

LONG RANGE OUTWARD MIGRATION OF GIANT PLANETS, WITH APPLICATION TO FOMALHAUT b

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ABSTRACT

Recent observations of exoplanets by direct imaging reveal that giant planets orbit at a few dozens to more than a hundred AU from their central star. The question of the origin of these planets challenges the standard theories of planet formation. We propose a new way of obtaining such far planets, by outward migration of a pair of planets formed in the 10 AU region. Two giant planets in mean motion resonance in a common gap in the protoplanetary disk migrate outward, if the inner one is significantly more massive than the outer one. Using hydrodynamical simulations, we show that their semimajor axes can increase by almost 1 order of magnitude. In a flared disk, the pair of planets should reach an asymptotic radius. This mechanism could account for the presence of Fomalhaut b; then, a second, more massive planet, should be orbiting Fomalhaut at about 75 AU.

Key words: methods: numerical – planetary systems: formation – planetary systems: protoplanetary disks

1. INTRODUCTION

Most of the known exoplanets are gaseous giants, with semimajor axes below 2 AU, or even as close to their parent stars as 0.01 AU. Inward planetary migration is generally considered a necessary phenomenon to explain this distribution because these planets should not form there. In the core-accretion model (Pollack et al. 1996), giant planets should form beyond the ice-line (located at about 4 AU for a solar type star). The general expectation is that planets form at distances comparable to those characterizing the orbits of the giant planets of our solar system, because further out, the dynamical (and accretional) timescales are too long.

However, giant planets have been recently observed by direct imaging at a few dozens to 120 AU from their host stars (Kalas et al. 2008; Marois et al. 2008). It has been proposed that these large semimajor axes may be due to scattering with other giant planets after formation in the 5–20 AU region (e.g., Scharf & Menou 2009; Veras et al. 2009). Boley (2009), Clarke (2009), and Rafikov (2009) suggest that planet formation by gravitational instability (see Durisen et al. 2007, for a review) can be effective beyond ~ 50 AU, because the cooling time relative to the dynamical time becomes short, and the Toomre Q parameter may be smaller in the colder, outer parts of the disk.

In this Letter, we explain how resonant interactions between two planets in a common gap can lead to outward migration, possibly explaining the presence of these planets. We outline the mechanism in Section 2, and present proof-of-concept simulations in Section 3. The effects of disk structure are discussed in Section 4, where we show that a pair of planets can be driven to an equilibrium radius in a flared disk. Caveats are discussed in Section 5, and our main conclusions and applications of this mechanism, paying special attention to the case of Fomalhaut, are presented in Section 6.

2. OUTWARD MIGRATION IN RESONANCE

Many authors have observed that two planets migrating in the same disk are often trapped in mean motion resonance (e.g., Snellgrove et al. 2001; Papaloizou 2003; Kley et al. 2004;

Pierens & Nelson 2008; Crida et al. 2008). Masset & Snellgrove (2001) found that if the outer planet is significantly less massive than the inner one, and if the two planets open overlapping gaps, then the migration of the pair can proceed outward. In this situation, the inner planet lies close to the inner edge of the common gap, and far from its outer edge; therefore, it feels a positive torque from the inner disk, and no torque from the outer disk. Symmetrically, the outer planet mostly feels a negative torque from the outer disk. If the inner planet is more massive, it feels a larger torque, in absolute value, than the outer planet, and the total torque applied to the pair of planets is positive. The pair of planets therefore moves outward.

In order for this process to last on the long term, the material lying outside of the common gap must be funneled toward the inner disk. Otherwise, it piles up at the outer edge of the gap, which eventually reverses the torque balance. In addition, the inner disk must be refilled. In fact, assuming a local damping of the outer planet's wake, one finds in the simulations an outward drift rate of the disk material significantly smaller than that of the planets. Thus, the gas just outside the gap is necessarily caught up with the outer separatrix of the outer planet and thrown inward.

The whole process may be enhanced by the horseshoe drag due to gas of the outer disk passing through the common gap—like in type III migration (Masset & Papaloizou 2003). However, the outward migration of the pair is observed at all disk mass in numerical simulations. Since the corotational effects as in type III migration become virtually negligible at small disk mass, it is primarily the wake torque imbalance that drives the outward migration.

Another clue of the importance of the Lindblad torque imbalance on the process is the sensitivity of the drift rate on the disk's aspect ratio H/r (H being the scale height of the disk and r the distance to the star), as the one-sided Lindblad torque is proportional to $(H/r)^{-3}$. Masset & Snellgrove (2001) and Morbidelli & Crida (2007) find that the migration rate is a decreasing function of the aspect ratio of the disk. This effect is further enhanced by the fact that the tidal truncation of the outer edge of the common gap by the outer planet is less pronounced in thicker disks.

3. LONG-TERM SIMULATION

3.1. Code Presentation

Accurate numerical simulation of the evolution of giant planets requires simultaneous computation of planet–disk interactions and global evolution of the disk. Therefore, we use the 2D1D version of FARGO (Masset 2000a, 2000b),⁵ in which the standard two-dimensional polar grid is surrounded by a one-dimensional grid to compute the disk evolution on all of its physical extension (Crida et al. 2007).

The two-dimensional grid extends from $0.45 L$ to $12.61 L$, where L is the length unit (taken as 10 AU). The one-dimensional grid—that is, the disk—extends from $0.05 L$ to $20 L$. The resolution is $\delta r/r = 0.01$, constant in the two grids. In the two-dimensional grid, $\delta\theta = \delta r/r$.

3.2. Disk and Planets Settings

The inner regions of protoplanetary disks are poorly constrained, and disk profiles must be assumed for typical inner disk migration studies. In contrast, millimeter interferometry (e.g., Piétu et al. 2007) places strong constraints on disk structures between 30 and 200 AU.

Of particular relevance to scenarios of migration is the temperature profile, which can be recast as $H/r = c_s/r\Omega$, where c_s is the sound speed and Ω is the angular velocity. The aspect ratio follows $H/r = h_0 \times (r/L)^\beta$. The flaring index β is typically 0.25 and ranges from 0.17 to 0.32 (Piétu et al. 2007). The Piétu et al. results are compatible with $h_0 = 0.045$, for $\mu = 2.4$ (mean molecular weight) and $T = 30 K$ at 100 AU. Because we use a locally isothermal equation of state in our simulations, the temperature and aspect ratio are a function of radius only, independent of time.

For the surface density profile, we assume $\Sigma(r) = \Sigma_0 \times (r/L)^{-p}$. Detection of dust in the continuum constrains the profile of the dust column density multiplied by its emissivity (hence its temperature). Assuming a uniform dust to gas ratio, this yields $1 - 2\beta + p \approx 2$, thus $p \approx 1.5$, which is the value adopted in this work (see, e.g., Piétu et al. 2007). Σ_0 is much less constrained. However, unless the disk mass is large enough for type III migration to affect the dynamics, our results should be essentially independent of Σ_0 . More precisely, the drift rate should scale with Σ_0^{-1} ; this is approximately what is seen in our numerical simulations with Σ_0 varying by 1 order of magnitude. We therefore adopt values of the surface density that offer a good trade off between speed and the onset of type III migration of the outer planet that would take it away from the resonance with the inner one.

Calculations are run with a Shakura & Sunyaev (1973) α -coefficient between 0.001 and 0.01 in order to explore the importance of the turbulent viscosity $\nu = \alpha c_s H$.

The two planets have masses $M_1 = 3 \times 10^{-3} M_*$ and $M_2 = 10^{-3} M_*$, i.e., $2 M_{\text{Jupiter}}$ if $M_* = 2 M_\odot$. The planets are initially on circular orbits at $a_1 = L$, $a_2 = 2L$, and do not feel the disk potential for the first 100 (Sections 3.3 and 5.3) or 400 (Section 4) orbits of the inner planet, so the disk can adapt to their presence. Denoting $r_H = r(M_p/3M_*)^{1/3}$ the Hill radius of a planet, the region inside $0.6 r_H$ for a given planet is not taken into account when calculating the force of the disk on the planet, using a smooth filter, in agreement with Crida et al. (2009).

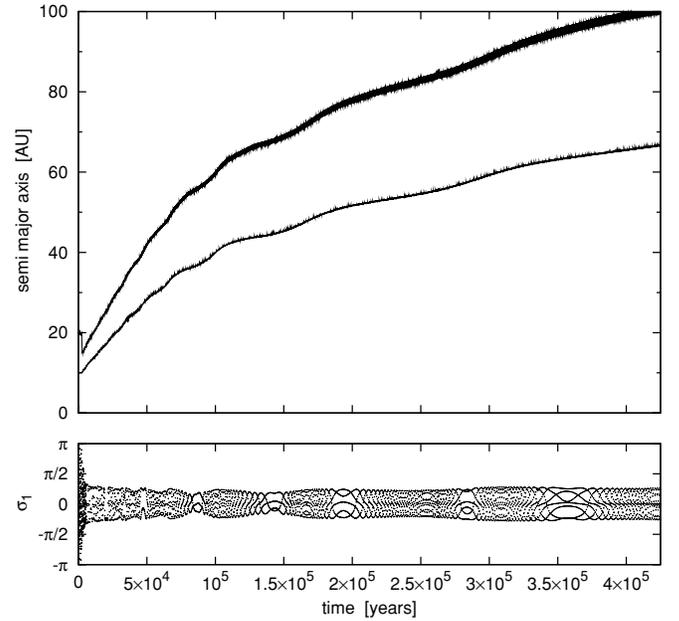


Figure 1. Top: semimajor axes a_1 and a_2 as a function of time (assuming $M_* = 2 M_\odot$). Bottom: resonant angle $\sigma_1 = -\lambda_1 + 2\lambda_2 - \omega_1$ as a function of time.

Table 1
Parameters of the Disk Used in Section 3.3

r (AU)	T (K)	H/r	Σ (kg m^{-2})	Q
10	104	0.045	2666	9.55
20	73.5	0.0535	943	8.0
50	46.5	0.0673	238	6.4
100	32.9	0.08	84.3	5.37

3.3. Result

Consider the case $\alpha = 0.01$, $H/r = 0.045(r/L)^{1/4}$, and $\Sigma_0 = 1.5 \times 10^{-3} M_* L^{-2}$. Assuming $M_* = 2 M_\odot$ (like Fomalhaut) and $L = 10$ AU, this is 6.6 times denser than the Minimum Mass Solar Nebula (MMSN; Hayashi 1981); the disk parameters are summarized in Table 1. The migration path of the planets in this disk is plotted in the top panel of Figure 1. After the release, the outer planet migrates rapidly inward, and is captured in 2:1 mean motion resonance. Then, the two planets migrate smoothly outward together. The resonant angle $\sigma_1 = -\lambda_1 + 2\lambda_2 - \omega_1$ is displayed in the bottom panel. The outer planet reaches $10 L$ (100 AU) in 4×10^5 years, and the migration speed decreases as the planets move further from the star. At distances larger than 100 AU for the outer planet, the orbits seem to converge to a stable equilibrium radius.

Assuming that the migration speed is proportional to the gas density, the outer planet would reach 100 AU in 4 million years in a disk that is only two-thirds the mass of an MMSN. Because this time is roughly the lifetime of a protoplanetary disk, a disk that is 10 times lighter than considered above represents the minimum mass required to migrate the outer planet beyond 100 AU. Such a disk would be light for a $2 M_\odot$ star, so we conclude that there is sufficient time for the resonance mechanism to transport a massive planet to more than 100 AU.

4. EXTENSION OF CONCEPT

To understand this process in general, we have performed additional short-term simulations, using a smaller two-

⁵ <http://fargo.in2p3.fr>

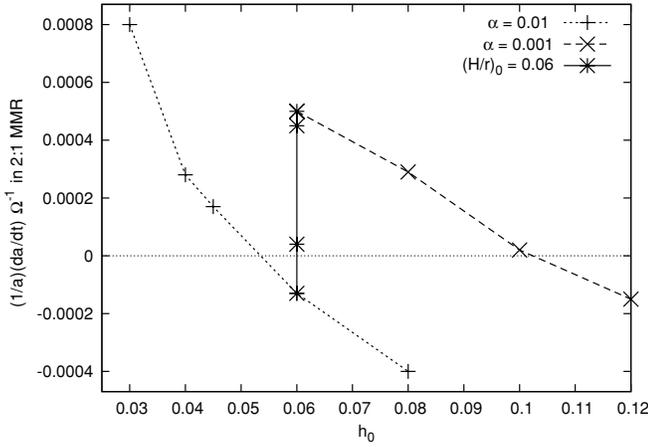


Figure 2. Migration speed of a resonant pair of planets, for various values of α and h_0 . “+” symbols: $\alpha = 10^{-2}$. “x” symbols: $\alpha = 10^{-3}$. Starred symbols: $h_0 = 0.06$, and $\alpha = 10^{-3}$, 2×10^{-3} , 5×10^{-3} , and 10^{-2} from top to bottom.

dimensional grid (extending from 0.4 to 5 L), with $\beta = 0.25$, $p = 1.5$, $\Sigma_0 = 10^{-3} M_* L^{-2}$ (1778 kg m^{-2} in the case $M_* = 2 M_\odot$, $L = 10$ AU), and with varying h_0 and α . In each case, the migration speed of the pair of planets is measured after the resonance capture. The result is displayed in Figure 2. The figure is almost filled with symbols, which means that outward migration is possible for a wide range of parameters. More precisely, the starred symbols (connected with the vertical solid line) display the migration rates obtained with aspect ratio $H/r = 0.06(r/L)^{1/4}$, for various values of α between 10^{-2} (bottom) and 10^{-3} (top); $d \ln(a)/dt$ is a decreasing function of α , and there exists a critical α_c such that the migration is directed outward for $\alpha < \alpha_c$ and inward for $\alpha > \alpha_c$.

Reciprocally, for a fixed α , $d \ln(a)/dt$ is a decreasing function of H/r (see the short-dashed ($\alpha = 10^{-2}$) and long-dashed ($\alpha = 10^{-3}$) curves). In both cases, there exists a critical aspect ratio h_c such that the migration is directed outward for $h_0 < h_c$ and inward for $h_0 > h_c$. This explains partly the slowing down of the migration as the semimajor axes increase in Figure 1, and suggests that in a flared disk with uniform α , the pair of planets should reach the location where $H/r = h_c$. This is a stable equilibrium point as far as migration is concerned: in the inside, $H/r < h_c$, and migration is directed outward; further from the star, $H/r > h_c$ and migration is directed inward. If H/r increases with r , there should be convergence toward the place where $H/r = h_c$. The evolution of the density profile also plays a role, but the study of this parameter is beyond the scope of this Letter.

One can expect that the efficiency of the Masset & Snellgrove (2001) mechanism is directly related to the size of the gap opened by the outer planet. To demonstrate this, we plot in Figure 3 the same points and curves as in Figure 2, but the migration speed is expressed in viscous time $\tau_v = r^2/\nu$, and the x -axis reports the parameter P , defined as

$$P = \frac{3}{4} \frac{H}{r_H} + \frac{M_*}{M_p} \frac{50\nu}{r^2 \Omega}. \quad (1)$$

This parameter is precisely related to the width and depth of the gap of the outer planet (Crida et al. 2006). The strong correlation is obvious. For a broad range of H/r and α , the dependence of the migration speed on these two parameters can be approximated by a dependence on the sole parameter P . The

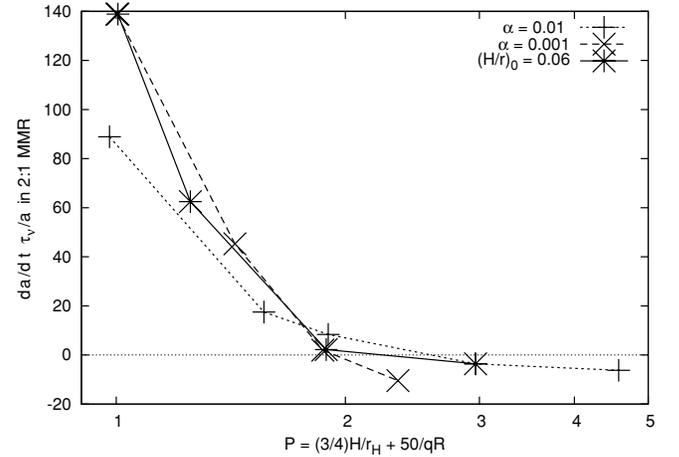


Figure 3. Migration speed of a resonant pair of planets, normalized to the viscous time, as a function of P . Same key as in Figure 2.

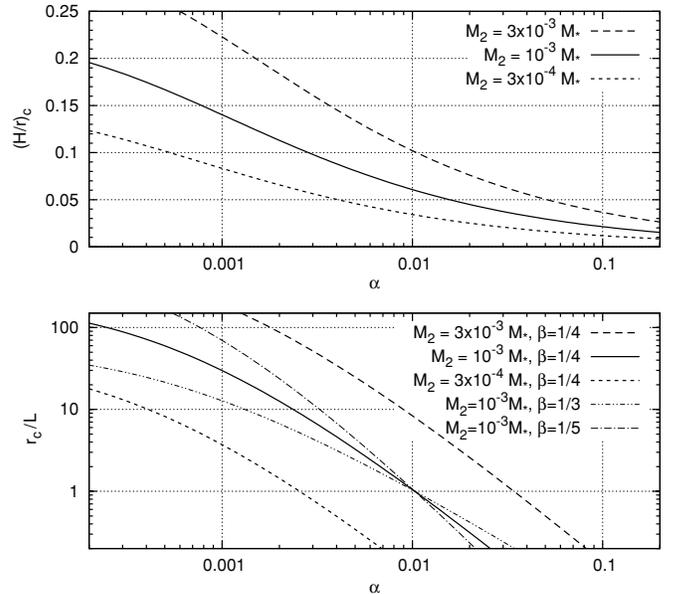


Figure 4. Top: solutions of the equation $P = 2.5$ for three values of M_2/M_* . Bottom: corresponding r_c , for $h_0 = 0.06$.

migration rate is close to stationary for $2 \lesssim P \lesssim 2.5$, whatever the values of α and H/r that combine into this value of P .

Then, using Equation (1), $P = 2.5$ gives the critical aspect ratio that halts the migration as a function of α , or α_c for a given H/r . The top panel of Figure 4 shows the value of the aspect ratio that makes $P = 2.5$ for the outer planet, as a function of α , for $2 \times 10^{-4} < \alpha < 0.2$. The radius at which the outer planet would stop is then

$$r_c = \left[\frac{(H/r)_c}{h_0} \right]^{1/\beta} L.$$

This is displayed in the bottom panel for $h_0 = 0.06$, so that L can be understood as the radius in the disk where $H/r = 0.06$. The equilibrium radius r_c depends on α , β , and M_2 . Since β is generally small, r_c can be very large with respect to L .

Note that for a given value of P , the migration speed may depend on other parameters. Therefore, the critical value for stationary migration $P \approx 2.5$ holds only in the case considered here, that is with $M_1/M_2 = 3$, $\beta = 0.25$, $\Sigma_0 = 10^{-3} M_* L^{-2}$,

and $p = 1.5$. A full analysis of all the parameter space is beyond the scope of this Letter.

5. DISCUSSION

5.1. Gas Accretion on the Planets

A shortcoming of the mechanism presented here is that it disregards the accretion of gas onto the planets. As the pair of planets proceeds outward, the gas proceeds inward through the common gap. The outer planet should presumably accrete most of the incoming material, narrowing the mass difference with the inner planet, so that the torque balance could cancel or reverse at some point. Accretion of the nebular gas onto a giant planet is a complex process. It is dependent on the global structure of the flow, on the local properties of the circum-planetary disk or envelope (poorly described in numerical simulations), and on the micro-physics of the nebula, such as the opacity of the grains. Most detailed studies of gas accretion onto giant planets cannot be applied to the present case, because the conditions are different from the conditions that prevail at 10 AU, where these studies apply. This appeals for a thorough investigation of gas accretion in the conditions of outward migration in the outskirts of the disk, which is beyond the scope of this work.

5.2. Self-Gravity of the Gas and Three-dimensional Effects

The gas self-gravity may yield, depending on the equation of state of the fluid and the mass of the disk, to a vertical compression of the fluid in the vicinity of the shock (e.g., Boley & Durisen 2006). This would increase the Lindblad torque, more efficiently on the side of the most massive planet, thus it would actually enhance the mechanism of outward migration. But three-dimensional effects apply mostly at high-order resonances, close to the planet, which are depleted in our case. In addition, three-dimensional semianalytic calculations performed by Tanaka et al. (2002) show that the angular momentum is essentially exchanged with waves that have no vertical structure. Therefore, owing to the size of the gap that our pair of planets carves in the disk, and owing to the relatively large value of the Toomre parameter (see Table 1), we feel confident that non-self-gravitating, two-dimensional isothermal simulations capture the main features of our mechanism and provide an acceptable order of magnitude of the outward drift rate.

5.3. Equation of State

In the above simulations, the equation of state was locally isothermal. This is realistic at large distances from the star, where the opacity is low and the cooling time is short with respect to the dynamical time. However, at ~ 10 AU from the star, the cooling time can be larger than the orbital period. This can affect the aspect ratio, because of the heating of the disk by the wakes launched by the planets.

Additional simulations have been run, with the settings of Section 3.3, and computing the full energy equation, with viscous heating Q_+ , and no heat diffusion in the disk plane, but vertical radiative cooling $Q_- = 2\sigma_R T^4 / \kappa \Sigma$ (where σ_R is the Stefan–Boltzmann constant). The opacity κ is constant and tuned for the unperturbed disk to be in thermal equilibrium (see Equation (6) of Crida 2009). This gives a cooling time of $111 \Omega^{-1}$ initially. The aspect ratio increases to about 0.06 in the planets region after 100 orbits of the inner planet. This makes the migration rate after the resonance capture to decrease to $\sim 10^{-4}$ AU year $^{-1}$ instead of $\sim 4 \times 10^{-4}$ AU year $^{-1}$.

With a realistic opacity, given by Bell & Lin (1994, the Appendix), T and H/r are increased for $r \lesssim 10$ AU, and shrink for $r \gtrsim 15$ AU because we do not take heating from the star into account. At 10 AU, the cooling time is then $\sim 400 \Omega^{-1}$. Migration proceeds outward at a speed initially similar to the locally isothermal case (which was expected as H/r is not much changed around 10–15 AU), slightly accelerating while reaching the outer regions where H/r is smaller.

So, it seems that migration can proceed outward on the long range also in non-locally-isothermal disks, even if a self-consistent radiative disk model still has to be tested.

6. APPLICATIONS

We find that a pair of planets formed in the 5–20 AU region can reach large semimajor axes, if the outer planet is lighter than the inner one and they orbit in resonance in a common gap in the protoplanetary disk. The pair of planets tends to reach regions in the disk where H/r has a critical value (of the order of 0.1–0.15, depending on the other parameters); this can be 10 times further from the star than where they formed. After the disk has dissipated, the evolution of the planets may lead to the disruption of their original resonant configuration, due to various processes: (1) the eccentricities can rise too much if the planets are too massive, (2) a third planet could destabilize the system at some point, and (3) the outer planet may enter a debris disk, in which it migrates independently of the inner planet by planetesimals scattering. Such a phenomenon may account for the HR 8799 system, where the outermost planet is the lightest one, but no resonance has been identified so far (Marois et al. 2008).

Otherwise, the planets should remain on orbits in resonance far from the star. Their eccentricities should therefore be nonzero but moderate: during the outward migration, e is excited by the resonance and simultaneously damped by the disk (e.g., Crida et al. 2008). In the simulation presented in Figure 1, the eccentricities are about 0.01 and 0.02–0.03 for the inner and outer planet, respectively. In contrast, the scattering of giant planets leads to very eccentric orbits (up to $e = 0.8$). The typical final eccentricity of planets formed by gravitational instability is not known.

The mechanism presented here could account for the case of Fomalhaut b. Indeed, this planet has a mass smaller than $1.5 \times 10^{-3} M_*$ ($3 M_{\text{Jupiter}}$) (Kalas et al. 2008; Chiang et al. 2009), while Quillen (2006) predicted a planet smaller than Saturn. Thus, a more massive, inner planet could exist. In addition, Kalas et al. (2008) noticed that Fomalhaut b is not apsidally aligned with the dust belt, which suggests the presence of additional perturbers in the system. Observations in the M band by Kenworthy et al. (2009) rule out the presence of a planet more massive than $2 M_{\text{Jupiter}}$ in the range 8–40 AU from Fomalhaut. However, if our mechanism occurred in the Fomalhaut system, then the second planet should be orbiting at about 75 AU from the star, with a mass of the order of 1–10 Jupiter masses.

Simulations have been performed on the hyades cluster of the D.A.M.T.P., on the on the hpc–bw cluster of the *Rechenzentrum* of the University of Tübingen, and on a 92 core cluster funded by the program “*Origine des Planètes et de la Vie*” of the French *Institut National des Sciences de l’Univers*. A. Crida acknowledges STFC and J. Papaloizou. We acknowledge enlightening discussions with V. Piétu. We also thank the anonymous referee for his/her insightful remarks, which improved the Letter.

REFERENCES

- Bell, K. R., & Lin, D. N. C. 1994, *ApJ*, 427, 987
- Boley, A. C. 2009, *ApJ*, 695, L53
- Boley, A. C., & Durisen, R. H. 2006, *ApJ*, 641, 534
- Chiang, E., Kite, E., Kalas, P., Graham, J. R., & Clampin, M. 2009, *ApJ*, 693, 734
- Clarke, C. 2009, *MNRAS*, 396, 1066
- Crida, A. 2009, *ApJ*, 698, 606
- Crida, A., Baruteau, C., Kley, W., & Masset, F. 2009, *A&A*, 502, 679
- Crida, A., Morbidelli, A., & Masset, F. 2006, *Icarus*, 181, 587
- Crida, A., Morbidelli, A., & Masset, F. 2007, *A&A*, 461, 1173
- Crida, A., Sándor, Z., & Kley, W. 2008, *A&A*, 483, 325
- Durisen, R. H., Boss, A. P., Mayer, L., Nelson, A. F., Quinn, T., & Rice, W. K. M. 2007, *Protostars and Planets V*, ed. B. Reipurth, D. Jewitt, & K. Keil (Tucson, AZ: Univ. Arizona Press), 607
- Hayashi, C. 1981, *Prog. Theor. Phys. Suppl.*, 70, 35
- Kalas, P., et al. 2008, *Science*, 322, 1345
- Kenworthy, M. A., Mamajek, E. E., Hinz, P. M., Meyer, M. R., Heinze, A. N., Miller, D. L., Sivanandam, S., & Freed, M. 2009, *ApJ*, 697, 1928
- Kley, W., Peitz, J., & Bryden, G. 2004, *A&A*, 414, 735
- Marois, C., Macintosh, B., Barman, T., Zuckerman, B., Song, I., Patience, J., Lafrenière, D., & Doyon, R. 2008, *Science*, 322, 1348
- Masset, F. 2000a, *A&AS*, 141, 165
- Masset, F. 2000b, in ASP Conf. Ser. 219, *Disks, Planetesimals, and Planets*, ed. G. Garzón et al. (San Francisco, CA: ASP), 75
- Masset, F., & Papaloizou, J. C. B. 2003, *ApJ*, 588, 494
- Masset, F., & Snellgrove, M. 2001, *MNRAS*, 320, L55
- Morbidelli, A., & Crida, A. 2007, *Icarus*, 191, 158
- Papaloizou, J. C. B. 2003, *Celest. Mech. Dyn. Astron.*, 87, 53
- Piétu, V., Dutrey, A., & Guilloteau, S. 2007, *A&A*, 467, 163
- Pierens, A., & Nelson, R. P. 2008, *A&A*, 482, 333
- Pollack, J. B., Hubickyj, O., Bodenheimer, P., Lissauer, J. J., Podolak, M., & Greenzweig, Y. 1996, *Icarus*, 124, 62
- Quillen, A. C. 2006, *MNRAS*, 372, L14
- Rafikov, R. R. 2009, *ApJ*, 704, 281
- Shakura, N. I., & Sunyaev, R. A. 1973, *A&A*, 24, 337
- Scharf, C., & Menou, K. 2009, *ApJ*, 693, L113
- Snellgrove, M. D., Papaloizou, J. C. B., & Nelson, R. P. 2001, *A&A*, 374, 1092
- Tanaka, H., Takeuchi, T., & Ward, W. R. 2002, *ApJ*, 565, 1257
- Veras, D., Crepp, J. R., & Ford, E. B. 2009, *ApJ*, 696, 1600