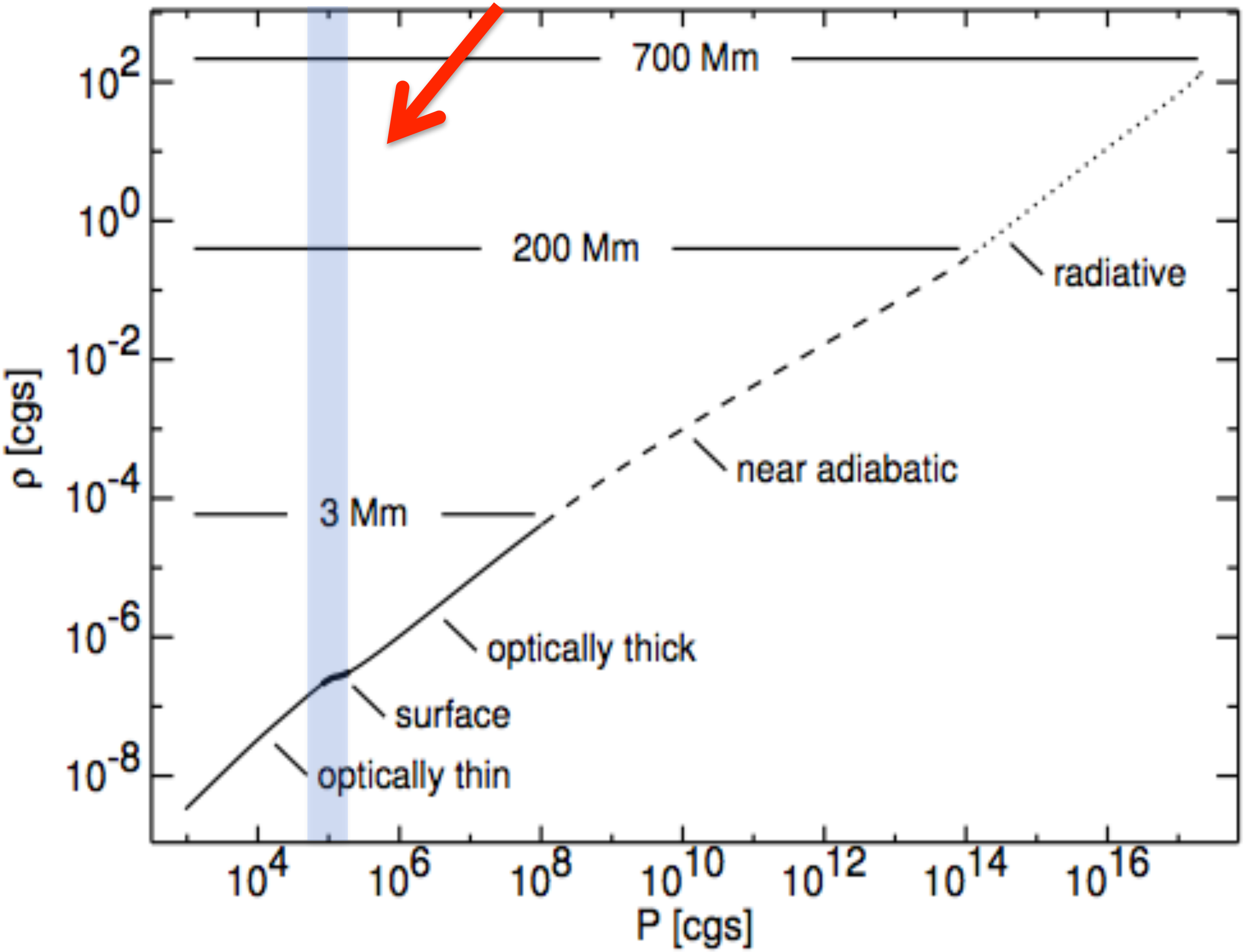
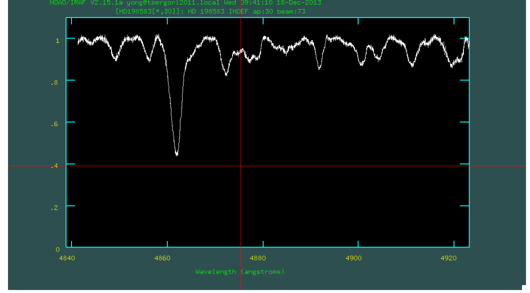
spectroscopy - relies on models of stellar atmospheres and radiation transport



Stellar atmosphere



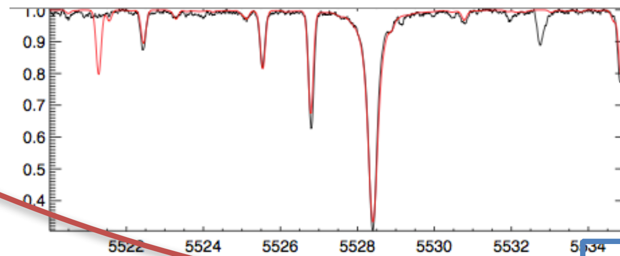
Nordlund et al. 2009



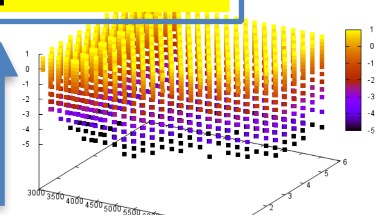
Observed spectrum

Stellar parameters:
T_{eff},
surface gravity (P)
velocities,
chem. abundances
up to 100 dimensions!

Comparison



Model spectrum



Model atmosphere

```
READ DECK6 72 RHOX,T,P,XNE,ABROSS,ACCRAD,VTURB, FLXCNV,VCONV,VEL:
 1.11438611E-03   4303.3 1.114E+01 1.188E+09 1.197E-04 5.762E-03
 1.46933430E-03   4325.0 1.469E+01 1.499E+09 1.309E-04 5.781E-03
                  4338.1 1.906E+01 1.806E+09 1.404E-04 5.788E-03
                  4355.4 2.443E+01 2.200E+09 1.541E-04 5.798E-03
                  4372.8 3.093E+01 2.664E+09 1.704E-04 5.816E-03
                  4391.0 3.874E+01 3.217E+09 1.900E-04 5.851E-03
                  4409.4 4.8
                  4428.0 5.911E+01 4.629E+09 2.393E-04 5.939E-03
 7.22091430E-03   4446.5 7.221E+01 5.515E+09 2.698E-04 5.995E-03
 8.76903374E-03   4464.8 8.769E+01 6.544E+09 3.046E-04 6.061E-03
 1.05970558E-02   4482.7 1.060E+02 7.729E+09 3.440E-04 6.141E-03
 1.27556538E-02   4500.1 1.276E+02 9.088E+09 3.885E-04 6.240E-03
 1.53040028E-02   4517.4 1.530E+02 1.066E+10 4.389E-04 6.356E-03
 1.83113451E-02   4534.4 1.831E+02 1.246E+10 4.961E-04 6.489E-03
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Spectrum synthesis

Basic model atmosphere theory (usually tailored to a given class of stars): non-LTE (NLTE), 3D hydrodynamics, magnetic fields, winds, sphericity, molecular opacities, binarity, chromospheres, etc

1D LTE MODEL ATMOSPHERES AND SPECTRUM SYNTHESIS

1D LTE (hydrostatic) models

$$\cos\theta \frac{dI_\nu}{dz} = \kappa_\nu I_\nu - \eta_\nu$$

radiative transport equation in the static (t-independent case)

$$F = \frac{L}{4\pi R^2} = \sigma T_{\rm eff}^4,$$

flux conservation —> div(F) = 0

$$\boxed{F = F_{conv} + F_{rad}}$$

$$F_{conv} \sim \frac{\alpha_{MLT}}{H_p},$$

convective flux - based on mixing-length theory

$$\int_0^\infty J_\nu(\tau) d\nu = \int_0^\infty S_\nu(\tau) d\nu$$

radiative equilibrium equation

$$\nabla P_{tot} = -\rho \frac{GM_r}{r^2},$$

hydrostatic equilibrium equation

$$\boxed{\nabla P_{\rm tot} = \nabla P_{\rm g} + \nabla P_{\rm turb} + \nabla P_{\rm rad},}$$

$$\boxed{\nabla P_{rad} = -1/c \int_0^\infty (\kappa_\nu + \sigma_\nu) F_\nu d\nu}$$

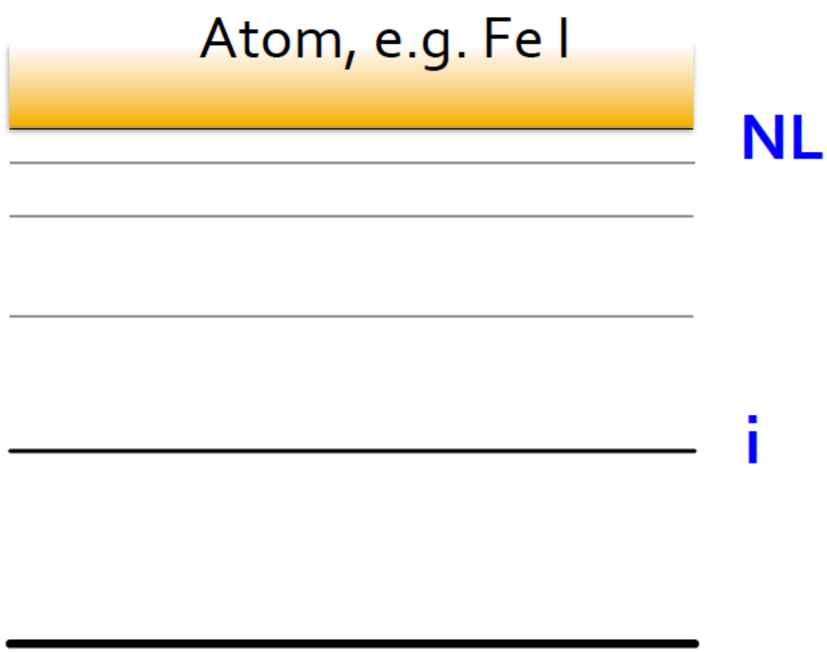
$$\boxed{\nabla P_{turb} \sim \rho \upsilon_t^2}$$

turbulent and radiative pressures

LTE

We neglect transitions in the atom caused by radiation

$n_{i,c}$ – level population, [atoms /cm³]
χ – ionization energy of an ion



$$[\, n_{c+1}/n_{i,c} \,] \sim N_e^{-1}\, T^{3/2}\, e^{(-\chi/KT)}$$

$$\downarrow$$

$$S_\nu,\ \kappa_\nu,\ \sigma_\nu$$

$$\downarrow$$

$$I_\nu = \int_0^\infty S_\nu e^{-\tau_\nu} d\tau_\nu ,$$

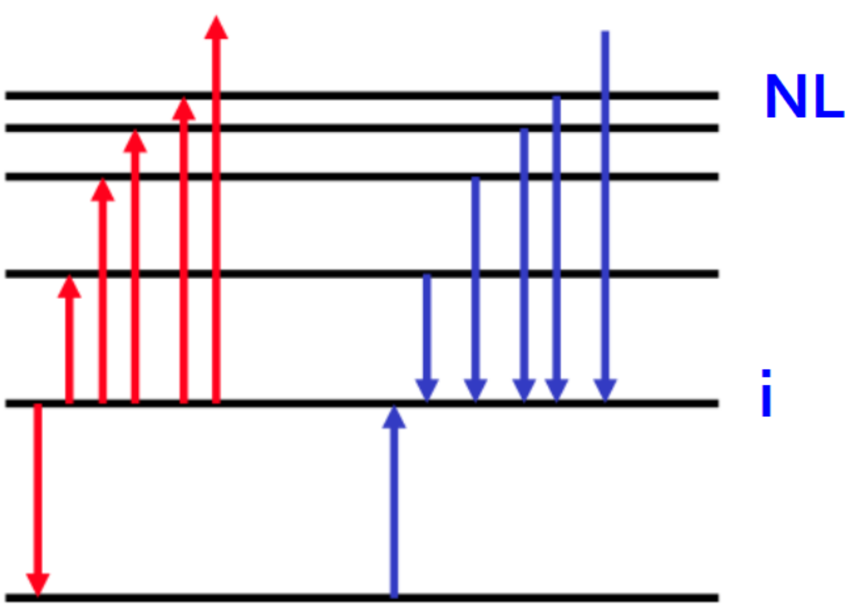
emergent intensity I_ν

Non-LTE

$$n_i \sum (C_{ij} + R_{ij}) = \sum n_j (C_{ji} + R_{ji}), \quad i = 1, \ldots, NL$$

Rates out = Rates in

C_{ij}, R_{ij} – transition rates [1/second/particle]



Statistical equilibrium
the number of atoms in each excitation level i and each ionization stage j

rate equations for N energy levels + radiation transfer

$$\sum_{n>m} N_n \left(A_{nm} + B_{nm}u_\nu + C_{nm}\right) + \sum_{k<m} N_k \left(B_{km}u_\nu + C_{km}\right) + N_\mathrm{e}\left(R_m + Q_m\right) - N_m \left\{ \sum_{k<m}\left(A_{mk} + B_{mk}u_\nu + C_{mk}\right) + \sum_{n>m}\left(B_{mn}u_\nu + C_{mn}\right) + \left(P_m + S_m\right) \right\} = 0$$

$$P_m = 4\pi \int \frac{a_\nu J_\nu}{h\nu}\, d\nu$$

spontaneous radiative emission A_{nm}

photo-ionisation P_m

recombination R_m

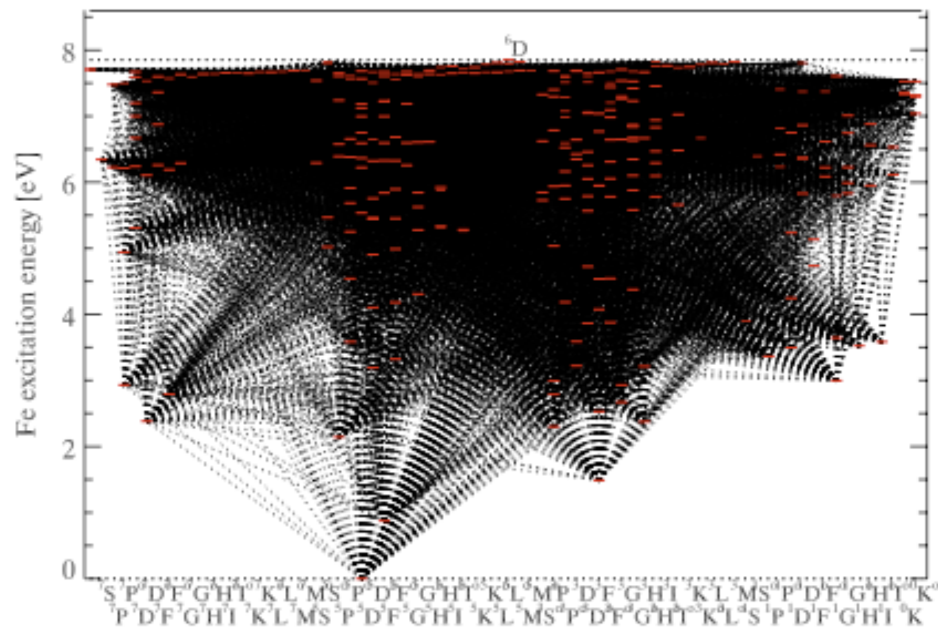
collisional excitation C_{mk}

charge transfer …

photons, electrons,

H atoms …

Fe I model atom



Non-LTE (statistical equilibrium)

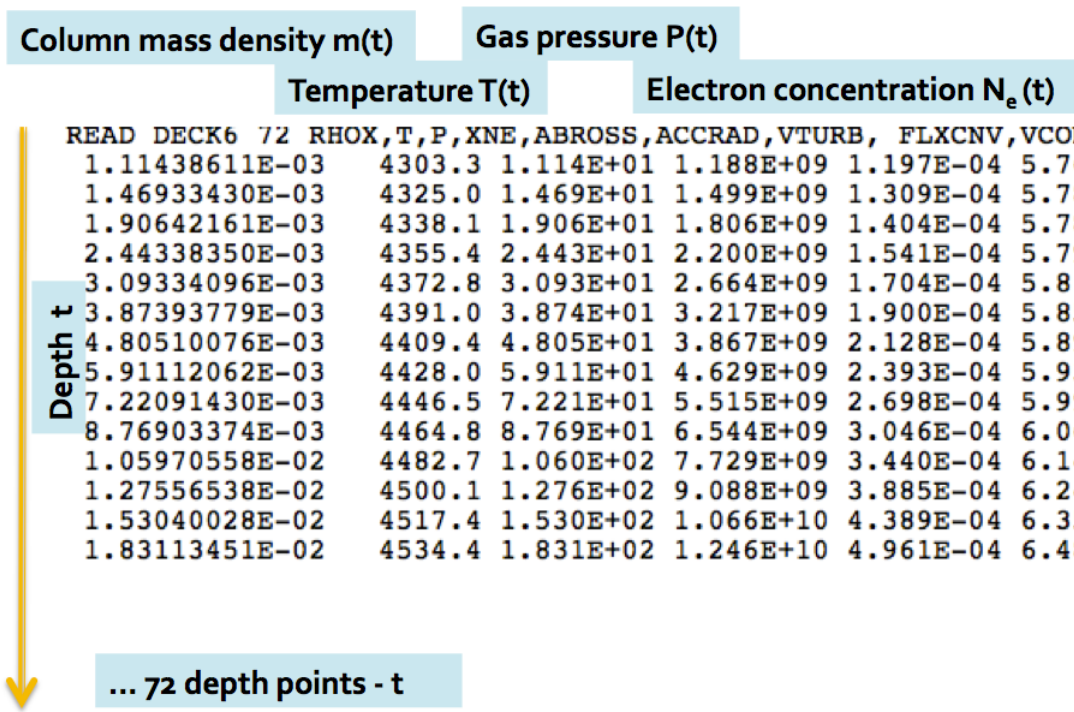
SE is usually solved using a dedicated code assuming a fixed model atmosphere

MULTI / 1D
https://folk.uio.no/matsc/

DETAIL (not public)

MULTI / 3D (not public)

Balder (not public)

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 1.53040028E-02    4517.4 1.530E+02 1.066E+10 4.389E-04 6.3
 1.83113451E-02    4534.4 1.831E+02 1.246E+10 4.961E-04 6.4
```

LTE and NLTE spectrum synthesis

the profile function

$$\psi(\nu-\nu_0)=\phi(\nu-\nu_0)=\frac{H(a,v)}{\sqrt{\pi}\Delta\nu_D} \quad \text{with} \quad a=\frac{\gamma_R+\gamma_3+\gamma_4+\gamma_6}{4\pi\Delta\nu_D} \qquad v=\frac{\nu-\nu_0}{\Delta\nu_D}$$

line absorption coefficient

$$\kappa_\lambda^l=\frac{\pi e^2}{m_e c}\frac{\lambda}{c}b_i\frac{N_i^{\rm LTE}}{N_{\rm El}}N_{\rm H}\log\varepsilon f_{ij}\frac{H(a,v)}{\Delta\lambda_D}\left(1-\frac{b_j}{b_i}{\rm e}^{-hc/\lambda kT}\right) \quad \boxed{\kappa_\lambda=\kappa_\lambda^l+\kappa_\lambda^c,}$$

$$S_\nu\equiv\frac{2hc^2}{\lambda^5}\frac{1}{{\rm e}^{hc/kT\lambda}-1}=B_\nu$$ **Line source function (= Planck function in LTE)**

$$I_\lambda(\tau_\nu=0,\mu)=\int_0^\infty S_\lambda(\tau_\lambda)e^{-\tau_\lambda/\mu}d\tau_\lambda/\mu$$ **Emergent intensity**

$$F_\lambda(0)=2\pi\int_0^\infty S_\lambda(T(\tau_\lambda))E_2(\tau_\lambda)d\tau_\lambda$$ **Emergent flux**

3D NLTE LINE FORMATION

3D RHD models

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \boldsymbol{v}),$$

mass conservation

$$\frac{\partial \rho \boldsymbol{v}}{\partial t} = -\nabla \cdot (\rho \boldsymbol{v} \boldsymbol{v}) - \nabla P - \rho \nabla \Phi - \nabla \cdot \tau_{visc},$$

momentum conservation

$$\Phi = -\frac{GM}{(r_0^4 + r^4/\sqrt{1+(r/r_1)^8})^{1/4}} \qquad d\Phi/dz = g,$$

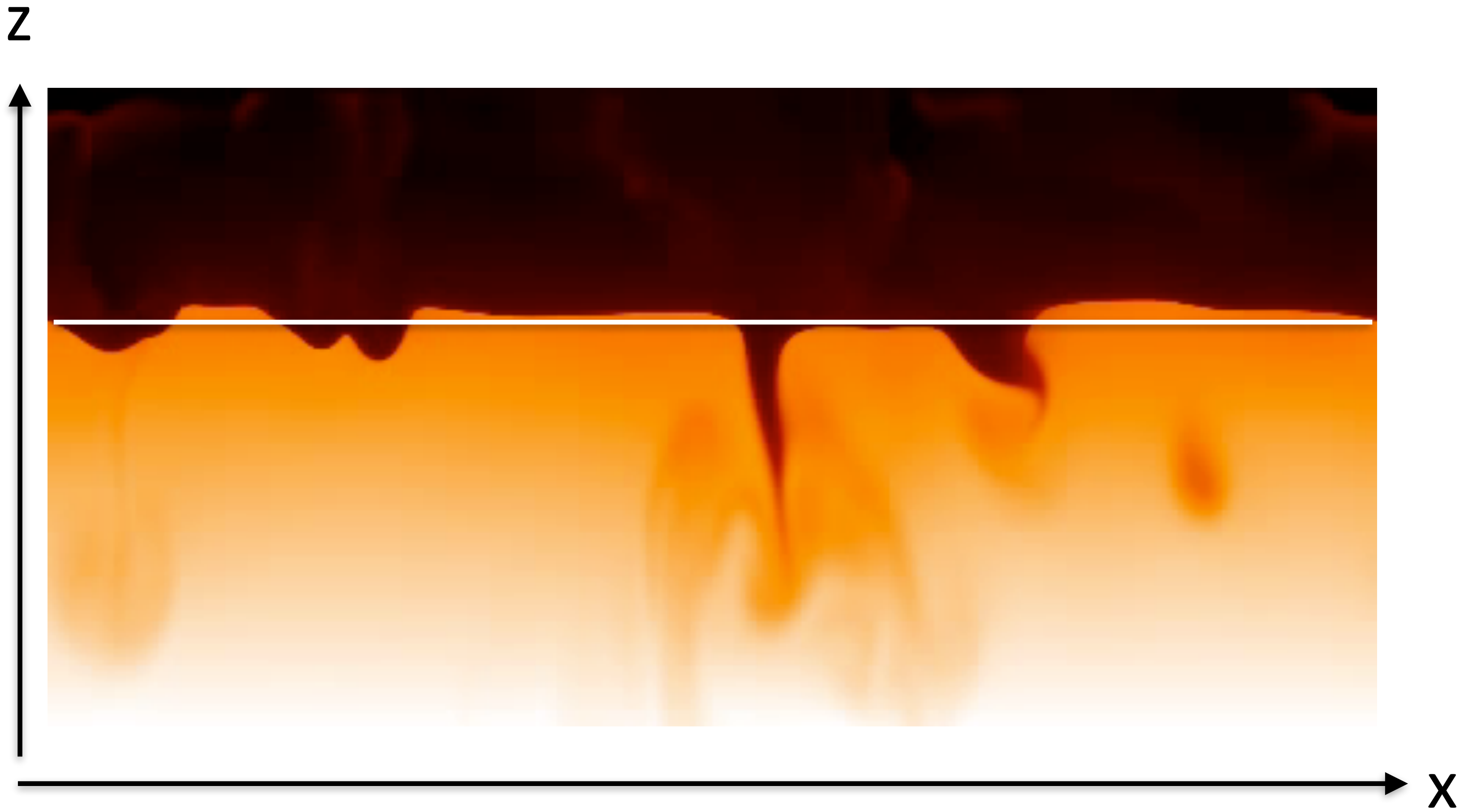
$$\frac{\partial e}{\partial t} = -\nabla \cdot (e \boldsymbol{v}) - P(\nabla \cdot \boldsymbol{v})/\rho + Q_{\mathrm{rad}} + Q_{\mathrm{visc}}$$

energy conservation

$$Q_{\mathrm{rad}} \equiv \sum_{n=1,n_bins} (J_{bin} - B_{bin}) w_{bin},$$

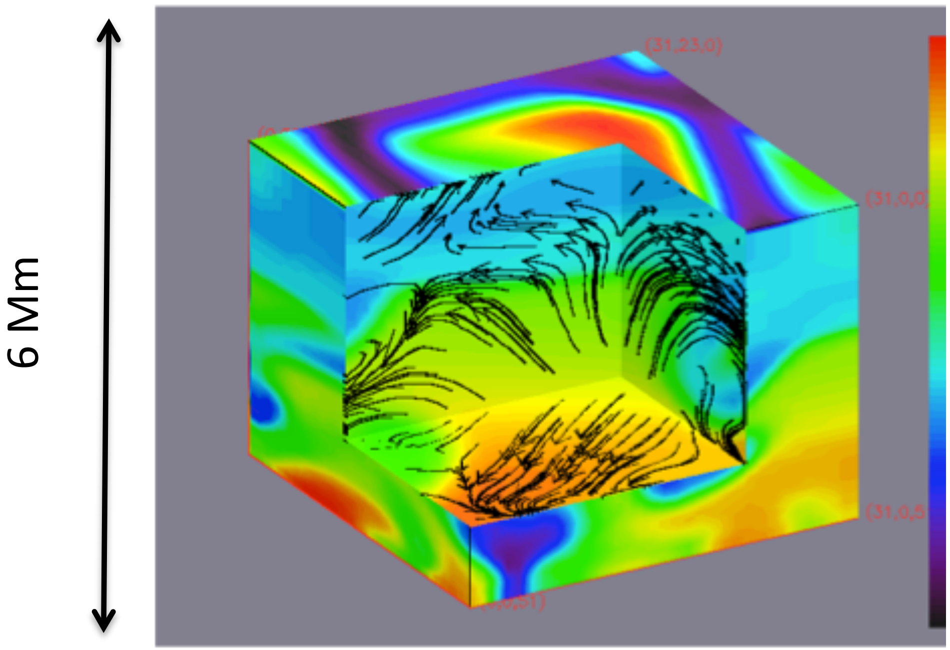
radiative energy exchange
(see 1D LTE case of Fconv =0)

$$\int_0^\infty J_\nu(\tau) d\nu = \int_0^\infty S_\nu(\tau) d\nu$$



colour - temperature

3D RHD models



velocity and temperature structure of a granule in a numerical 3D RHD simulation of the solar convection

Nordlund et al. 2009

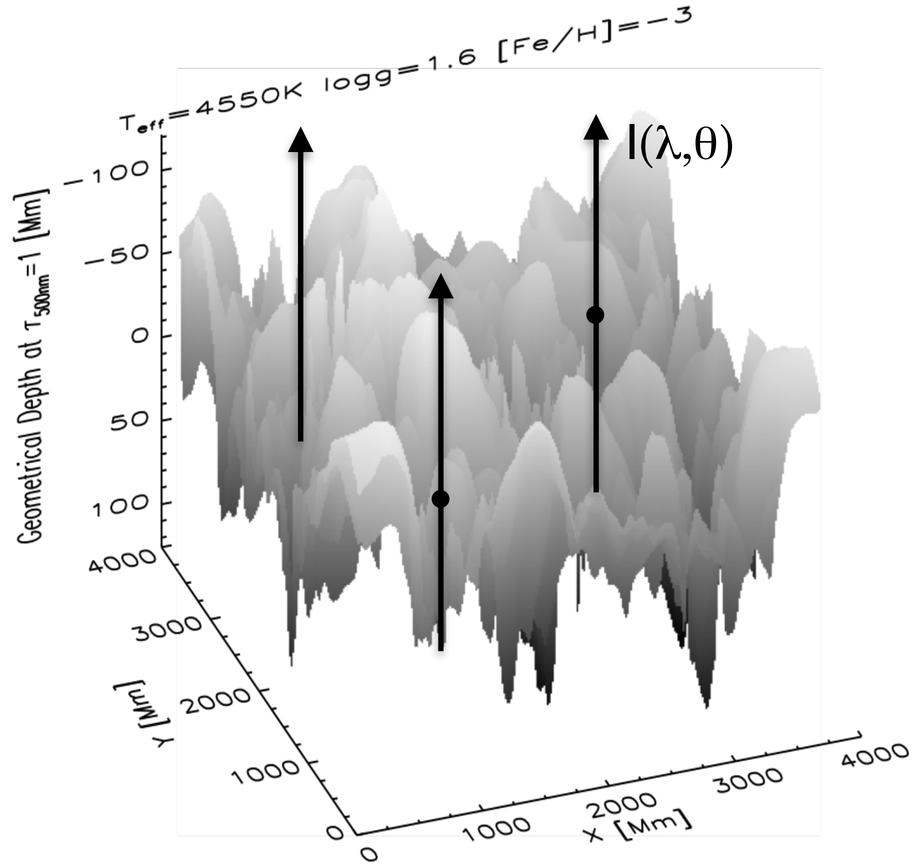
1D hydrostatic model

3D Non-LTE radiative transfer

✓ Non-LTE (statistical equilibrium, t-independent)

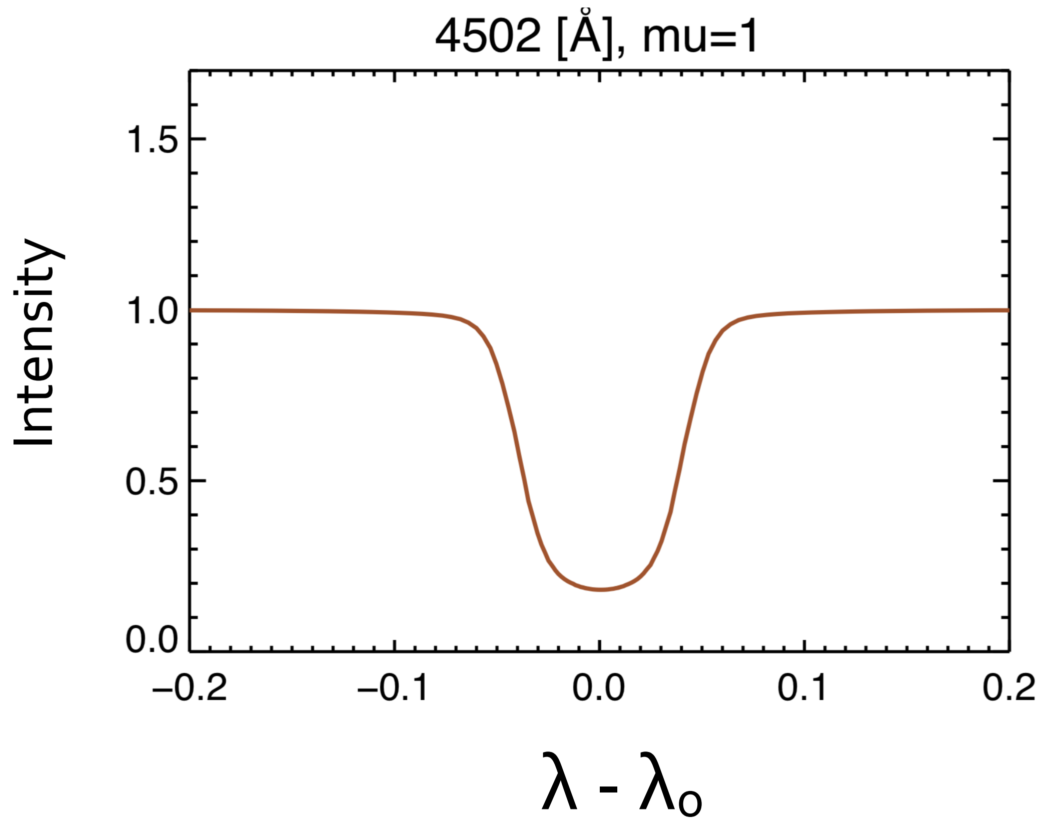
✓ Hydrodynamical models as fixed thermodynamic structures (T, P_e, velocities)

- 'trace element' assumption one element at a time
- no effect on the structure of a model atmosphere
- usually vertical RT sufficient
- typical resolution (30,30,230)



1D LTE radiative transfer

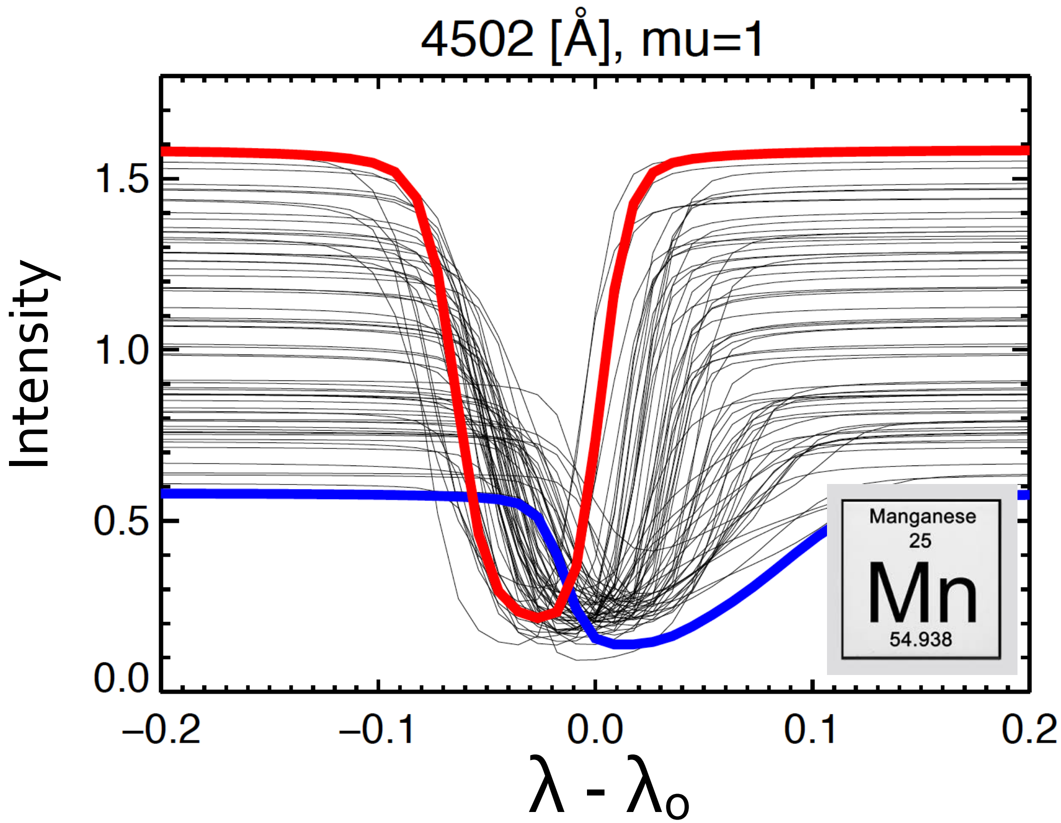
Mn line in the Sun

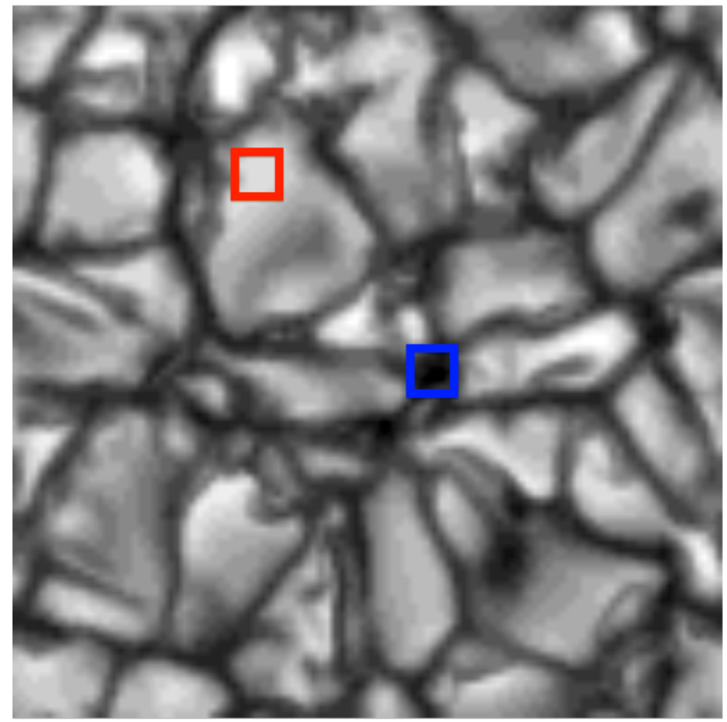


Bergemann et al. (2019, arXiv:1905.01835)
Gallagher et al. in prep.

3D Non-LTE radiative transfer

Mn line in the Sun





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