Spacetime, geometry, gravitation

Ideas and some peculiar aspects

Bertrand Chauvineau

Observatoire de la Côte d'Azur
Lagrange laboratory
Nice-Sophia Antipolis University
Nice, France

Aims / contents / topics

Spacetime & geometry

Gravitation & spacetime geometry

General Relativity, the large scale Universe & Λ

General Relativity & ways to alternatives

Metric theories, $f(R)$ theories, Scalar-Tensor theories

Some keys ideas related to spacetime & to gravitation (« classical » ideas) :

Newton (1687) :

- necessity of a precise definition of the spacetime (space & time) properties before doing physics
- Newton spacetime → gravity as a « force » phenomenon

Maxwell (~1865) : electromagnetism equations versus the galilean relativity principle

The Michelson-Morley experiment (1881) & the special relativity theory (1905)

Minkowski (1908) : the relativity theory versus **Minkowski's spacetime**

→ a **drastic change** in the way of thinking in theoretical physics

Einstein (1915) : gravity as a spacetime geometry effect (→ General relativity)
(& later : alternatives to General Relativity)

I - Newton versus Minkowski spacetimes (in a nutschell) and gravity :

	Newton's spacetime	Minkowski's spacetime
Invariant quantity between two close events	dt (or $-dt^2$ …) (absolute time)	$-c^2dt^2+dx^2+dy^2+dz^2$ (relativity of time)
(fundamental) constant	none	c (a priori nothing to do with light !)
Spacetime geometry ?	**NO** (but space geometry -euclidean-)	YES (pseudo-euclidean)
Inertial motions (just determined by the spacetime characteristics)	Rectilinear & uniform	Rectilinear & uniform

→ in both, the gravity phenomenon requires « something else » than the spacetime's properties (something like a « force »)

Newton's spacetime has no spacetime geometry (unlike Minkowski's), but its properties are (to some extent) **as precise** --while different-- as the ones of Minkowski's spacetime.

→ consider physical problems like simultaneity, causality, …

flat space & uniform time

or

flat spacetime

& no force → inertial motions = rectilinear and uniform

Newton : the cause of planetary motions is a specific **force** (spacetime properties unchanged)

→ Matter generates a force field : **Newton's universal gravity theory**

But another possibility could be to **reject the idea of a gravitational force** (Riemann)

→ Matter « modifies » the spacetime properties

→ Motion under gravity = **inertial** motion in a **non-newtonian spacetime**

Starting from the **newtonian spacetime** : how changing the **space** geometry to recover the Newton's gravity theory equations ? (Riemann)

→ **FAILS !!!**

Starting from the **minkowski spacetime** :

the geodesics (extremal « length » curves ←→ inertial motions) of the spacetime of metric

$$ds_N^2 = -\left(1-\frac{2U}{c^2}\right)c^2dt^2 + dx^2 + dy^2 + dz^2 \quad \text{(Newton's metric)}$$

are functions $x(t)$, $y(t)$, $z(t)$ that satisfy, at lowest order (in U and v^2), the Newtonian's law of motion under a gravity potential U, ie

$$\frac{d^2x^i}{dt^2} = \partial_i U$$

$$\delta\int ds_N = 0 \quad \Leftrightarrow \quad \frac{d^2x^i}{dt^2} = \partial_i U + O\left(U^2, Uv^2, v^4\right)$$

Remark : the « time part », and only it (at lowest order) of the Minkowski spacetime is affected

→ Riemann could not success !!!

Obviously : the previous metric is **not** a « geometric gravity theory », but just a geometrical reinterpretation of Newton's theory !

A « geometric gravity theory » is expected to give **directly** **the link between the spacetime geometrical objects and** its **matter** (ie non gravitational) content

→ locally minkowskian ? (local spatial isotropy ?)

The metric tensor in (very) brief

$Q(x^i + dx^i)$ ✦
$P(x^i)$ ✦ (2 close points)

Riemannian variety :

$$ds^2 = \left(\sum_{a,b}\right) g_{ab}\left(x^c\right) dx^a dx^b \equiv g_{00}\left(x^c\right) dx^0 dx^0 + 2g_{01}\left(x^c\right) dx^0 dx^1 + \ldots + g_{11}\left(x^c\right) dx^1 dx^1 + \ldots$$

interval — metric tensor components (symmetric) ➔ should define an **invertible matrix**

$$\left(g_{ab}\right) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \ \& \ \left(g^{ab}\right) = \left(g_{ab}\right)^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$ds^2 = dx^2 + dy^2$ euclidean plane (cart. coord.)

$$\left(g_{ab}\right) = \begin{pmatrix} 1 & 0 \\ 0 & \sin^2\theta \end{pmatrix} \ \& \ \left(g^{ab}\right) = \begin{pmatrix} 1 & 0 \\ 0 & \sin^{-2}\theta \end{pmatrix}$$

$ds^2 = dx^2 + \sin^2 x dy^2$ sphere (2 dim)  $(d\theta^2 + \sin^2\theta d\varphi^2)$

$ds^2 = dx^2 + x^2 dy^2$ (locally) euclidean plane (pol. coord.) $(dr^2 + r^2 d\theta^2)$ $\left(g_{ab}\right) = \begin{pmatrix} 1 & 0 \\ 0 & r^2 \end{pmatrix} \ \& \ \left(g^{ab}\right) = \ldots$

$(X = x\cos y, Y = x \sin y \rightarrow ds^2 = dX^2 + dY^2)$

$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2$ Minkowski spacetime (cart. coord.)

$ds^2 = dudv + dy^2 + dz^2$ Minkowski spacetime ($u = ct + x$, $v = -ct + x$)

$$\left(g_{ab}\right) = \begin{pmatrix} 0 & 1/2 & 0 & 0 \\ 1/2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Flat spacetime/variety (metric connexion meaning) : one can find coordinates in which all the metric tensor components are constant (or : Riemann-Christoffel curvature tensor = 0)

Plane (no local curvature) : euclidean plane, Minkowski spacetime, …

Local curvature (not plane) : sphere, Newton's metric $ds_N^2 = -\left(1 - \frac{2U}{c^2}\right) c^2dt^2 + dx^2 + dy^2 + dz^2, \ldots$

What is *c* ? How interpreting it ? How naming it ?

Minkowski spacetime's definition has nothing to do with the light/Maxwell theory !
(despite the fact it was discovered thanks to some « strange » properties of the light, first revealed by the Michelson & Morley experiment, motivated by Maxwell's equations, …).

c is just the modulous of **a family of *modulous-invariant* speeds**, that exists from the mere fact that **the spacetime is equipped with a** ((– + + +) pseudo-euclidean) **geometry**

Requiring the validity of the **Maxwell equations** (in their usual form, and in the usual formulation of the theory), ie of the electromagnetic theory (that is not inherent to special relativity, but that can be « stick » in it) in **all galilean coordinate systems** requires :

Speed of electromagnetic waves in vacuum (as soon as Maxwell equations apply) ← $\frac{1}{\sqrt{\varepsilon_0 \mu_0}} = c$ → Speed of gravitational waves in general relativity & ST (in general, *c* related to GW)

inherent to the mere **spacetime** definition, as soon as it is **dynamical** (at least in General Relativity & Scalar-Tensor gravity)

Refering to *c* as the « speed of gravity » (rather than « speed of light ») :

would it be a better motivated choice for the terminology ?

II – General Relativity (GR) (& motivations to go beyond ?) :

Newton : gravitational potential ← matter content : $\Delta U = 4\pi G\rho$ (Poisson equation)

Gravitational potential

Matter content

Einstein (GR) : spacetime geometry ←→ matter content :

GR Einstein's equation : $R_{ab} - \frac{1}{2} R g_{ab} = \frac{8\pi G}{c^4}\left(\overset{(1)}{T}_{ab} + \overset{(2)}{T}_{ab} + \ldots \right)$ & matter field eqs

(depends on) spacetime's geometry

Matter content (& spacetime's geometry)

$\nabla^a \overset{(1)}{T}_{ab} = 0$

$\nabla^a \overset{(2)}{T}_{ab} = 0$

$\ldots$

(conservation eqs)

where :

$R_{ab} = \partial_c \Gamma^c_{ab} - \partial_b \Gamma^c_{ac} + \Gamma^e_{ce}\Gamma^c_{ab} - \Gamma^e_{ac}\Gamma^c_{be}$ → Ricci curvature tensor

$R = g^{ce} R_{ce}$ → (Ricci) curvature scalar

$\Gamma(g)^c_{ab} \equiv \frac{1}{2} g^{ce}\left(\partial_a g_{be} + \partial_b g_{ae} - \partial_e g_{ab}\right)$ → metric/Christoffel connexion

g^{ab} defined by $g_{ab} g^{bc} = \delta^c_a$ → contravariant vs covariant metric components

GR in the weak field case :

→ a weak (gravitational) field can not generate velocity changes of the order of c

the spacetime's metric is **close to Minkowski** (∃ a coord system)

$$g_{ab} = m_{ab} + h_{ab} \quad \text{with} \quad |h_{ab}| << 1$$

$$m_{ab} = diag(-c^2, 1, 1, 1)$$

Linearized GR's Einstein equation :

$$m^{ce}\partial_c\partial_e h_{ab} = -\frac{16\pi G}{c^4}\left(T_{ab} - \frac{1}{2}Tm_{ab}\right)$$

$m^{ce}\partial_c\partial_e$ (usual Dalembertian)

← using relevant conditions on the coordinate system — Einstein equation

In vacuum

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)h_{ab} = 0$$

→ c = speed of gravity

→ gravitational waves

Stationarity (besides weak field) → Poisson equation

→ back to Newton's gravity

$$ds_N^2 = -\left(1 - \frac{2U}{c^2}\right)c^2dt^2 + dx^2 + dy^2 + dz^2$$

is the lowest order (0PN) solution of GR

U (interpreted in relativistic terms)

GR and the Universe (try to describe the Universe as a whole) :

1917 (Einstein) : no stationary dust-filled (finite, (hyper)**spherical**) Universe …

$$ds^2 = -c^2 dt^2 + \frac{dr^2}{1-\frac{r^2}{R^2}} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

with R constant (radius of the Universe)

… but if the field equation is completed by a cosmological term …

$$R_{ab} - \frac{1}{2} R g_{ab} + \Lambda g_{ab} = \frac{8\pi G}{c^4}\left(\overset{(1)}{T}_{ab} + \overset{(2)}{T}_{ab} + \ldots\right)$$

with Λ constant

… thence a stationary dust-filled solution exists :

$$R_E = \frac{1}{\sqrt{\Lambda}} \quad \& \quad \rho_E = \frac{\Lambda c^2}{4\pi G} \quad \text{satisfies} \quad R_{ab} - \frac{1}{2} R g_{ab} + \Lambda g_{ab} = \frac{8\pi G}{c^4}\begin{pmatrix} \rho_E c^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

comoving dust

1920-30 Slipher, Hubble, Lemaître : Universe's expansion

Natural behaviour of GR cosmological solutions without (or with) Λ
→ **Λ no longer required !!!** (but may exist …)

- **1998-9** Perlmutter, Riess, Schmidt → accelerated expansion

- dust filled Universe
- RW cosmology $ds^2 = -c^2dt^2 + a(t)^2\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right]$
- GR

} **requires Λ !!!** (asymptotically …)

What is Λ ? How interpreting it ?

A new fundamental constant ? Something else ?

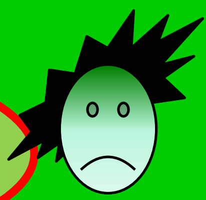
An appealing point :

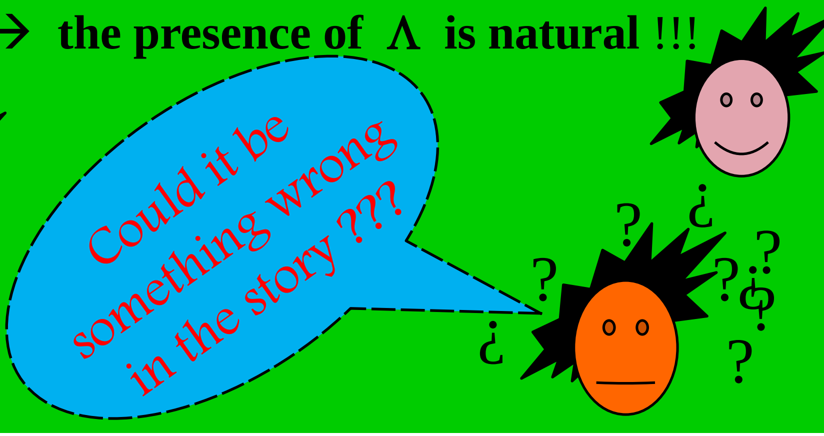
$$R_{\alpha\beta} - \frac{1}{2}Rg_{\alpha\beta} + \Lambda g_{\alpha\beta} = 8\pi T_{\alpha\beta} \quad \leftrightarrow \quad R_{\alpha\beta} - \frac{1}{2}Rg_{\alpha\beta} = 8\pi T_{\alpha\beta} + 8\pi\tilde{T}_{\alpha\beta}\left(\tilde{P} = -\tilde{\varepsilon}\right)$$

→ the **cosmological term** is ←→ to a perfect fluid with the **vacuum eq of state** !!! (QFT)

As a (QFT) field, the **vacuum should gravitate** → **the presence of Λ is natural** !!!

… BUT … $\frac{\Lambda_{QFT}}{\Lambda_{observed}} \sim 10^{120}$!!!!!!





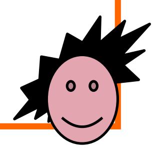
Going beyond GR ?

➔ … could be worth looking for an alternative « story » !!!

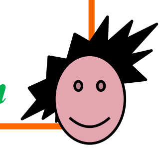
The « equivalence » :

$$R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} + \Lambda g_{\alpha\beta} = 8\pi T_{\alpha\beta} \quad \leftrightarrow \quad R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} = 8\pi T_{\alpha\beta} + 8\pi \tilde{T}_{\alpha\beta} \left(\tilde{P} = -\tilde{\varepsilon} \right)$$

suggests :
don't change the Universe's matter content, but the gravity theory → **alternative gravity theories**
reminiscent from
Mercury's perihelion's shift problem



suggests :
don't change the gravity theory, but the Universe's matter content → **dark energy theories**
reminiscent from
Uranus' orbit's anomalies problem



other options : change neither theory nor matter, but allow for
- voids (local inhomogeneities effect) ;
- remove large scale symmetries ;
- …

The **main topic of the following** … (just some alternatives)

GR lagrangian : $\ell_{GR} = \frac{c^4}{16\pi G}\sqrt{-g}\,(R-2\Lambda) + \sqrt{-g}\,\overset{(1)}{L}_{NG}\left(\overset{(1)}{\Psi}; g_{ab}\right) + \sqrt{-g}\,\overset{(2)}{L}_{NG}\left(\overset{(2)}{\Psi}; g_{ab}\right) + \ldots$

$\overset{(1)}{L}_{NG} \rightarrow \overset{(1)}{T}_{ab}$, $\overset{(2)}{L}_{NG} \rightarrow \overset{(2)}{T}_{ab}$

… it suggests some possible ways for alternatives

gravity sector | **« matter » sector**

$$\ell_{GR} = \underbrace{\frac{c^4}{16\pi G}\sqrt{-g}\,(R-2\Lambda)}_{\text{gravity sector}} + \underbrace{\sqrt{-g}\,L_{NG}(\Psi; g_{ab})}_{\text{« matter » sector}}$$

Could the Newton's constant be upgraded as a scalar field ?

* More intricate dependence in the metric ?
* Metric as the alone geometrical field ?

Does the matter necessarily couple with the metric only ?

- pure geometric theories (metric &/or independent connection)

  ➔ **pure metric theories**

- **scalar**(s)**-tensor theories**

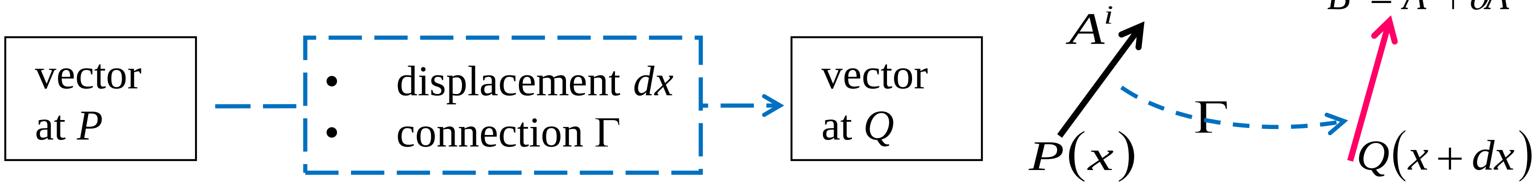
- others (scalar-vector-tensor, bimetric, massive gravity, …)

III – Purely metric gravity theories (MGT) :

Let us identify 2 basic geometric concepts :

- the metric g : defines intervals (generalizes the « length » concept) $\quad ds^2 = g_{ab}dx^a dx^b$

- the connection Γ : defines (local) parallelism



$B^i = A^i + \delta A^i$ with : $\delta A^a = -\Gamma^a_{bc}\, A^b\, dx^c$

The metric's connexion $\quad \Gamma(g)^c_{ab} \equiv \frac{1}{2} g^{ce}\left(\partial_a g_{be} + \partial_b g_{ae} - \partial_e g_{ab}\right)$

is just **one peculiar** connexion (that nevertheless possesses some fair properties)

But if these objects are considered as independent (metric-affine approach)

- varying wrt the metric → an equation (the « Einstein's equation » of the theory)
- varying wrt Γ → an equation that links the connexion to the metric

GR's action $\xrightarrow[\text{formalism}]{\text{metric-affine}}$ $\Gamma = \Gamma(g)$ **!!!**

From now on, we do the following <u>choices</u> :

- **MGT** : the gravitational sector of the theory depends on the **metric only**
- the **connexion** is a priori the metric's one $\Gamma = \Gamma(g)$ (second order formalism)

The most general MGT's lagrangian one could imagine :

$$\ell_{metric} = \frac{c^4}{16\pi G}\sqrt{-g}\,F\left(R, g^{ab}\partial_a R\partial_b R, g^{ab}\nabla_a\partial_b R, R_{ab}R^{ab}, R_{abcd}R^{abcd}, \ldots\right) + \sqrt{-g}\,L_{NG}\left(\Psi; g_{ab}\right)$$

« covariant » derivative (includes curvature effects)

Put all the metric (only) dependent terms you can imagine …

where F is required to be a **scalar** (invariant in all coordinate transforms)

ensures the « covariance » of the theory (relativity principle)

The simplest choice (for a non trivial theory) **:** $\boldsymbol{F(\ldots) = R}$ → **GR !!!**

<u>GR is the simplest MGT !</u> (the following step being GR with Λ)

What is the Einstein's equation of an MGT expected to look like ?

Preliminaries : **some generalities on lagrangian systems**

1rst order lagrangian $L_F(q,q')$

$$\underbrace{\frac{d}{dt}\frac{\partial L_F}{\partial q'} = \frac{\partial L_F}{\partial q}}_{\text{Euler-Lagrange}}$$

→ second order eq : $q'' = f(q,q')$

The associated « energy » reads $E_F = -L_F + q'\dfrac{\partial L_F}{\partial q'}$ and one shows that this energy **may be bound**, in which case the **theory is « stable »** (Ostrogradski stability)

2cd order lagrangian $L_S(q,q',q'')$

$$\frac{d}{dt}\frac{\partial L_S}{\partial q'} - \frac{d^2}{dt^2}\frac{\partial L_S}{\partial q''} = \frac{\partial L_S}{\partial q}$$

→ fourth order eq : (nondegeneracy) $q'''' = f(q,q',q'',q''')$

The associated « energy » reads $E_S = -L_S + q'\left(\dfrac{\partial L_S}{\partial q'} - \dfrac{d}{dt}\dfrac{\partial L_S}{\partial q''}\right) + q''\dfrac{\partial L_S}{\partial q''}$

and one shows that this energy is **generically unbound** → the **theory is « unstable »** (Ostrogradski instability)

R. Woodard (2007)

MGT lagrangians depend on R, thence **on the <u>metric's second derivatives</u>**

related to the prior hypothesis $\Gamma = \Gamma(g)$

expect

- the corresponding Einstein equation is of **fourth order** ;
- **Ostrogradski <u>instability of the theory</u> !!!**

<u>BUT :</u>

GR

If : $\boldsymbol{F(\ldots) = R}$ (or R+cst) **:** **the second derivatives terms → surface terms**

- GR Einstein's eq is of 2cd order
- no Ostrogradskian instability

First order effective lagrangian

***F*(*R*)**

In the other cases : fourth order Einstein's equation ...

… but **NO Ostrogradski instability if** $\boldsymbol{F(\ldots) = f(R)}$ **and in this case only !!!**

R. Woodard (2007)

→ A strong argument **supporting the** $\boldsymbol{f(R)}$ **theories among the whole MGT family**

Field equations : $$f'R_{ab}-\frac{1}{2}fg_{ab}-\nabla_a\partial_b f'+g_{ab}\nabla_c\partial^c f'=\frac{8\pi G}{c^4}T_{ab}$$

& the usual matter equations

generate fourth order terms (as soon as *f* is not affine)

A result on RW cosmology in this *f*(*R*) framework :

$$ds^2=-c^2dt^2+a(t)^2\left[\frac{dr^2}{1-kr^2}+r^2\left(d\theta^2+\sin^2\theta d\varphi^2\right)\right]$$

→ One can choose (reconstruct) a function *f*(*R*) in such a way that *a*(*t*) fits any prior Universe's history !!!

R. Woodard (2007)

f(*R*) theories would be of weak interest **if limited to this physical problem**

(but also : Solar System, stellar's structure, cosmological perturbations, …)

IV – (Usual) scalar-tensor (ST) theories :

$$\ell = \frac{c^4}{16\pi}\sqrt{-g}\left[\phi R - \frac{\omega(\phi)}{\phi} g^{ab}\partial_a\phi\partial_b\phi - 2\phi U(\phi)\right] + \underbrace{\sqrt{-g}\, L_{NG}\left(\Psi; g_{ab}\right)}$$

minimal matter-gravity coupling
→ Metric only (choice)

Motivations : - some attempts to quantize gravity (or unify with other interactions)
→ Brans-Dicke (BD) like theory (BD = ST with ω = constant)
- in some cases, close to GR in some sense (see later)
→ OK solar Systems' tests

Field equations :

$$R_{ab} - \frac{1}{2} R g_{ab} + U g_{ab} = \frac{8\pi T_{ab}}{c^4 \phi} + \frac{\omega}{\phi^2}\left[\partial_a\phi\partial_b\phi - \frac{1}{2} g_{ab}\partial_c\phi\partial^c\phi\right] + \frac{1}{\phi}\left(\nabla_a\partial_b\phi - g_{ab}\nabla_c\partial^c\phi\right)$$

$$(3+2\omega)\nabla_a\partial^a\phi + \frac{d\omega}{d\phi}\partial_a\phi\partial^a\phi + 2\phi\left(U - \phi\frac{dU}{d\phi}\right) = \frac{8\pi}{c^4} T$$

& the usual matter equations

ST may be locally interpreted as a gravity theory with varying effective Newton's constant (in a Cavendish-like experiment) :

$$G_{eff} = \frac{2\omega + 4}{2\omega + 3}\frac{1}{\Phi}$$

Remarks : - 2cd order field equations
- no Ostrogradskian instability, as soon as $\omega > -3/2$

f(*R*) versus ST :

$$f(R)\,\text{gravity}: \quad \underbrace{f' R_{ab}} - \frac{1}{2} f g_{ab} = T_{ab} + \underbrace{\nabla_a \partial_b f' - g_{ab} \nabla_c \partial^c f'}$$

$$(\omega = 0)\text{- BD gravity}: \quad \begin{cases} \underbrace{\phi R_{ab}} - \frac{1}{2}\phi(R - 2U) g_{ab} = T_{ab} + \underbrace{\nabla_a \partial_b \phi - g_{ab} \nabla_c \partial^c \phi} \\ 3\nabla_a \partial^a \phi + 2\phi(U - \phi U') = T \quad \text{or} \quad \frac{d(2\phi U)}{d\phi} = R \end{cases}$$

The ressemblance of the (Einstein's) equations is suggestive !

→ can go further ?

Let us define, from *f*(*R*) : $\phi = f'(R)$ & $2U(\phi) = R - \frac{f(R)}{f'(R)}$ $\left(R = f'^{-1}(\phi) \right)$

(excludes RG)

➔ The ***f*(*R*)** theory can be seen **as a peculiar BD** (thence ST) **theory**

→ *f*(*R*) Ostrogradskian stability, this correspondance requiring $\omega = 0$ ($> -3/2$)

P. Teyssandier, P. Tourrenc (1983)

ST versus GR :

The ST Einstein's equation in the constant scalar case :

$$\partial\phi = 0 \xrightarrow{\text{finite } \omega} R_{ab} - \frac{1}{2} R g_{ab} + U g_{ab} = T_{ab} \qquad \text{(with } U \text{ constant)}$$

ie GR gravity. But the scalar eq yields $\quad 2\phi \dfrac{U - \phi U'}{3+2\omega} = \dfrac{T}{3+2\omega}$

Consider (for simplicity) the case without potential. The scalar equation then requires :

$$\frac{T}{3+2\omega} = 0 \;\rightarrow\; \omega = \infty \quad (\text{if } T \neq 0\,)$$

Thence, the convergence of ST to GR generically requires $\omega \rightarrow \infty$

BUT : $$R_{ab} - \frac{1}{2} R g_{ab} = \frac{T_{ab}}{\phi} + \underbrace{\frac{\omega}{\phi^2}\left[\partial_a\phi\partial_b\phi - \frac{1}{2} g_{ab}\partial_c\phi\partial^c\phi\right]}_{\rightarrow \infty \times 0 \;\; !!!} + \underbrace{\frac{1}{\phi}\left(\nabla_a\partial_b\phi - g_{ab}\nabla_c\partial^c\phi\right)}_{\rightarrow 0 \;\; \text{OK}}$$

limit = GR, but with an extra matter term : massless scalar field

originating in the
BD scalar vanishing part !
(turns out to be = 0 in stationary cases…)

Physical relevance : ask a physical question :

Consider a flat RW dust-filled universe, with the observational constraint that H_0 is known.

→ What can be said on its age in the frameworks of (1) GR ; (2) (infinite ω)-BD ?

GR's answer : $T_{GR} = \dfrac{2}{3H_0}$

BD's answer : $T_{\omega BD} \in \left]\dfrac{1}{H_0}\dfrac{1+\omega+\sqrt{1+2\omega/3}}{4+3\omega}, \dfrac{2}{H_0}\dfrac{1+\omega}{4+3\omega}\right]$

Thence the (infinite ω)-BD's answer : $T_{BD\lim} \in \left]\dfrac{1}{3H_0}, \dfrac{2}{3H_0}\right]$

different answers → (infinite ω)-BD differs from GR

… but $]1/(3H_0),2/(3H_0)]$ corresponds to the GR's answer got

for a **dust + massless scalar** filled flat RW universe (in accordance with …)

BC (2007)

Rmks : - the residual scalar field is zero considering stationary solutions/problems

- the same conclusions essentially work, in some sense, for general ST

Experiments/propagation of light (Cassini) : ST pass Solar System tests if $\omega >$ 40 000

C. Will (2014)

Some large scale (cosmology, …) studies are grounded on ST theories effects that are by far more important than expected from solar system constraints

Conciliating the two ? A possibility could be the so called **chameleon mechanism**

(in brief)

The scalar has an effective « mass » increasing with local density

ϕ range $\propto$ mass^{-1}

- large range in galactic, cosmological, … mediums
- weak range (then no effect) in planetary, solar systems, … mediums

J. Khoury, A. Weltman (2004)

→ Yukawa like (spatial) damping

V – Scalar-tensor theories with an external scalar (EST) :

The ST theories' lagrangian

$$\ell = \frac{c^4}{16\pi}\sqrt{-g}\left[\phi R - \frac{\omega(\phi)}{\phi} g^{ab}\partial_a\phi\partial_b\phi - 2\phi U(\phi)\right] + \sqrt{-g}\, L_{NG}(\Psi; g_{ab})$$

Vary wrt metric g_{ab} → Einstein's equation
Vary wrt scalar field ϕ → Scalar equation (after combining with Einstein's equation)
Vary wrt matter fields Ψ → Matter field equations

conservation equations (energy, Euler, …)

However, considering that :
- physical consideration sometimes lead to scalar fields that are imposed in the theory (external, ie not varied in the lagrangian)

M. Reuter, H. Weyer (2004)

- resorting to external fields is sometimes required in physics (unimodular gravity, bimetric/massive gravity, …)

S. Weinberg (1989) ; C. de Rham et al (2011) ; C. Böhmer, N. Tamanini (2013)

… it may be worth to take a close look at the consequences if the scalar field is not varied at the action (lagrangian) level …

Digression : the « role » of the geometric identities

GR with 2 independent matter fields Ψ_1 & Ψ_2

$$R_{ab} - \frac{1}{2} R g_{ab} = \frac{8\pi G}{c^4}\left(\overset{(1)}{T}_{ab} + \overset{(2)}{T}_{ab} + \ldots \right) \quad \& \quad \nabla^a \overset{(1)}{T}_{ab} = 0 \quad \& \quad \nabla^a \overset{(2)}{T}_{ab} = 0$$

got varying the action w.r.t. the metric

got thanks to variation w.r.t. Ψ_1

got thanks to variation w.r.t. Ψ_2

Take the divergence and get (thanks to some geometrical identities) : $\nabla^a \left(\overset{(1)}{T}_{ab} + \overset{(2)}{T}_{ab} \right) = 0$

(nothing new)

The same theory (ie same action), but with Ψ_2 **not to be varied** (external/non-dynamical field)

$$R_{ab} - \frac{1}{2} R g_{ab} = \frac{8\pi G}{c^4}\left(\overset{(1)}{T}_{ab} + \overset{(2)}{T}_{ab} + \ldots \right) \quad \& \quad \nabla^a \overset{(1)}{T}_{ab} = 0 \quad \text{and that's all !!!}$$

got varying the action w.r.t. the metric

got thanks to variation w.r.t. Ψ_1

Take the divergence and get : $\nabla^a \left(\overset{(1)}{T}_{ab} + \overset{(2)}{T}_{ab} \right) = 0$ → $\nabla^a \overset{(2)}{T}_{ab} = 0$ is back !!!

Not a new equation, but made explicit thanks to the geometrical identities (showing that Einstein + conservation eqs are the same in both theories)

ϕ not varied → the scalar equation is lost

Vary wrt metric g_{ab} → Einstein's equation
~~Vary wrt scalar field ϕ → Scalar equation~~
Vary wrt matter fields Ψ → Matter field equations

Is the resulting theory « less constrained » ?

One could be tempted to claim :

NO, because geometrical identities & conservation equations ensure that the scalar equation is back

Ricci id : $\nabla_a g_{bc} = 0$

contracted Bianchi id : $\nabla_a \left(R^{ab} - \frac{1}{2} R g^{ab} \right) = 0$

Let us check !!! Two points : (a) conservation equations, (b) use geometrical identities

(a) Conservation equations :

not a trivial task, but **OK here** since :

- the matter action is a scalar
- all matter fields (Ψ) in the matter sector
- ϕ does not enter the matter sector
- no external field in the matter sector

$$\nabla_a T^{ab} = 0$$

(b) Use geometrical identities : it yields

$$\left[\nabla_a \partial^a \phi - \frac{8\pi c^{-4} T - \omega' \partial_a \phi \partial^a \phi - 2\phi(U - \phi U')}{3+2\omega}\right] \partial_b \phi = 0$$

$= 0$ in (usual) ST theories

leads to GR-like solutions if $= 0$

The EST theory equations admit :
- usual ST solutions
- (but also) GR solutions
- (and even !!!) **some mixes of the two !!!**

An (unexpected) cosmological solution :

Flat ($k = 0$) RW $\rightarrow$ $ds^2 = -c^2 dt^2 + a(t)^2 \left[dx^2 + dy^2 + dz^2\right]$

(ω = cst & U = 0)- EST equations in the dust filled case :

$$\begin{cases} 3\phi\left(\frac{a'}{a}\right)^2 = 8\pi\varepsilon + \frac{\omega}{2}\frac{\phi'^2}{\phi} - 3\frac{a'}{a}\phi' \\ 2\frac{a''}{a} + \frac{\phi''}{\phi} = -\frac{\omega}{2}\left(\frac{\phi'}{\phi}\right)^2 - 2\frac{a'}{a}\frac{\phi'}{\phi} - \left(\frac{a'}{a}\right)^2 \\ \left(\varepsilon a^3\right)' = 0 \end{cases}$$

($X' \equiv \frac{dX}{dt}$)

…and that's all !!!

Let us remark the form of the ***induced* equation** : $\phi' \times \left[\left(a^3 \phi'\right)' - \frac{8\pi\varepsilon a^3}{2\omega+3} \right] = 0$

GR-like

$$\phi = C$$
$$a = A t^{2/3}$$

BD-like

$$\phi = \phi_0 \left(t - t_+\right)^{p_+} \left(t - t_-\right)^{p_-}$$
$$a = B \left(t - t_+\right)^{q_+} \left(t - t_-\right)^{q_-}$$

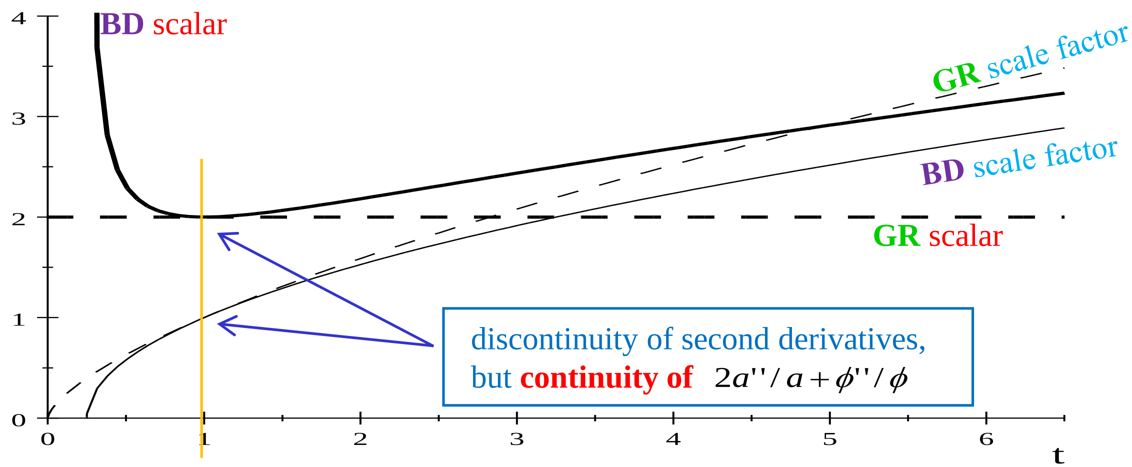
$$p_\pm = \frac{1 \pm 3s\sqrt{1 + 2\omega/3}}{4 + 3\omega}$$
$$q_\pm = \frac{1 + \omega \mp s\sqrt{1 + 2\omega/3}}{4 + 3\omega}$$

$\frac{\phi_0}{C} = \ldots$ & $\frac{B}{A} = \ldots$ ⟹ continuity of a, a', ϕ and ϕ' (and ε)

A possible solution → a GR phase followed by a Brans-Dicke one (with ω > –4/3)
(or the converse … → inflationary-like scenarios ?)

Illustration with numerical values :

- $\omega = 0$
- matching time = 1
- $A = 1$
- $C = 2$



BC, D. Rodrigues, J. Fabris (2015)

The EST gravity allows the coexistence in a ***same*** spacetime of (exact) GR regions & (exact) ST regions

→ some (new) kind of sceening mechanism for ST theories ?

<u>Some questions :</u>

- external scalar : does it mean it should be fixed a priori ?
  → not the case here, but happens in some theories with ext elements
- deterministic status of EST ? In the previous cosmological example :
  - the matching time (GR → BD) is arbitrary
  - the « jump GR → ST or ST → GR » is not ensured to occur

  → more data provided for the Cauchy problem ?
- …

<u>To be explored/in progress:</u>

- spherical symmetry : static, LTB-like, …
  → screening mechanism ?
- perturbed solutions in vacuum
  → gravitational waves ? Emission mechanisms ?
- perturbing about RW solution
  → revisiting cosmological perturbations ? « matching » ?
- other kinds of external fields ?

EST : just an « academic » curiosity ?
… or is there some physics to do with it ?


… thank you for your attention !!!
ϕ
zzzz…