# From comet dynamics to dark matter dynamics

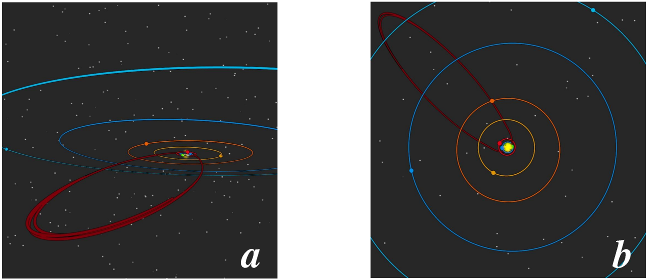
Guillaume Rollin[1], Pierre Haag[1], José Lages[1], Dima Shepelyansky[2]

Nice
15/09/2015

[1] Team PhAs - Physique théorique et Astrophysique
Institut UTINAM - UMR CNRS 6213
Observatoire de Besançon
Université de Franche Comté
[2] Laboratoire de Physique Théorique - UMR CNRS 5152
Université Paul Sabatier, Toulouse

References :
G. Rollin, J. Lages, D. L. Shepelyansky, Chaotic enhancement of dark matter density in binary systems, A&A 576, A 40 (2015)
G. Rollin, P. Haag, J. Lages, Symplectic map description of Halley's comet dynamics, Phys. Letters A, 379, 14–15 (2015)
G. Rollin, J. Lages, D. L. Shepelyansky, Strange repeller structures for the Jacobi Hamiltonian of restricted three-body problem (in prep.)

# Comets orbit




Orbit of Halley's comet : *a*) arbitrary point of view *b*) Projection on the ecliptic plane.

**Idea**

The Comet's trajectory is quasi-parabolic $\rightarrow$ we want to study the dynamics of this kind of trajectory in binary systems.

# From comet dynamics to DMP dynamics

## Comet

When it approaches the solar system the comet feels :

- the Sun (the first attractor).
- the planets (Jupiter is the more massive one).

## Dark matter particle

When it approaches binary system DMP feels :

- main body (the most massive).
- secondary body.

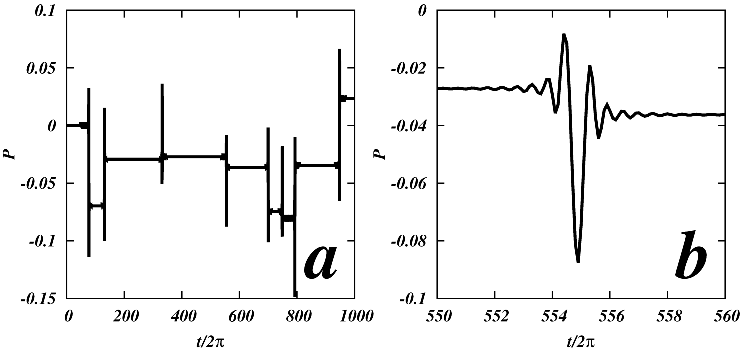
## Finally

→ the method used for comets can be used for DMPs.

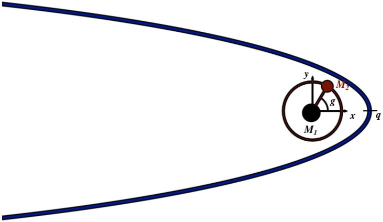
## Kicked evolution

At each passage the comet is perturbed close to his perihelion. The energy evolves by short jump it's a kicked evolution...

ref : J.-L. Zhou, Y.-S. Sun, J.-Q. Zheng, and M.J. Valtonen, Astron. Astrophys. 364, 887–893 (2000)



# Petrosky & Broucke solution (1988)




Comets in SS have a large eccentricity $\rightarrow$ solution for quasi-parabolic case can be found :
The classical Hamiltonian formulation for restricted 3 body problem is :

$$H = \frac{1}{2}\left(p_r^2 + \frac{p_\theta^2}{r^2}\right) - \sigma p_\theta - \frac{(1-\mu)}{R_1} - \frac{\mu}{R_2} \quad (1)$$

after one change of variable the new hamiltonian is :

$$H = \frac{P}{2} - \sigma G + \mu V \quad (2)$$

where $P = 1/L^2 \propto 1/a \propto w$ where $a$ is the semi-major axis and $G$ is the Delauney's variable (angular momentum).

**Petrosky & Broucke solution (1988)**

$\rightarrow$ We can treat $V$ as a perturbation with respect to $\mu$.

ref : T. Y. Petrosky and R. Broucke, Celestial Mechanics, 42, 53-79 (1988)

# Perturbation theory

## Solution

In asymptotic limit the perturbation theory leads to :

$$P_{as}^{+} = P_0 + 2\sigma\mu \sum_{j=1}^{\infty} \Delta_j \sin(jg_0) \quad (3)$$

where $g_0$ is the second body phase. For moderatelty large $q$ we can neglect terms with $j \geq 2$.

## Kepler map

Therefore by the Kepler's third Law :

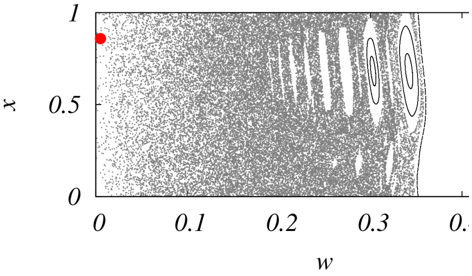
$$\begin{aligned} P_{n+1} =& P_n + 2\sigma\mu \sum_{j=1}^{\infty} \Delta_j \sin(jg_0) \\ g_{n+1} =& g_n - \frac{2\pi\sigma}{(-P_{n+1})^{3/2}} \end{aligned} \quad (4)$$

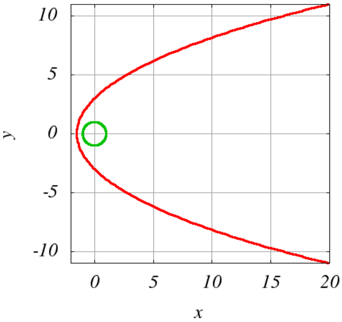
→ The symplectic map.

## Mapping

The mapping leads to Poincaré section :

$$w_{n+1} = w_n + F(x_n)$$
$$x_{n+1} = x_n + w_{n+1}^{-3/2}$$



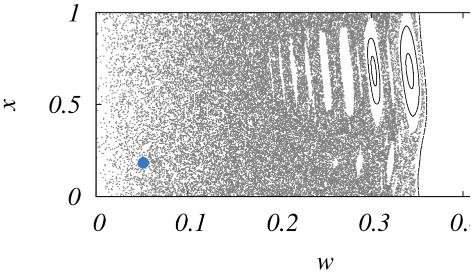



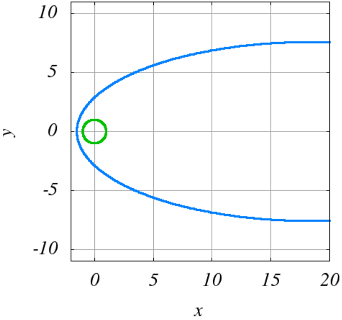
# Dynamics with the Kepler map

## Mapping

The mapping leads to Poincaré section :

$$w_{n+1} = w_n + F(x_n)$$
$$x_{n+1} = x_n + w_{n+1}^{-3/2}$$



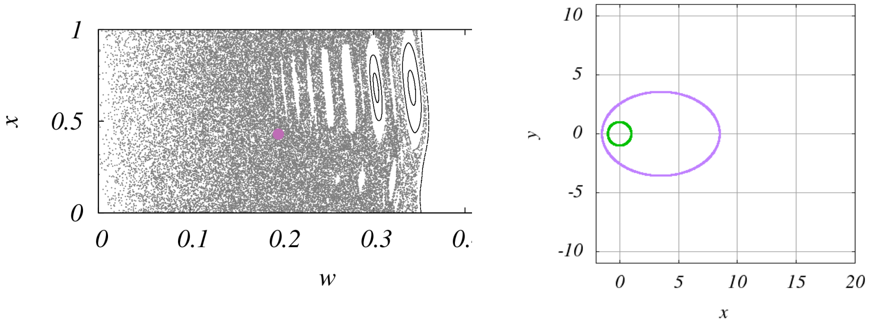
# Dynamics with the Kepler map

## Mapping

The mapping leads to Poincaré section :

$$w_{n+1} = w_n + F(x_n)$$

$$x_{n+1} = x_n + w_{n+1}^{-3/2}$$



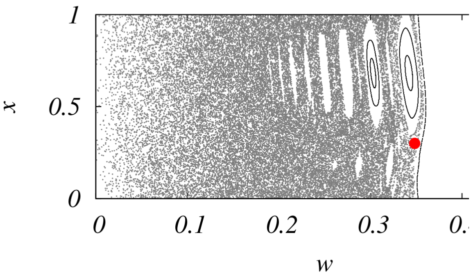
# Dynamics with the Kepler map

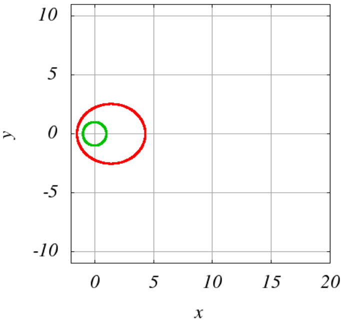
## Mapping

The mapping leads to Poincaré section :

$$w_{n+1} = w_n + F(x_n)$$

$$x_{n+1} = x_n + w_{n+1}^{-3/2}$$



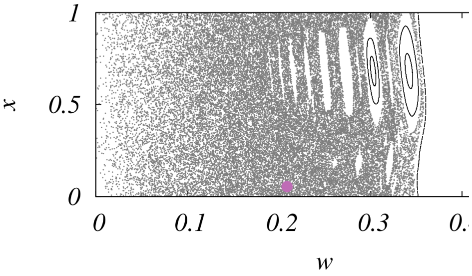



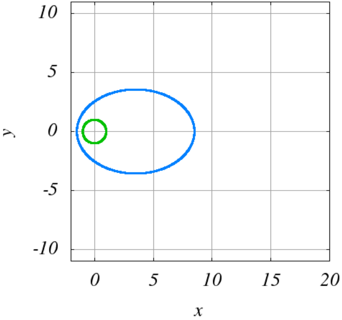
# Dynamics with the Kepler map

## Mapping

The mapping leads to Poincaré section :

$$w_{n+1} = w_n + F(x_n)$$
$$x_{n+1} = x_n + w_{n+1}^{-3/2}$$



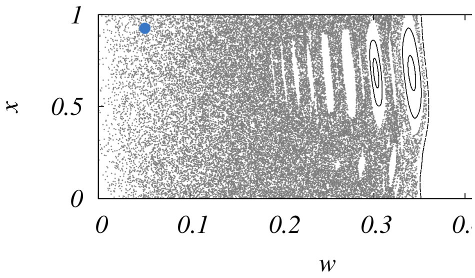



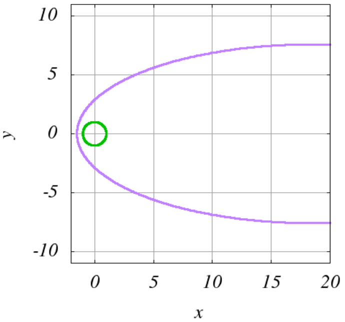
# Dynamics with the Kepler map

## Mapping

The mapping leads to Poincaré section :

$$w_{n+1} = w_n + F(x_n)$$
$$x_{n+1} = x_n + w_{n+1}^{-3/2}$$

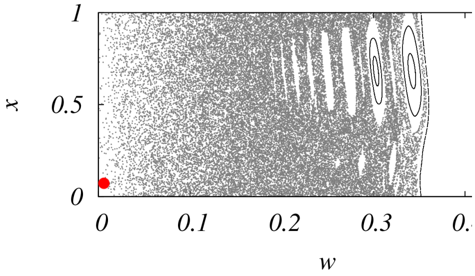


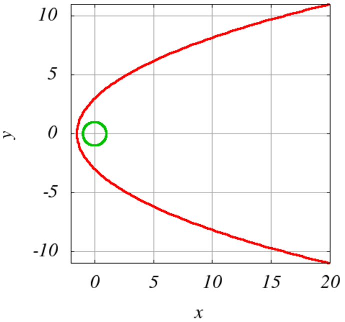



## Mapping

The mapping leads to Poincaré section :

$$w_{n+1} = w_n + F(x_n)$$
$$x_{n+1} = x_n + w_{n+1}^{-3/2}$$



# Liu and Sun method (1992)

## Question

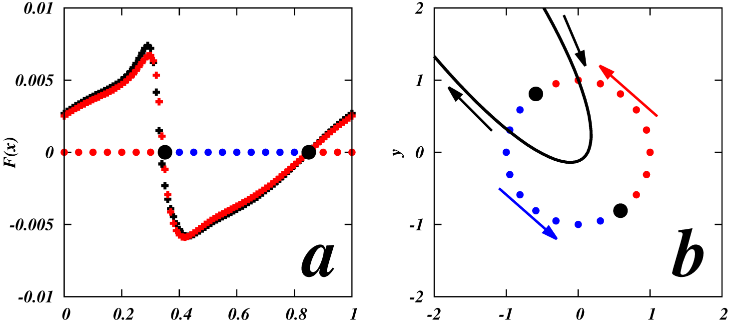
What is the form of the kick function ?

## Solution

Liu and Sun (1992) solution to compute the kick function.

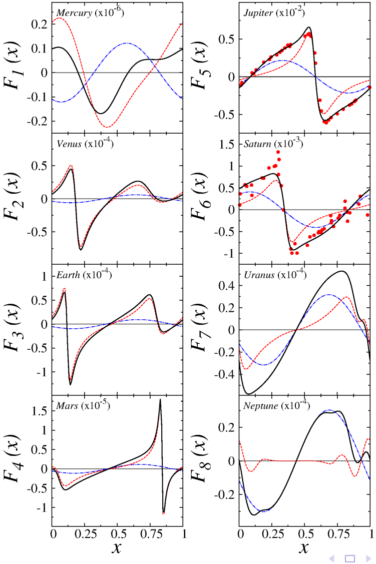
$$\Delta w = 2 \int_{-\infty}^{+\infty} \boldsymbol{p} \cdot \boldsymbol{\nabla} \left( H - H_0 \right) dt \tag{5}$$

→ it's just the work on an parabolic orbit (osculating orbit).



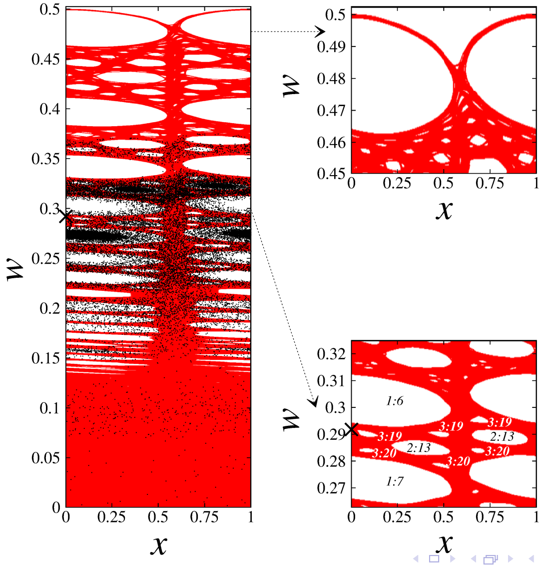
# Liu and Sun method (1992)

- Kick of each planet of the Solar System



# Kepler map iteration

- Poincaré section $(w, x)$



# Statistical results

- Survival times with Jupiter only

| | Jupiter réel | Sawtooth approx | Chirikov and Vecheslavov |
|---|---|---|---|
| $\bar{t}$ (years) | $\sim 1 \times 10^7$ | $\sim 4 \times 10^7$ | $\sim 6 \times 10^7$ |
| $\bar{n}$ | $\sim 4 \times 10^4$ | $\sim 4 \times 10^5$ | $\sim 6 \times 10^5$ |

- Survival times with Jupiter + Saturn

| | Jupiter + Saturn | Jupiter | Chirikov and Vecheslavov |
|---|---|---|---|
| $\bar{t}$ (years) | $\sim 9 \times 10^6$ | $\sim 1 \times 10^7$ | $\sim 4 \times 10^6$ |
| $\bar{n}$ | $\sim 9 \times 10^4$ | $\sim 4 \times 10^4$ | $\sim 2 \times 10^4$ |

ref : B. V. Chirikov and V. V. Vecheslavov, Astron. Astrophys., 221, 146-154 (1989)
G. Rollin, P. Haag, J. Lages, Phys. Letters A, 379, 14–15 (2015)

# DMPs dynamics

## Number of particles

At infinity DMPs have Maxwell's velocity distribution :

$$f(v)dv = \sqrt{\frac{54}{\pi}}\frac{v^2}{u^3}\exp\left(\frac{-3v^2}{2u^2}\right)dv \quad (6)$$

where $u \approx 220km/s$ with our units : $G = 1$, $v_2 = 1$, $r_2 = 1$ and $M_1 + M_2 \sim M_1 = 1$ $u = 17$ for $v_2 = 13km/s$ (Jupiter) and $u = 0.035$ for $v_2 = 6000km/s$ (star and black hole).

## $\kappa$ ratio

Let us define

$$\kappa = \frac{N_{tot}}{N_{0-J}} = \frac{1}{1 - \exp\left\{\frac{-3J}{2u^2}\right\}} \quad (7)$$

where $J$ is the kick amplitude. If $J \ll u$ (Jup.) $\kappa \simeq 2u^2/(3J)$ but if $J \sim u$ (S-BH) $\kappa \simeq 1$ all particles are capturable.

## $\zeta$ ratio

Let us define $\zeta$ and $\zeta_g$ ratio : $\zeta = \frac{\rho_{r_2}}{\rho_{gJ}}$, $\zeta_g = \frac{\rho_{r_2}}{\rho_g}$
where $\rho_{r_2}$ is particle density in a sphere of radius $r_2$, $\rho_{gJ}$ is mean galactic particles density in an energy range and $\rho_g = 4 \times 10^{-25}\ g.cm^{-3}$ is mean galactic particles density.

# DMP dynamics (Dark map results)

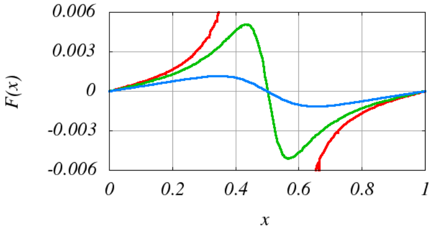
## Sympletic map

We assume that DMP dynamics is governed by the symplectic map.

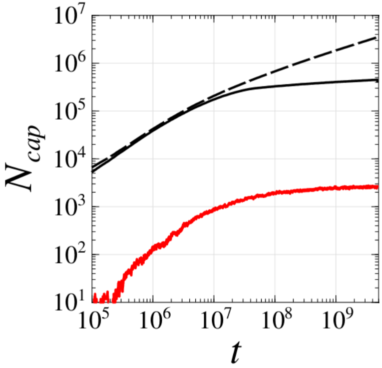
$$w_{n+1} = w_n + F(x_n)$$
$$x_{n+1} = x_n + w_{n+1}^{-3/2}$$

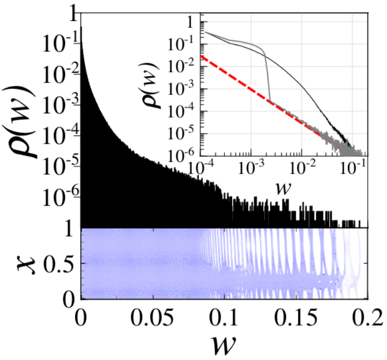
The "real" kick function used in the "dark map" (Lages & Shepelyansky). $\rightarrow$

complicated function with 3 parameters $(q, \theta, \phi)$.



- Captured particles




ref : J. Lages, D. L. Shepelyansky, MNRAS, 430, L25-L29

# DMP dynamics (Dark map results)

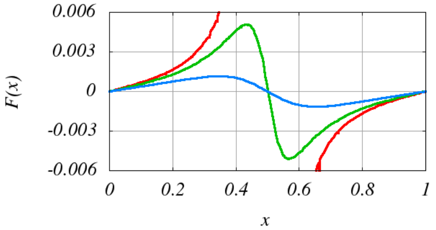
## Sympletic map

We assume that DMP dynamics is gouverned by the symplectic map.

$$w_{n+1} = w_n + F(x_n)$$
$$x_{n+1} = x_n + w_{n+1}^{-3/2}$$

The "real" kick function used in the "dark map" (Lages & Shepelyansky). $\rightarrow$

complicated function with 3 parameters $(q, \theta, \phi)$.



# DMP dynamics

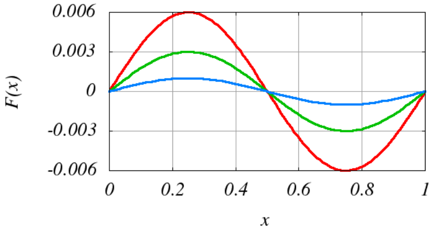
## Sympletic map

We assume that DMP dynamics is gouverned by the symplectic map.

$$w_{n+1} = w_n + F(x_n)$$
$$x_{n+1} = x_n + w_{n+1}^{-3/2}$$

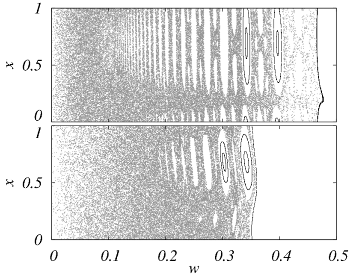
The Kick function used in the "Kepler-Petrosky map" (Rollin, Lages & Shepelyansky).

$$F(x_n) = \begin{cases} A\sin(2\pi x_n) \text{ if } q < 1.5 \\ A\exp(-\alpha(q-1.5))\sin(2\pi x_n) \text{ if } 1.5 < q < \infty \end{cases}$$

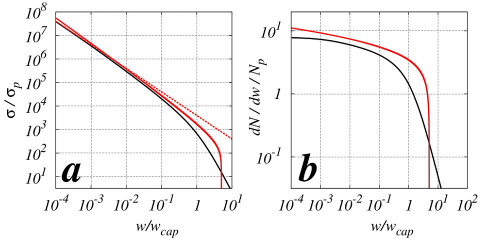


# DMP dynamics (Kepler-Petrosky map results)

- Poincaré sections :

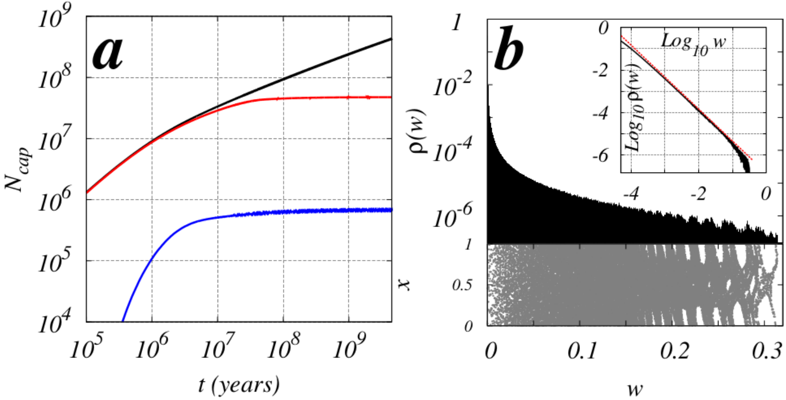



- Capture cross sections :

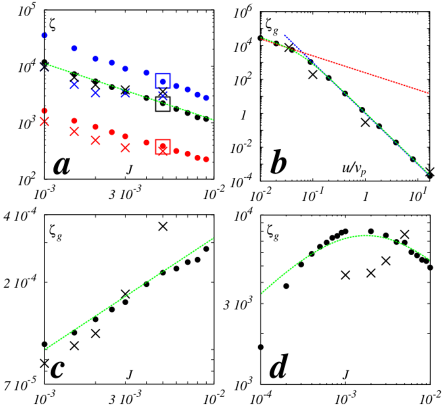


# DMP dynamics (Kepler-Petrosky map results)

- Captured particles :



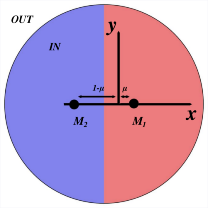
# DMP dynamics (Kepler-Petrosky map results)




ref : G. Rollin, J. Lages, D. L. Shepelyansky, A&A, 576, A 40 (2015)

# Work in progress...

## Open three body problem

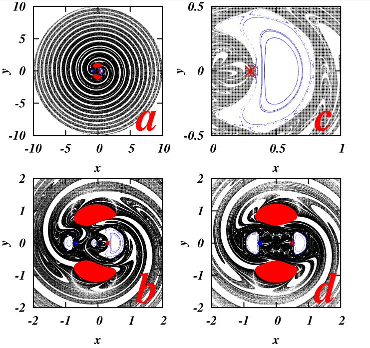


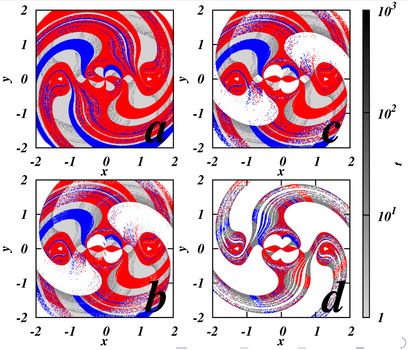

With an ionization wall around the system

→ open system

→ attractor/repeller.


ref :
Jan Nagler, Phys. Rev. E, 69, 066218 (2004)
Jan Nagler, Phys. Rev. E, 71, 026227 (2005)
G. Rollin, J. Lages, D. L. Shepelyansky, (in prep.)

# Conclusion

- Symplectic mapping can be used in a first approach of capture-evolution-ejection process in restricted three body problem.
- Dynamics of comets is kicked by the 2 contributions of the kick function.
- Symplectic mapping analysis show an huge enhancement of local dark matter density around fast binary systems.
- Ejection process has a fractale structure in phase-space.
- Mapping technics could be used to study matter transfers between binary system...