

THE INTERNAL STRUCTURE OF THE SUN INFERRED FROM G MODES AND LOW-FREQUENCY P MODES

Y. Elsworth¹, F. Baudin², W. Chaplin¹, B. Andersen³, T. Appourchaux², P. Boumier², A.-M. Broomhall¹, T. Corbard⁴, W. Finsterle⁵, C. Fröhlich⁵, A. Gabriel², R.A. García⁶, D.O. Gough^{7,8}, G. Grec⁴, A. Jiménez⁹, A. Kosovichev¹⁰, J. Provost⁴, T. Sekii¹¹, T. Toutain^{3,4}, and S. Turck-Chièze⁶

¹*School of Physics and Astronomy, University of Birmingham, Edgbaston, Birmingham B15 2TT, UK*

²*Institut d'Astrophysique Spatiale, CNRS-Université Paris XI UMR 8617, 91405 Orsay Cedex, France*

³*Norwegian Space Center, N-0212 Oslo, Norway*

⁴*Département Cassiopée, UMR CNRS 6202, Observatoire de la Côte d'Azur, BP 4229, 06304 Nice Cedex 4, France*

⁵*Physikalisch-Meteorologisches Observatorium Davos, World Radiation Center, CH-7260 Davos Dorf, Switzerland*

⁶*CEA/DSM/DAPNIA, CE Saclay, 91191 Gif sur Yvette, France*

⁷*Institute of Astronomy, University of Cambridge, Cambridge CB3 0HA, UK.*

⁸*Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge CB3 0WA, UK*

⁹*Instituto de Astrofísica de Canarias, E-38205 La Laguna, Tenerife, Spain*

¹⁰*W. W. Hansen Experimental Physics Laboratory, Stanford University, Stanford, CA 94305-4085*

¹¹*Solar & Plasma Astrophysics Division and Solar-B Project Office, National Astronomical Observatory of Japan, Natural Institutes of Natural Sciences, Mitaka, Tokyo 181-8588, Japan*

ABSTRACT

The Phoebus group is an international collaboration of helioseismologists, its aim being to detect low-frequency solar g modes. Here, we report on recent work, including the development and application of new techniques based on the detection of coincidences in contemporaneous datasets and the asymptotic properties of the g-mode frequencies. The length of the time series available to the community is now more than ten years, and this has reduced significantly the upper detection limits on the g-mode amplitudes. Furthermore, low-degree p modes can now be detected clearly at frequencies below 1000 μHz .

Key words: g modes - SOHO - Sun.

1. INTRODUCTION

The Phoebus group was formed in 1997, with the aim of detecting g modes. The group set an upper limit to the g-mode amplitude of 10 mm s^{-1} at 200 μHz (Appourchaux et al., 2000). The Phoebus group reported its activity at the Big Bear Lake (Appourchaux, 2003), Tenerife (Appourchaux et al., 2001) and Boston meeting (Appourchaux, 1998; Appourchaux et al., 1998). Members of the GOLF team recently joined us at the beginning of 2005. The group is also now supported by the International Space Science Institute¹.

¹www.issibern.ch/teams/GModes

Since the SOHO/GONG meeting in Big Bear Lake, there have been new developments in detection techniques, both from work undertaken as part of the Phoebus programme, and from collaborations with the group. In what follows, we begin by discussing techniques developed over the past few years, and illustrate these with results. We then go on to give current upper limits on the g-mode amplitudes, and finally summarize where things stand.

2. G-MODE DETECTION TECHNIQUES.

Appourchaux (2003) noted that any recipe for g-mode detection may be comprised of combinations of the following: *Spectrum estimators*, *Mode masking*, *Statistical testing*, *Patterns*, and *Data combinations*. Each of the 'steps' can be combined to give a different search methodology. For instance *Statistical testing* is required on any *Spectrum Estimators* derived from any *Mode masking*. This is one possible example. One step that must always be included is *Statistical testing*, for it provides an essential safeguard against over-interpretation of the data.

Since our last review, a novel technique has been developed by Duvall (2004). It is based on time-distance analysis of p modes, and seeks to uncover and measure flows induced by g modes at the base of the convection zone. This is a very promising technique that uses several of the aforementioned steps, these being spectrum estimators, mode masking, statistical techniques, and patterns. The potential of this technique is yet to be fully realized. Since it aims at observing the g modes deep in the convection zone, this technique might more easily detect greater

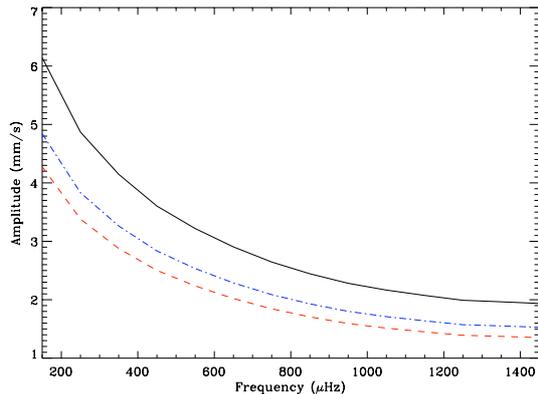


Figure 1. Threshold levels for a 1% probability that anything detected over a range of $100 \mu\text{Hz}$ is noise for single peak (black thick line), multiplet (blue) and for two peaks in consecutive bins (red). The levels are specific to GOLF and BiSON data as they depend on the amount of common noise, which varies amongst spectra.

velocity changes.

Data combination. George Isaak always advocated use of coincidence technique (of the type applied in, for example, nuclear physics). This seminal idea in our field has been used in Fig. 1 to derive the $1\text{-}\sigma$ detection level using a contemporaneous 3071-day combination of GOLF and BiSON data. Proper allowance must be made for the presence of common background noise from the solar granulation. This gives rise to important modifications to the probability thresholds (See A.-M.Broomhall *et al.*, these proceedings).

Patterns: asymptotic properties. The use of the asymptotic behaviour of the g-mode frequencies (or periods) was pioneered by Delache (Delache and Scherrer, 1983; Fröhlich and Delache, 1984). The asymptotic approach is of relevance only to high-order g modes (of frequency below $100 \mu\text{Hz}$) for which the asymptotic behaviour applies. Unfortunately, the solar noise increases towards lower frequencies, and the mode spacing dramatically decreases; matters are further complicated by the rotational splitting (Fröhlich and Andersen, 1995) which also contributes to the overall pattern.

Following discussions in the Phoebus group, we have resumed the work using this technique. As the theoretical mode amplitude increases with increasing frequency (see e.g. Fig. 4) the asymptotic frequencies have to be corrected with e.g. the results from the detailed model calculations (Provost *et al.*, 2000). Moreover, the rotational splitting has to be corrected not only for the influence of the Coriolis force (Berthomieu *et al.*, 1978), but also for the effect of a possibly rapidly rotating core which has to agree with the results of the p modes in the outer layers of the radiative core. The method we use is not based upon bivariate analysis (Fröhlich and Andersen, 1995) but is

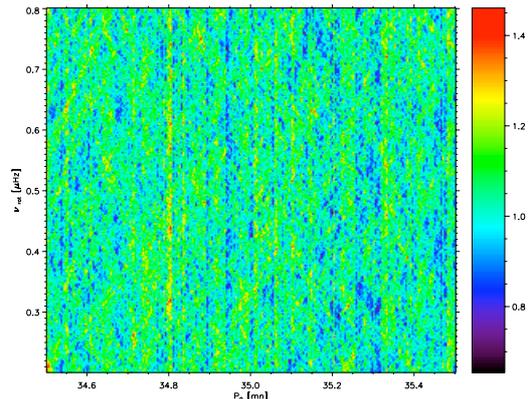


Figure 2. Total power in asymptotic g-mode frequencies as a function of the fundamental g-mode period P_0 and of rotational splitting for Blue SPM data. The total number of modes is about 100, the mean is 1 while the rms value is 0.1.

simpler to implement. We compute theoretical frequencies from the asymptotic formula and add up the power from each corresponding bin of the whitened power spectrum; the result is averaged over the number of added modes. The resulting statistics is χ^2 with $2n$ degrees of freedom with a mean of 1 by definition and rms value of $1/\sqrt{n}$, where n is the number of theoretical frequencies computed. Figure 2 show the results obtained with GOLF data.

Another technique based also on the asymptotic properties of the g modes comprises of computation of the Fourier transform of the power spectrum, and subsequent searches for regular patterns in period. This technique is explained in more detail by García *et al.* (these proceedings). In both cases, it is the use of combined information from many modes, and the patterns they give rise to in the spectrum, that decreases limits on the amplitude threshold required for detection. Figure 3 shows the results of the method of García *et al.* derived independently from their own prescription. The reproduction of their results allows one to design rejection-acceptance tests that can be applied to the disparity between their results. In any case, this ‘mode-averaged’ information does impose some restrictions on inferences made on the core structure. In short, detection on a mode-by-mode basis (i.e., of individual multiplets) is more desirable. Nevertheless, these pattern methods will provide invaluable input for the mode-by-mode searches.

Patterns: rotational splitting. Appourchaux *et al.* (2000) made use of the pattern created by the rotational splitting to detect low-frequency p modes: the so-called *collapsogramme* method. It has been reported by Appourchaux *et al.* (2006) that p modes below $1000 \mu\text{Hz}$ can now be detected in GONG and MDI data. It is important

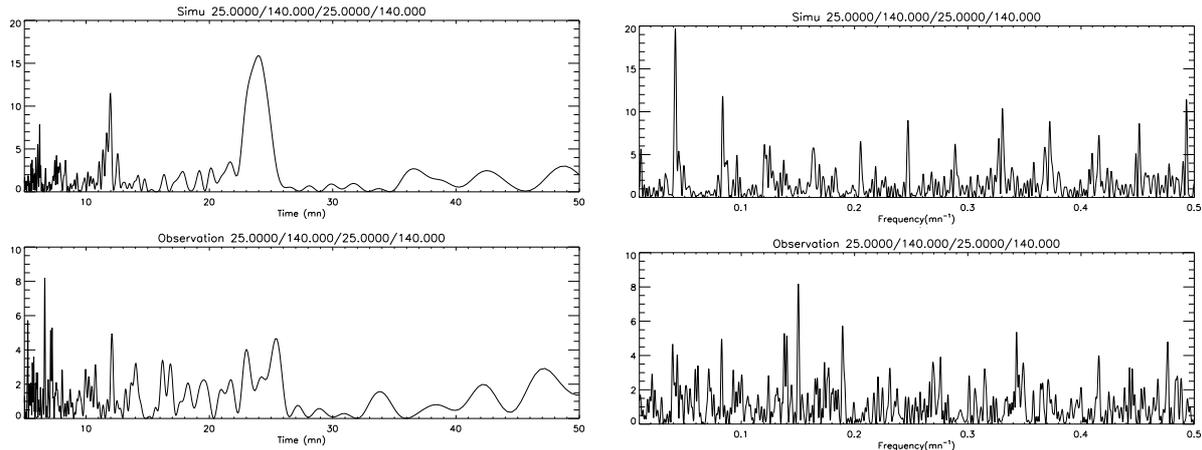


Figure 3. Spectrum of the spectrum as a function of time (left) and of frequency (right) for simulated data (top) and for GOLF data (bottom). The peak at 24 min (top, left) corresponds to the fundamental at 0.04 min^{-1} of the comb-like structure. In the case of the GOLF data, the comb-like structure is much less clear.

to point out that modes below $1000 \mu\text{Hz}$ were not undetectable 5 years ago. This technique has been recently used for attempting to detect mixed modes in the range $200\text{-}400\mu\text{Hz}$ with 10 years of GONG and MDI data, with no success so far.

A similar approach has been used by García et al. (2001) for the GOLF data. The collapsesogramme can then be adapted for the full-disc, ‘Sun-as-a-star’ instruments producing a single power spectrum: in this case it is called an *overlapogramme*. Multiplet search methods have also been developed by Chaplin et al. (2002) and by Turck-Chièze et al. (2004), which demonstrate that the detection limit can be lowered by searching for a pattern of closely separated ‘spikes’ (rotationally split compared), as opposed to individual spikes.

g-mode frequencies and activity. It has also been speculated that solar activity might modify the size of the resonant cavity of the g modes. A modulation of that cavity would then be observed as a frequency modulation. Gabriel (these proceedings) has explored ways and means to recover the modes despite this modulation.

3. ON THE UPPER LIMIT TO G-MODE AMPLITUDE

The canonical upper limit of 10 mm s^{-1} , given by the Phoebus (Appourchaux et al., 2000), has since been lowered, as a result of the collection of more data and the application of new analysis techniques (Gabriel et al., 2002).

Effect of time. As mentioned earlier by Appourchaux (1998), limits on the amplitude threshold for detection improve as $\sqrt{T}/\sqrt{\log(T)}$ (as opposed to \sqrt{T}). For a singlet, it can be derived that the upper limit at $200 \mu\text{Hz}$

is:

$$v_{\text{up}} = 4.3 \sqrt{\frac{10 + \log(T)}{T}} \quad (1)$$

where v_{up} is in mm s^{-1} , T is the observation time in years. Equation (1) is derived from Appourchaux et al. (2000). The limit for a 100-year timeseries would therefore be about 1.6 mm s^{-1} , not the 1-mm s^{-1} given by a simple square-root dependence on time. This discrepancy arises from the fact that the probability limit, at which the threshold is determined, must be kept constant for different T (Appourchaux, 1998). To do otherwise would increase the likelihood of making a false detection. The limit is about 4.8 mm s^{-1} obtained from a data set about 10 years long.

Effect of pattern. When searches are made for multiplets, in the manner of Chaplin et al. (2002); Turck-Chièze et al. (2004), or when optimal masks are applied, as was done by Wachter et al. (2003), one can gain even more, and lower the limit given above by a good factor 3.

Figure 4 compares the upper limit to the amplitude to theoretical limits. Time is obviously on our side. According to recent numerical simulations, g modes are likely to be excited by convective plumes (Dintrans et al., 2005) which could modify somewhat the amplitude estimate made by Kumar et al. (1996); work on estimating g-mode amplitudes that can be compared with the findings of Andersen (1996) is in progress (Dintrans, private communication).

4. CONCLUSION

Although the title of this paper was written at optimistic times, we have not yet succeeded in inferring the structure of the solar core. Anyway, the future seems to be

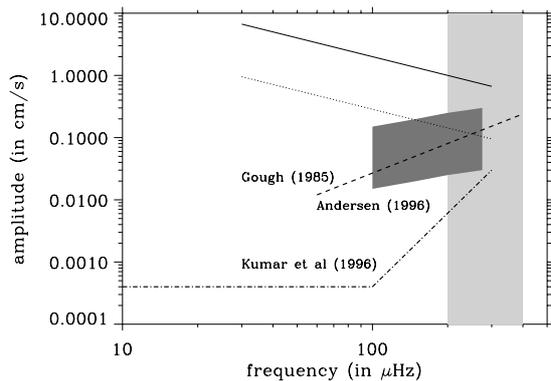


Figure 4. The black thin line is the limit for a singlet derived by Christensen-Dalsgaard (2002) from the upper limit given by Appourchaux et al. (2000) for a 2-year observing time. The dotted is the upper limit for a multiplet for a 10-year observing time scaled as in Eq. (1) assuming an improvement by a factor 3 with respect to a singlet. The thick black dotted-dashed and thick black dashed curves are the simplified version of the theoretical g-mode amplitude for $l = 1$ predicted by Gough (1985) and Kumar et al. (1996), respectively. The dark grey box shows the theoretical range of g-mode amplitude derived for $l = 1$ from calculation performed by Andersen (1996). The light grey box shows the location of the mixed modes.

brighter than ever. The SOHO mission has just been extended to the end of 2009. We are still more optimistic than we could have foreseen several years ago, and believe that at least individual mixed modes might be within reach of detection (given the *highest* theoretical amplitudes, and numerical calculations). One very promising avenue, which is yet to be fully exploited, is the technique pioneered by Duvall (2004), which would aim to detect the g modes below the observable surface using time-distance techniques.

A final word dedicated to the memory and the prescience of George Isaak. Isaak (1981) forecast the observation in the p-mode range of the cut-off frequency, of long mode lifetimes (longer than 12 days), of mode detection in total luminosity at about 10 ppm, of a noise level less than 1 cm s^{-1} , of measurement of rotational splitting, of the study of excitation and damping processes, and he was suggesting asteroseismology. The Phoebus group lost an invaluable inspirer, and we truly hope that one of his last suggestions will help us to find the g modes.

ACKNOWLEDGMENTS

The Phoebus group gratefully acknowledge support from the International Space Science Institute, Bern, Switzerland. Other financial support from the Centre National d'Etudes Spatiales and the European Space Agency has also been appreciated. SOHO is a project of international collaboration between ESA and NASA. This work utilizes data obtained by the Global Oscillation Network Group (GONG) project managed by the National Solar Observatory, a Division of the National Optical Astronomy Observatories, which is operated by AURA, Inc. under a cooperative agreement with the National Science Foundation. The GONG data were acquired by instruments operated by the Big Bear Solar Observatory, High Altitude Observatory, Learmonth Solar Observatory, Udaipur Solar Observatory, Instituto de Astrofísica de Canarias and Cerro Tololo Interamerican Observatory.

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