Is Cold Dark Matter really "cold" ?

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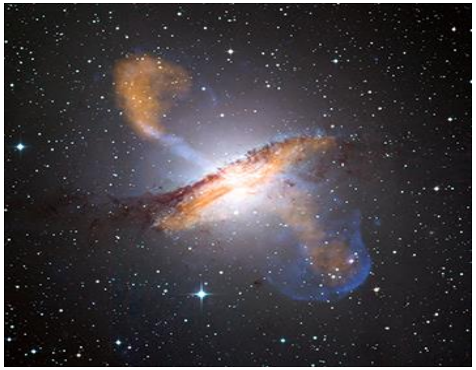


Outline

- *General aspects of dark matter*
- *Some DM models*
- *Cold DM – velocity dispersion effects*
- *DM in the Late Forming Scenario*

Dark Matter – successes and failures

- Dark matter explains adequately characteristics of the large scale structure of the Universe (cosmic WEB) as, for instance, the matter power spectrum in a wide range of scales.

- Without DM is difficult to reach the non-linear regime of growth of primordial fluctuations in an adequate timescale to explain the formation of galaxies

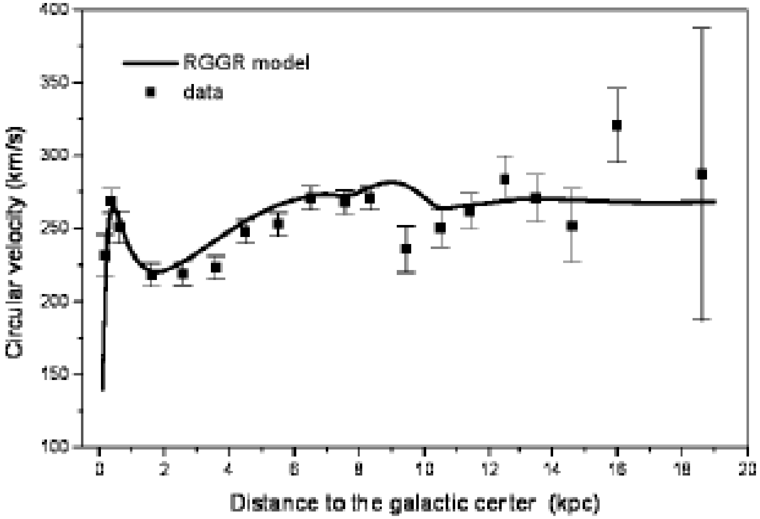
- Abundance of DM requires extensions of the Standard Model . However, no evidence for supersymmetry in LHC.

- Tension among some direct detection experiments (DAMA/LIBRA, CoGent, CRESS-II versus XENON 100) – only SSDM

- Difficulties with the predicted number of satellites and the central density profile of dark halos

Some Alternatives to DM

(Oliveira, de Freitas Pacheco & Reinisch, Gen.Rel.&Grav. 47,#12,2015)



RGGR theory better than MOND?

(Shapiro, Sola & Stefancic 2005)

$$V_c^2 = V_N^2 \left(1 - \frac{V_\infty^2}{\phi_N}\right) \Rightarrow V_\infty^2 = \alpha \nu c^2$$

Best model – fixed parameters

$$R_{thin} = 2.12 kpc \quad R_{thick} = 3.05 kpc$$

$$H_{thin} = 0.205 kpc \quad H_{thick} = 0.640 kpc$$

$$M_{gas} = 9.6 \times 10^9 M_\odot \quad H_{gas} = 100\, pc$$

Fixed constraint: Σ (8.3kpc)= 44 $M_\odot pc^{-2}$

Best fitted parameters

$$M_{bulge} = 7.8 \times 10^9 M_\odot \quad M_{thin} = 9.8 \times 10^9 M_\odot \quad M_{thick} = 2.6 \times 10^{10} M_\odot$$

RGGR parameter $\longrightarrow$ *V_∞ =226 km/s* *no universality of* $\alpha\nu$

DM as a Bose-Einstein Condensate (BEC)

All particles are in the fundamental state - Hartree approximation: $n(r) = |\Psi(r)|^2 = N\,|\phi(r)|^2$

Single particle wave function $\longrightarrow$ $-\dfrac{\hbar^2}{2m}\nabla^2\phi + mU(r)\phi(r) = E\phi(r)$

Effective potential $\longrightarrow$ $U(r) = -GmN\int d^3r'\,\dfrac{|\phi(r')|^2}{|\vec{r}-\vec{r}'|} + \dfrac{4\pi a\hbar^2}{m^2}N\,|\phi(r)|^2$

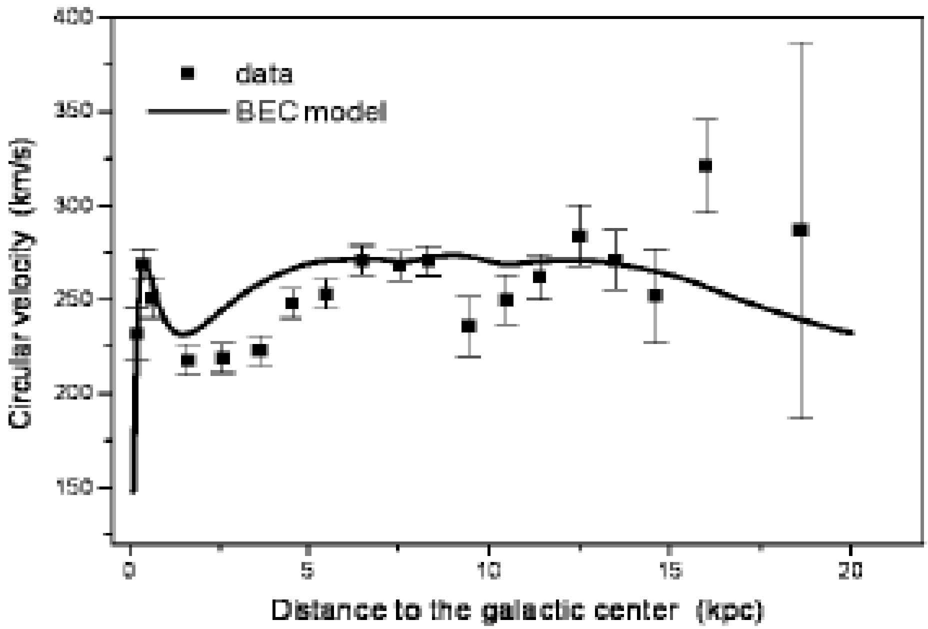
N $\longrightarrow \infty$ (Thomas-Fermi approximation) $\longrightarrow$ $\rho(r) = \rho_0\,\dfrac{\sin kr}{kr}$

With $\longrightarrow$ $kR_B = \pi \;\Rightarrow\; R_B = \pi\sqrt{\dfrac{a\hbar^2}{Gm^3}}$

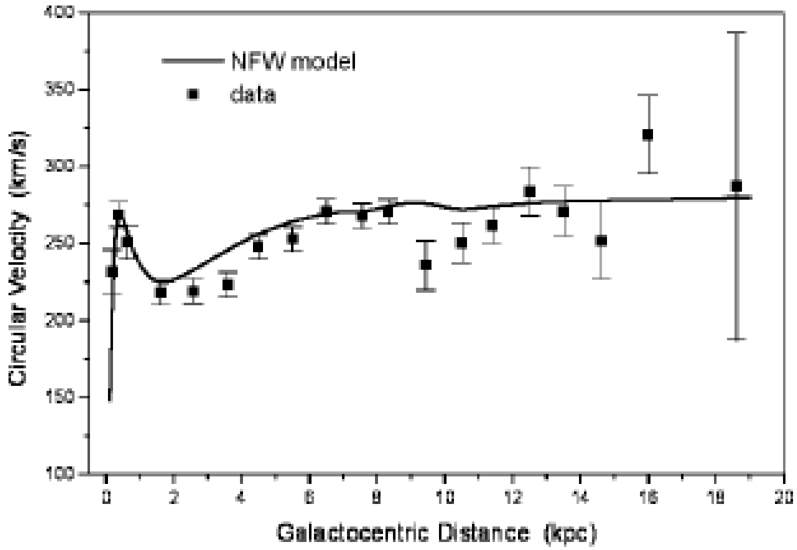
Best model :

$$M_{disk} = 5.0\times10^{10}\,M_\odot \quad M_{bulge} = 1.1\times10^{10}\,M_\odot$$

$$M_{BEC} = 1.69\times10^{11}\,M_\odot \qquad R_B = 15.7\,kpc$$



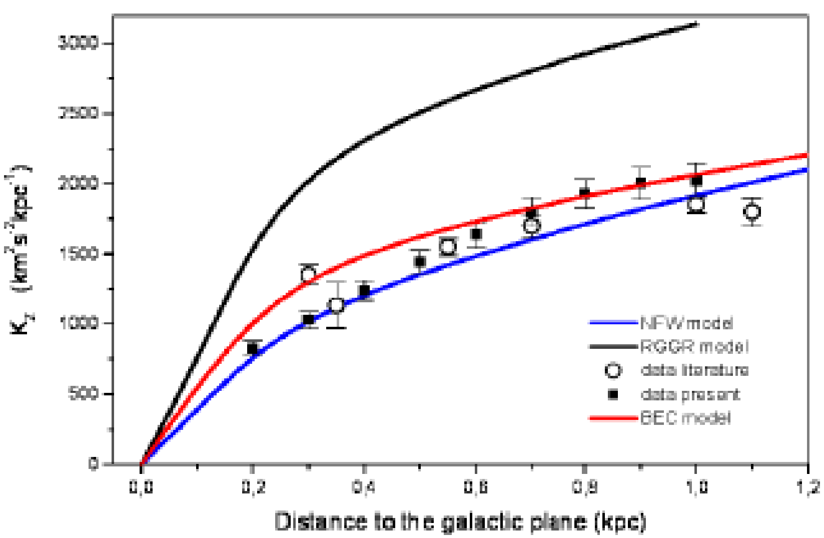
Comparison with a canonical NFW halo model



$$M_{thin} = 1.11\times10^{10} M_\odot \quad M_{thick} = 2.52\times10^{10} M_\odot$$

$$M_{bulge} = 1.07\times10^{10} M_\odot \quad M_h(<200kpc) = 2.1\times10^{12} M_\odot$$

$$\rho_{dm}c^2(8.3kpc) = 0.75\,GeVcm^{-3}$$



Testing the fit quality

Model	χ^2_R/ν	ν_R	χ^2_Z/ν	ν_Z
RGGR	1.83	15	-	-
BEC	4.61	13	10.09	14
NFW	2.17	14	5.27	14

Current Scenario for SSDM

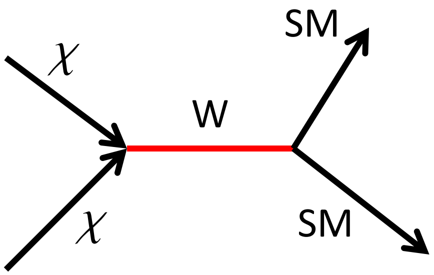
- Dark matter particles decouple when non-relativistic. Chemical decoupling occurs at $T \sim 25\, m_\chi$, fixing the relic abundance.
- After chemical decoupling, DM particles still interact with SM particles (mainly leptons) – this maintains their temperature close to the cosmic plasma temperature.
- Kinetic decoupling occurs around *T ~ 20-30 MeV* and depends on cross-sections fixed by weak interactions ("fingerprints" in small scales of the linear matter power spectrum?)
- After kinetic decoupling, the evolution is described by the Vlasov-Einstein equation.

Lee-Weinberg Bound

- thermal particles -

Chemical decoupling occurs when $\longrightarrow \quad n_\chi < \sigma v > = n_\gamma X_\chi < \sigma v > \approx H = \sqrt{\frac{8\pi G}{3c^2}\varepsilon_r}$

Weak interaction annihilation rate $\longrightarrow \quad < \sigma v > \approx \alpha^2 \frac{\hbar^2}{M_W^2 c}\left(\frac{m_\chi}{M_W}\right)^2$



From these equations $\longrightarrow \quad X_\chi = \left(\frac{2\pi^5}{90}\right)^{1/2} \frac{c^2 g_*^{1/2}}{\varsigma(3) k T_{cd}} \frac{M_W^4}{\alpha^2 m_\chi^2 M_P}$

DM relic abundance $\longrightarrow \quad \Omega_{dm} = \frac{8\pi G}{3H_0^2} m_\chi \left(\frac{g_0}{g_*}\right) n_{\gamma,0} X_\chi$ Use $\longrightarrow \quad m_\chi c^2 \simeq 25 k T_{cd}$

To obtain $\longrightarrow \Omega_{dm} h^2 \approx 0.0265 \left(\frac{g_0}{g_*^{1/2}}\right)\left(\frac{M_W}{m_\chi}\right)^2 \leq 0.12$

$g_0 = 2.92 \quad g_* = 93.25 \qquad \longrightarrow \quad m_\chi \geq 23$ GeV

The Vlasov-Einstein Equation

The one-particle distribution function obeys the Vlasov-Einstein equation

$$p^0 \frac{\partial f}{\partial t} + p^j \frac{\partial f}{\partial x^j} - \Gamma^j_{mn} p^m p^n \frac{\partial f}{\partial p^j} = 0$$

The particle density $\longrightarrow$ $n = \int f \, d^3 p$ and the matter tensor $\longrightarrow$ $T^{ik} = \int f \sqrt{|g|} p^i p^k \frac{d^3 p}{p^0}$

Define the tensor $\longrightarrow$ $\omega^{ik} = \langle V^i V^k \rangle = \frac{1}{n} \int f \frac{p^i p^k}{E^2} d^3 p$ with $\longrightarrow$ $V^i = a \frac{dx^i}{dt} = \frac{p^i}{E}$

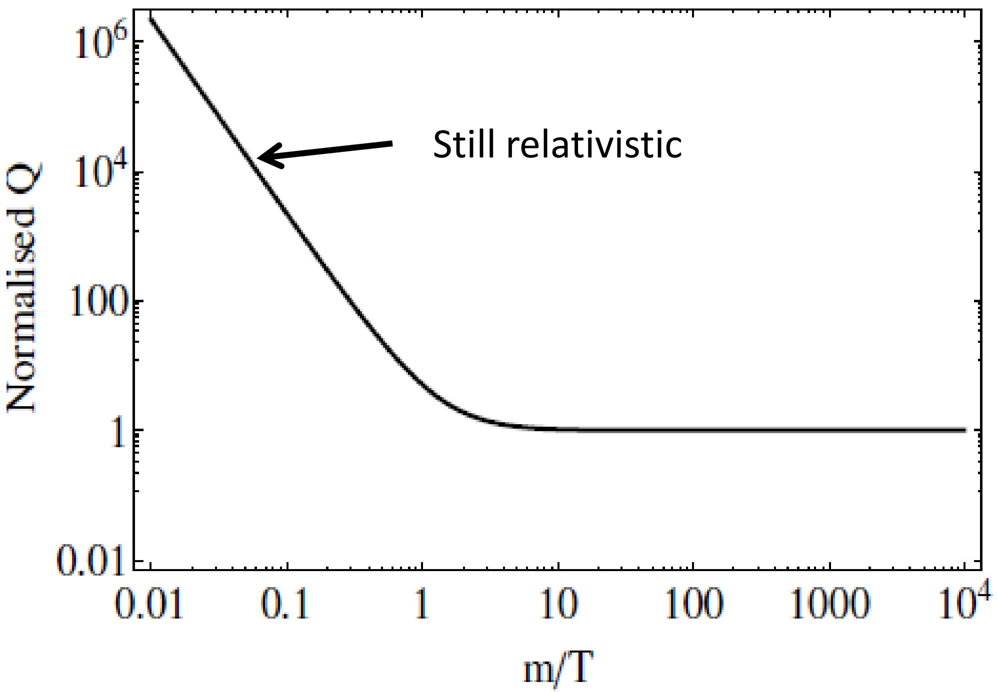
For a flat FRW cosmology, the VE equation reduces to $\longrightarrow$ $\frac{\partial f}{\partial t} - Hp \frac{\partial f}{\partial p} = 0$

General solution $\longrightarrow$ $f = f(ap)$ implying $\longrightarrow$ $n = \frac{4\pi}{a^3} I_2$ *and* $\sigma^2_{1D} = \frac{4\pi}{3na^3} \int_0^\infty f(x) \frac{x^4 dx}{(x^2 + m^2 a^2)}$

with the definition $\longrightarrow$ $I_n = \int_0^\infty f(x) x^n dx$

(see details in Piattella, Fabris & de Freitas Pacheco , JCAP 11, 002, 2013)

Define the phase-space indicator $\longrightarrow$ $Q = \frac{\rho}{(\sigma_{1D}^2)^{3/2}} = 4\pi\sqrt{27}m^4 I_2^{5/4} I_4^{-3/2}$ NR regime



In the non-relativistic regime, the phase-space density of DM remains constant

(Peirani & de Freitas Pacheco, PRD 77,064023, 2008)

One expects that even in the linear regime the phase density will remain constant up to violent relaxation or phase mixing become operative in the non-linear phase

A Fermi-Dirac distribution was assumed to compute the graphic

The linear regime

The perturbed metric (Newtonian gauge) $\longrightarrow$ $ds^2 = -(1+2\Psi)dt^2 + a^2(t)\delta_{ij}(1+2\Phi)dx^i dx^j$

The perturbed distribution function $\longrightarrow$ $f(t,x^i,p^i) = f^0(t,p^i) + f^1(t,x^i,p^i) + \ldots$

Integration of the DF over momenta $\longrightarrow$ $\int f d^3p = \int f^0 d^3p + \int f^1 d^3p = n_0 + n_1$

The perturbed bulk velocity requires some attention

$$V^i = \frac{1}{n}\int\left(a\frac{dx^i}{dt}\right) f d^3p$$

$$\frac{dx^i}{dt} = \frac{p^i}{p^0} = \left(1-\Phi+\Psi\right)\frac{p^i}{aE}$$

$$V^i = \frac{1}{(n_0+n_1)}\int\left(f^0 + f^1\right)(1-\Phi+\Psi)\frac{p^i}{E}d^3p$$

$$V^i \simeq \frac{1}{n_0}\int f^1 \frac{p^i}{E} d^3p$$

The term including f^0 does not contribute due to isotropy .
The bulk velocity is a pure first-order quantity, since it depends on f^1

Replace the perturbed DF into the VE equation and integrate over momenta

$$\dot{\delta} + \frac{1}{a}\partial_i V^i + 3\dot{\Phi} = 0$$

where $\longrightarrow$ $\delta = \frac{\rho_1}{\rho_0} = \frac{n_1}{n_0}$ for non relativistic particles

Multiply the VE equation by p^j/E and integrate over momenta to obtain

$$\partial_t V^i + HV^i + \frac{1}{a}\partial_j \omega_1^{ij} + \frac{1}{a}\partial_i \Psi = 0$$

Take the divergence of this equation and combine with equation for the density contrast

$$\ddot{\delta} + 2H\dot{\delta} + 6H\dot{\Phi} + 3\ddot{\Phi} - \frac{1}{a^2}\nabla^2\Psi + \frac{1}{a^2}\partial_i\partial_j\omega_1^{ij} = 0$$

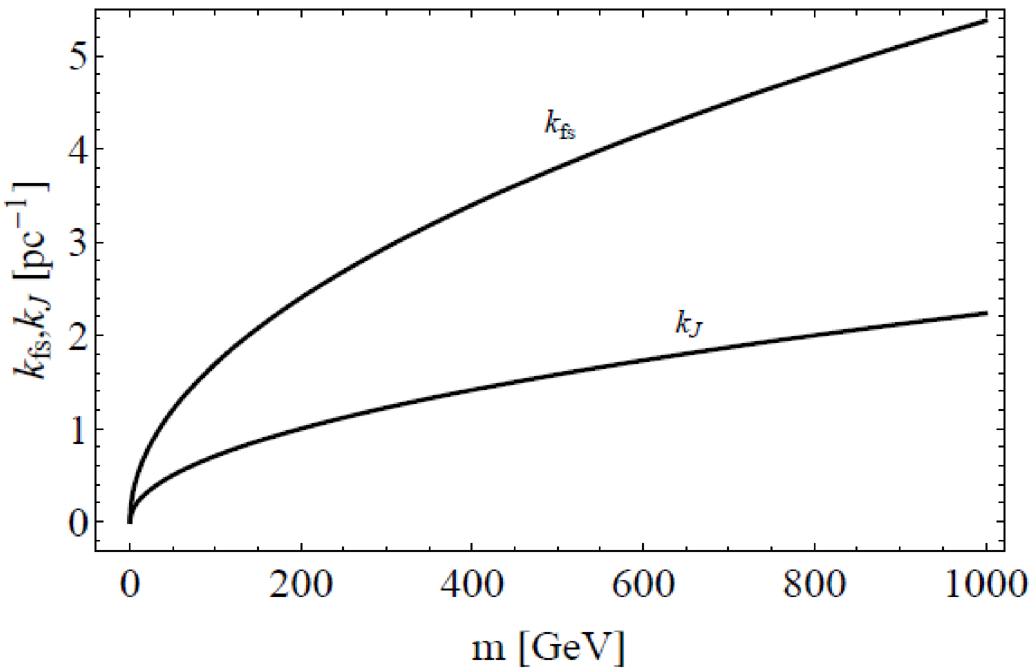
The metric potentials are derived from the linearized Einstein equations
Absence of anisotropic stresses $\longrightarrow$ $\Phi = -\Psi$

$$\begin{cases} 3H^2\Psi - 3H\dot{\Phi} + \frac{1}{a^2}\nabla^2\Phi = -4\pi G\rho_0\delta \quad (00\,comp.) \\ \nabla^2(\dot{\Phi} - H\Psi) = 4\pi G\rho_0 a\partial_i V^i \quad (0i\ comp.) \\ \ddot{\Phi} + 3H\dot{\Phi} - H\dot{\Psi} - (3H^2 + 2\dot{H})\Psi = 4\pi G(\delta P) \quad (ii\ comp.) \end{cases}$$

Next step – take Fourier components , consider sub-horizon scales ($k >> Ha$) and

isotropy such as $\longrightarrow$ $\omega_1^{ij} = \sigma_1^2 \delta^{ij}$ - then the density contrast evolves as

$$\ddot{\delta} + 2H\dot{\delta} - \left[4\pi G \rho_0 - \frac{5k^2}{a^2} \left(\frac{\rho_0}{Q} \right)^{2/3} \right] \delta = 0$$



Critical free-streaming and Jean wavenumber at matter-radiation equality

Gravitational instability if

$$\lambda_{J,dm}^2 \geq \frac{5\pi}{G} \rho_0^{-1/3} Q^{-2/3}$$

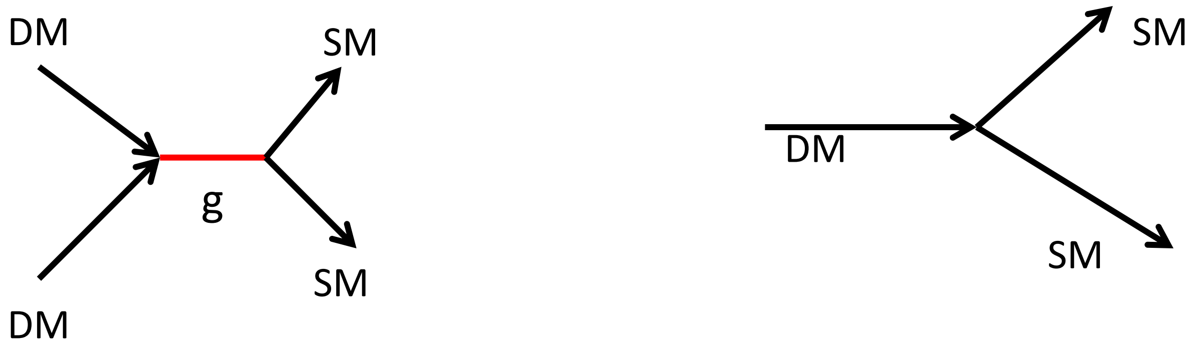
Jeans mass for dark halos

$$M_J = \frac{\pi^3}{2} \left(\frac{H_0^2 \Omega_{dm}}{k_J^3 G} \right) \approx 4.3 \times 10^{-6} M_\odot$$

No halos can be formed with masses less than the Jeans mass

Are there other particle model alternatives to SSDM?

Scalar, Vector and Fermionic particle models were considered by (Mambrini et al. arXiv:1508.06635)



Standard model decay products analyzed for 16 coupling models

Observational Constraints

- Proton-Antiproton spectrum up to 450GeV (AMS-02)
- Spectral lines in X and γ rays region (data from space satellites)
- Neutrino data (Ice Cube, Amanda, Super-K)
- Continuous γ-ray spectrum
- **Resulting limits** - scalar DM $\longrightarrow$ m < 100 keV
  vector DM $\longrightarrow$ m < 10 MeV
  fermionic DM $\longrightarrow$ m < 100 MeV

Radiation Effects

Perturbed metric $\longrightarrow$ $$ds^2 = -a^2(\tau)\left[d\tau^2 - \left(\delta_{ij} + h_{ij}\right)dx^i dx^j\right]$$

(where τ is now the conformal time)

The perturbation is written in terms of the inverse Fourier transform

$$h_{ij}(\vec{x},\tau) = \int d^3k \exp(i\vec{k}\cdot\vec{x})\left[k_i k_j h(\vec{k},\tau) + 6\left(k_i k_j - \frac{1}{3}\delta_{ij}\right)\eta(\vec{k},\tau)\right]$$

h and η are scalars corresponding to the trace and the traceless parts of the perturbation

Adopt the same procedure as before in order to obtain finally

$$\ddot{\delta}_n + \frac{(\dot{a} - 5\sigma_0^2/6)}{(a + 5\sigma_0^2/6)}\left[\dot{\delta}_n + \frac{\dot{h}}{2}\right] + k^2 \frac{(5\sigma_0^2/3)}{(3 + 5\sigma_0^2/2)}\delta_n + \frac{\ddot{h}}{2} = 0$$

(derivatives with respect to the conformal time)

See Piattella et al. arXiv: 1507.00982

The **C**ode for **A**nisotropies in the **M**icrowave **B**ackground (CAMB)

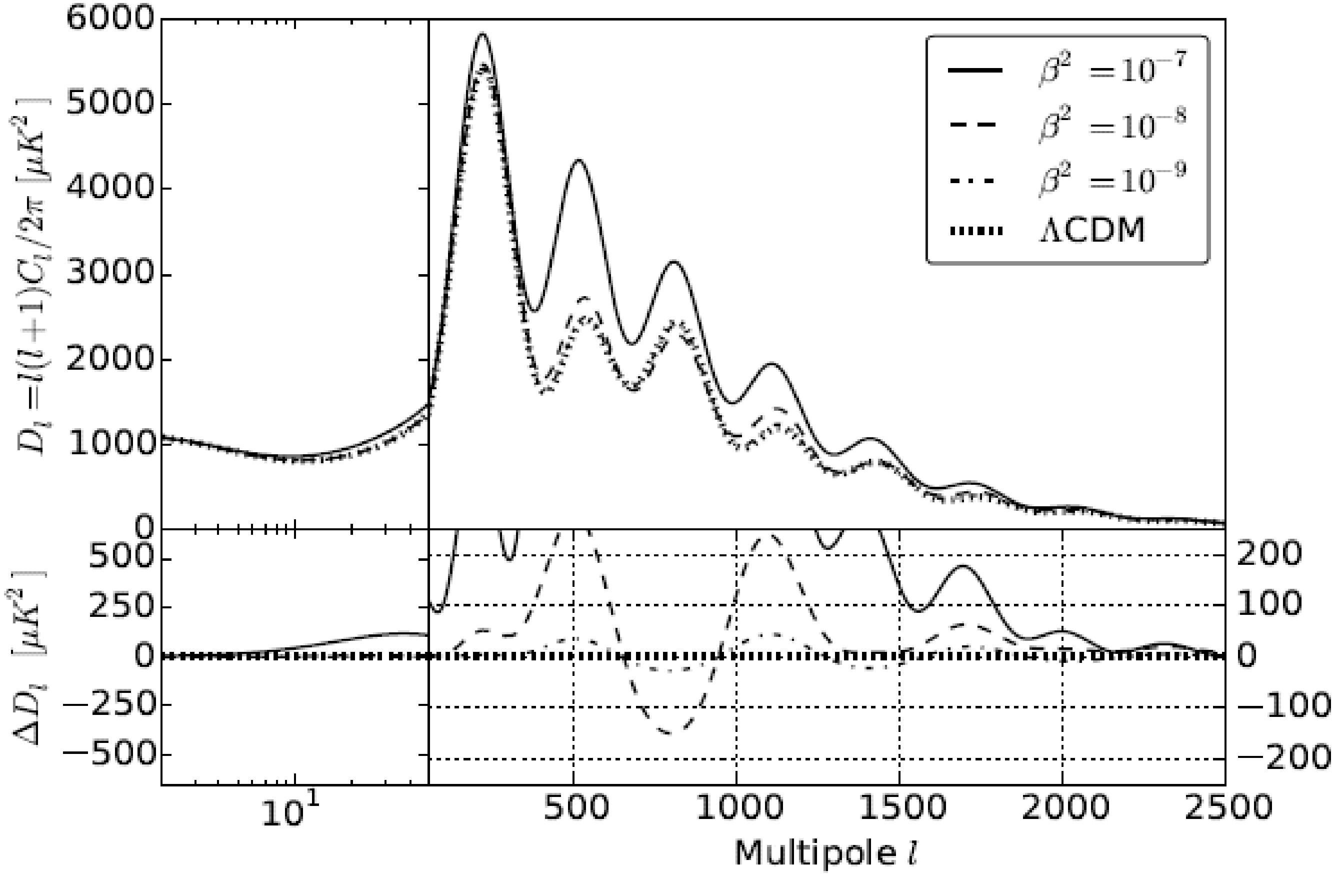
CAMB modified to include velocity dispersion effects

Solutions depend on the initial condition of the dispersion velocity

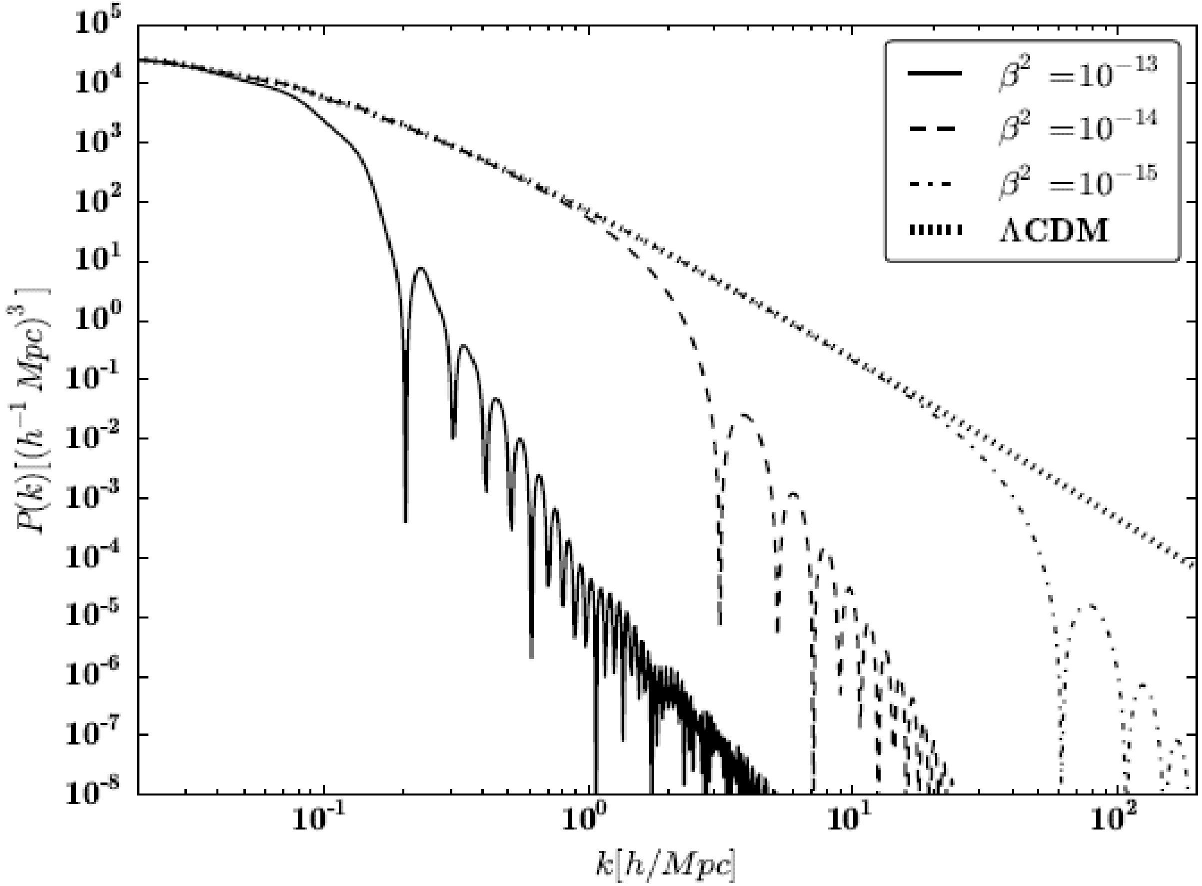
$$\sigma_0^2 = \sigma_{kd}^2 \frac{a_{kd}^2}{a^2} = \frac{\beta^2}{a^2}$$

Supersymmetric particle $\longrightarrow$ neutralino $\longrightarrow$ $\sigma_{kd}^2 = \frac{3T_{kd}}{m} \longrightarrow \frac{T_{kd}}{T_0} = \frac{a_0}{a_{kd}}\left(\frac{g_0}{g_{kd}}\right)^{1/3}$

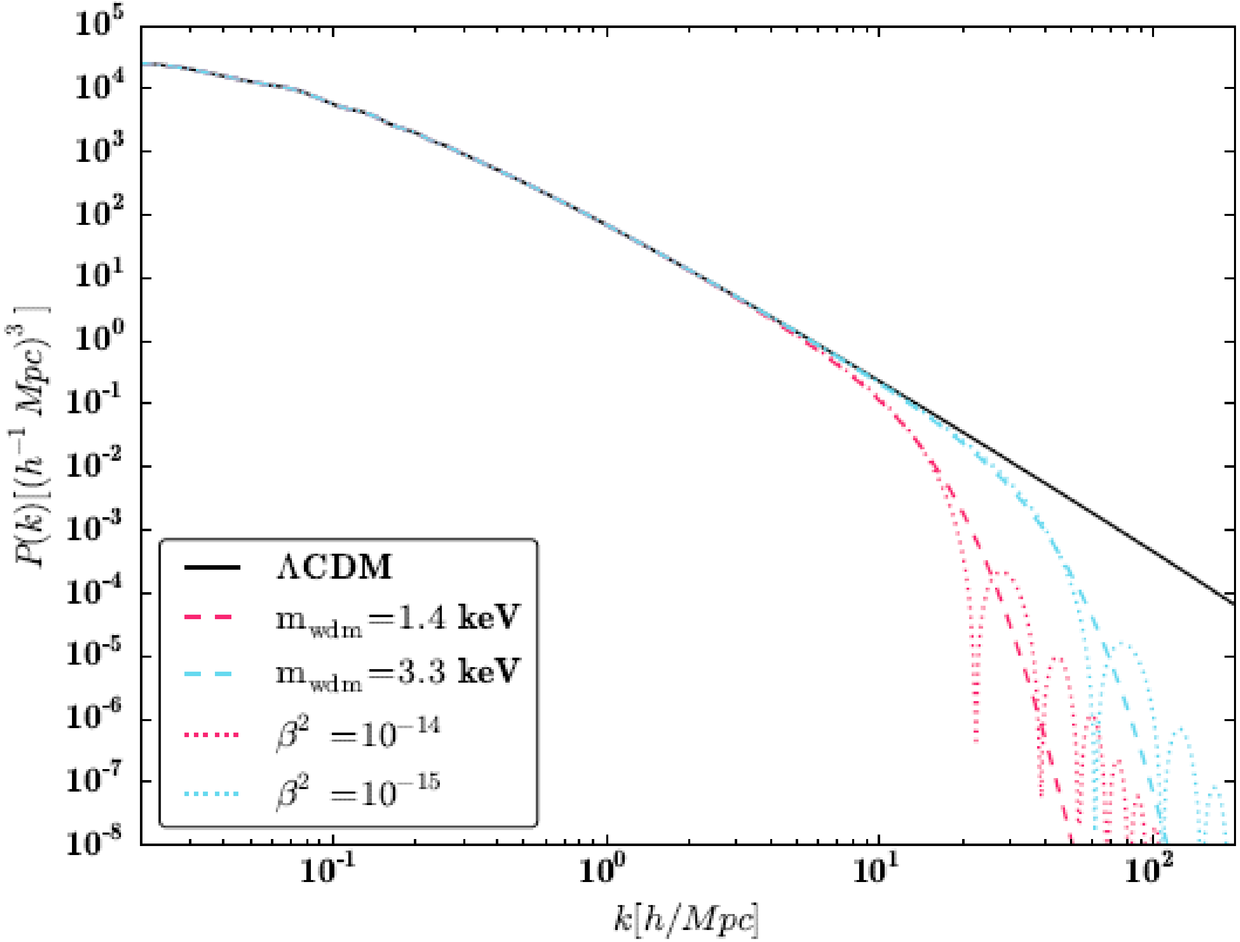
Using these relations $\longrightarrow$ $\beta^2 = \frac{3T_0^2}{mT_{kd}}\left(\frac{g_0}{g_{kd}}\right)^{2/3} = 8.28\times 10^{-26}\left(\frac{g_0}{g_{kd}}\right)^{2/3}\left(\frac{100GeV}{m}\right)\left(\frac{20MeV}{T_{kd}}\right)$



Constraint imposed by CMB angular power spectrum $\longrightarrow$ $\beta^2 < 10^{-9}$

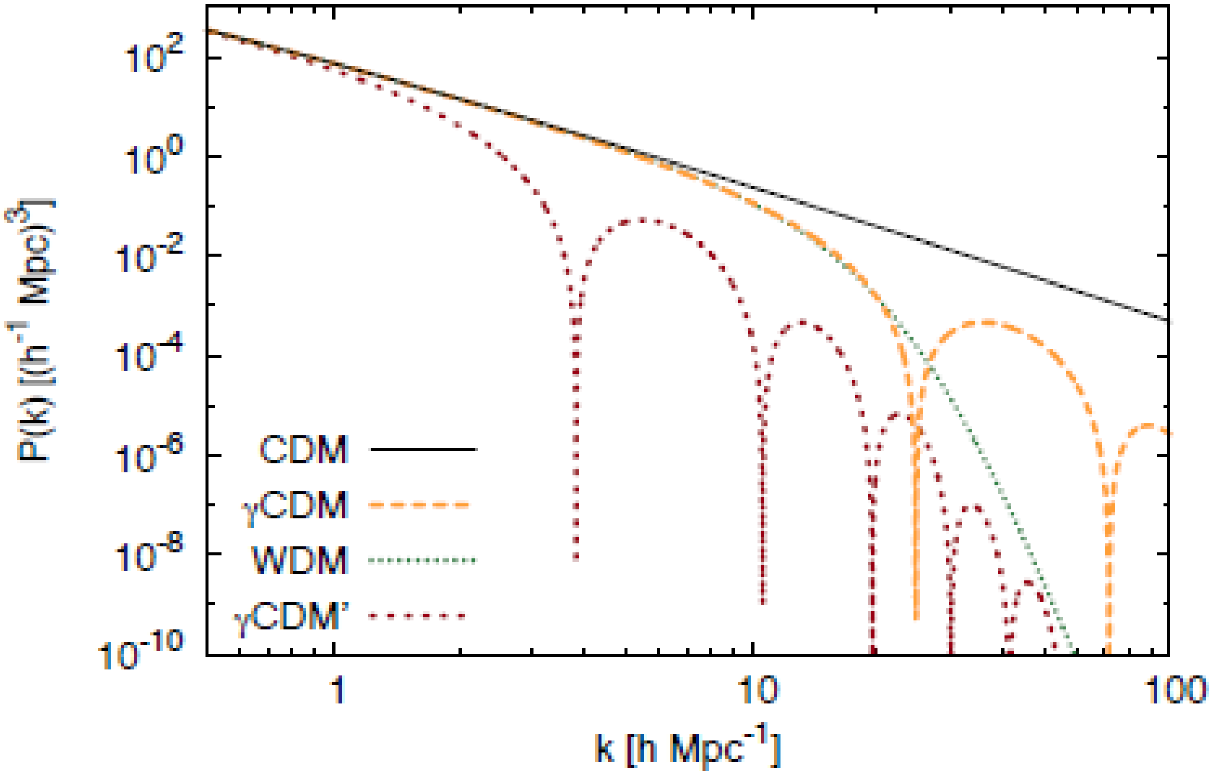


Linear matter power spectrum



Comparison with the linear matter power spectrum generated by WDM

$\beta^2 = 10^{-15} \equiv m = 3.3$ keV $\qquad \beta^2 = 10^{-14} \equiv m = 1.4$ keV



Linear matter power spectrum – radiation – dark matter interaction (Boehm et al. 2014)

$$\frac{\sigma_{dm\to\gamma}}{\sigma_T} = 2\times10^{-9} \quad and \quad 10^{-7}$$

Quasi-Cold DM vs WDM or IDM

- Velocity dispersion effects mimic WDM or IDM effects in the linear power spectrum of matter ($\beta^2 = 10^{-15} \longrightarrow$ m = 3.3 keV)
- Effects are of different nature – WDM $\longrightarrow$ "free streaming" , IDM $\longrightarrow$ "collisional damping while velocity dispersion effects are similar to "pressure" effects
- Supersymmetric particles originated from thermal equilibrium are unable to produce such a value of β^2 . A non thermal origin is required
- Particles are created or become non-relativistic at the redshift defining the value of β^2 that is

$$(1+z_t) = \sqrt{\frac{\sigma_t^2}{\beta^2}} \approx 10^7 \;\Rightarrow\; T \sim 2.5\,keV \;\; (\,just\ after\ nucleosynthesis)$$

The mass of non-thermal DM

The particle density and the velocity dispersion are derived from the distribution function

$$n = \frac{1}{2\pi^2}\int_0^\infty p^2 f(ap/p_0)dp = \frac{p_0^3}{2\pi^2 a^3} I_2$$

$$\sigma^2 = \frac{1}{2\pi^2 n}\int_0^\infty p^2 \left(\frac{p}{m}\right)^2 f(ap/p_0)dp = \frac{p_0^2}{m^2 a^2}\frac{I_4}{I_2}$$

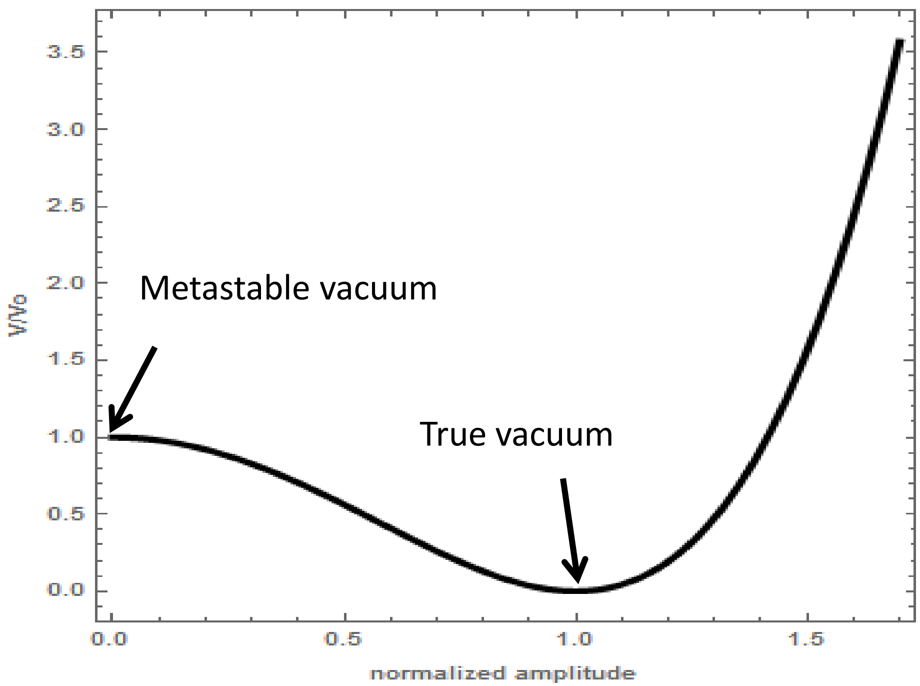
Then $\longrightarrow$ $\beta^2 = \sigma^2 a^2 = \frac{p_0^2}{m^2}\frac{I_4}{I_2}$

To eliminate p_0 use the DM abundance $\longrightarrow$ $\Omega_{dm} = \frac{8\pi G}{3H_0^2} m n_0 = \frac{4}{3\pi}\frac{G m p_0^3}{H_0^2} I_2$

To obtain finally $\longrightarrow$ $\beta^2 = 3.45\times 10^{-15}\frac{I_4}{I_2^{5/3}}\left(\frac{\Omega_{dm}h^2}{0.12}\right)^{2/3}\left(\frac{keV}{m}\right)^{8/3}$

Late Forming Dark Matter

- First proposed by Das & Weiner (Phys. Rev. D84, 123511, 2011)
- Scalar field rolls to a metastable vacuum state and oscillates, forming DM particles with mass $M_\phi = \sqrt{V''(\phi_0)}$
- Revised version – scalar field rolls to its true vacuum, oscillates and decays into DM particles



Field dynamics

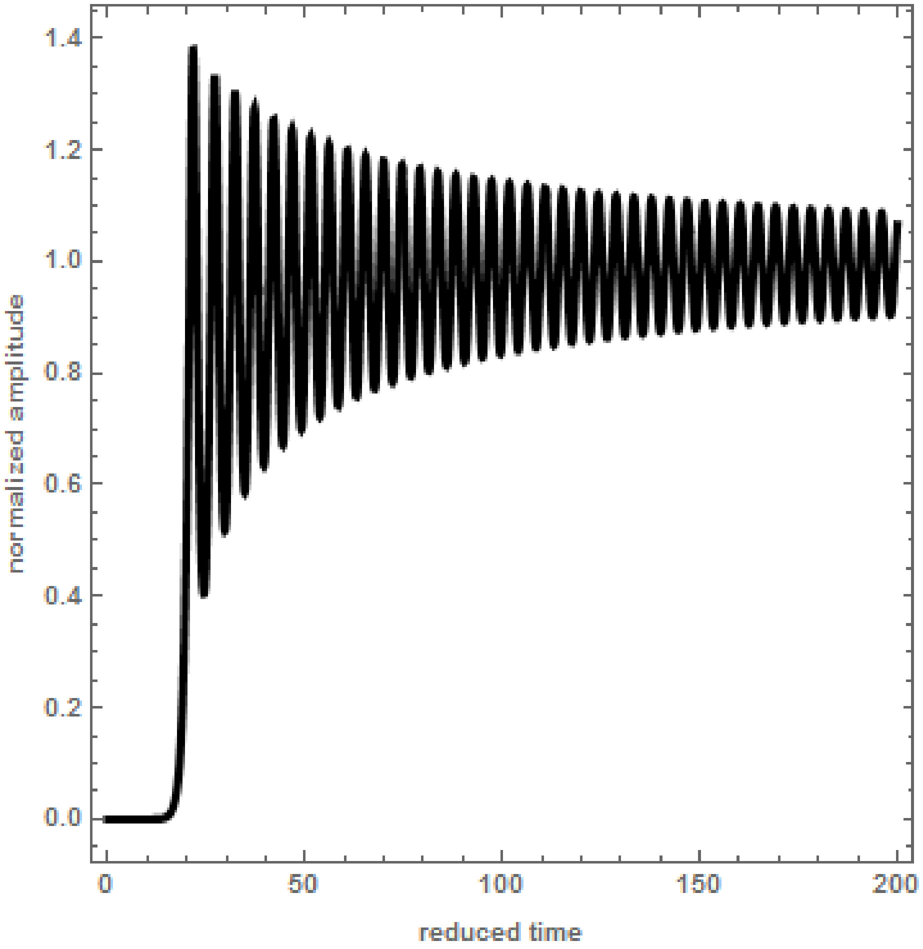
$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

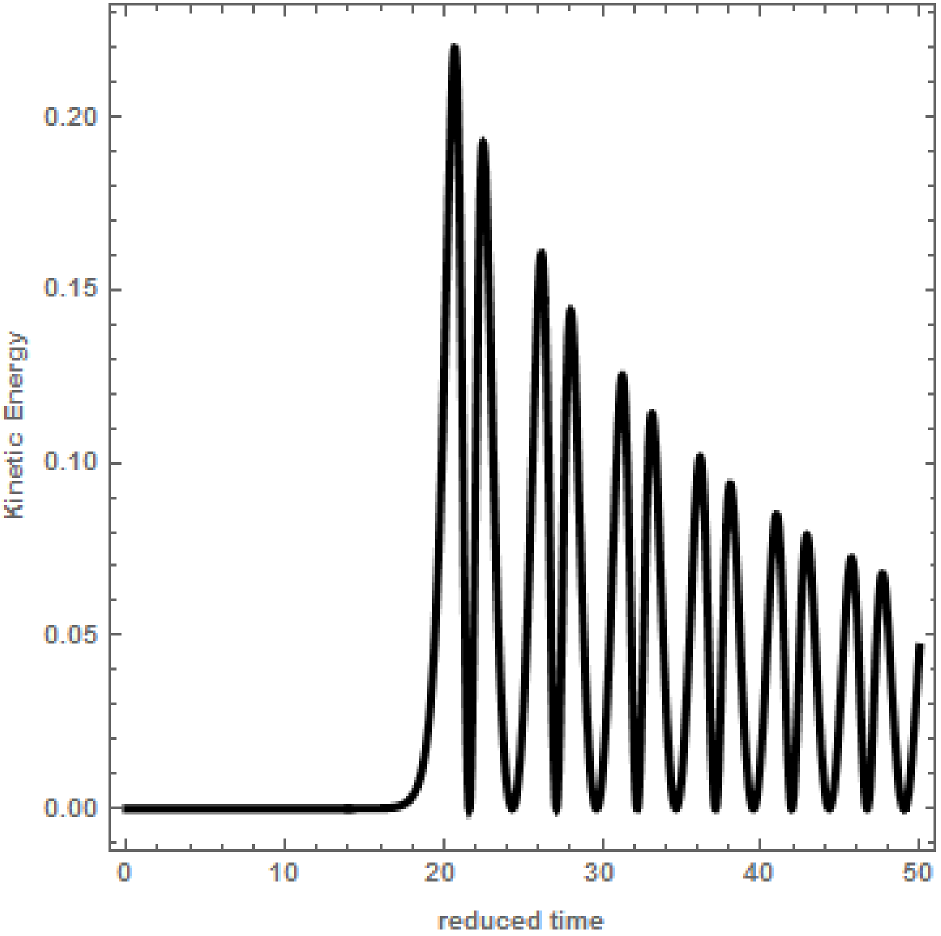
Toy model potential

$$V(\phi) = V_0\left[1 - \left(\frac{\phi}{\phi_0}\right)^2\right]^2$$

$$\text{with} \quad y = \phi/\phi_0 \quad x = tA \quad A = M_\phi/\sqrt{2}$$

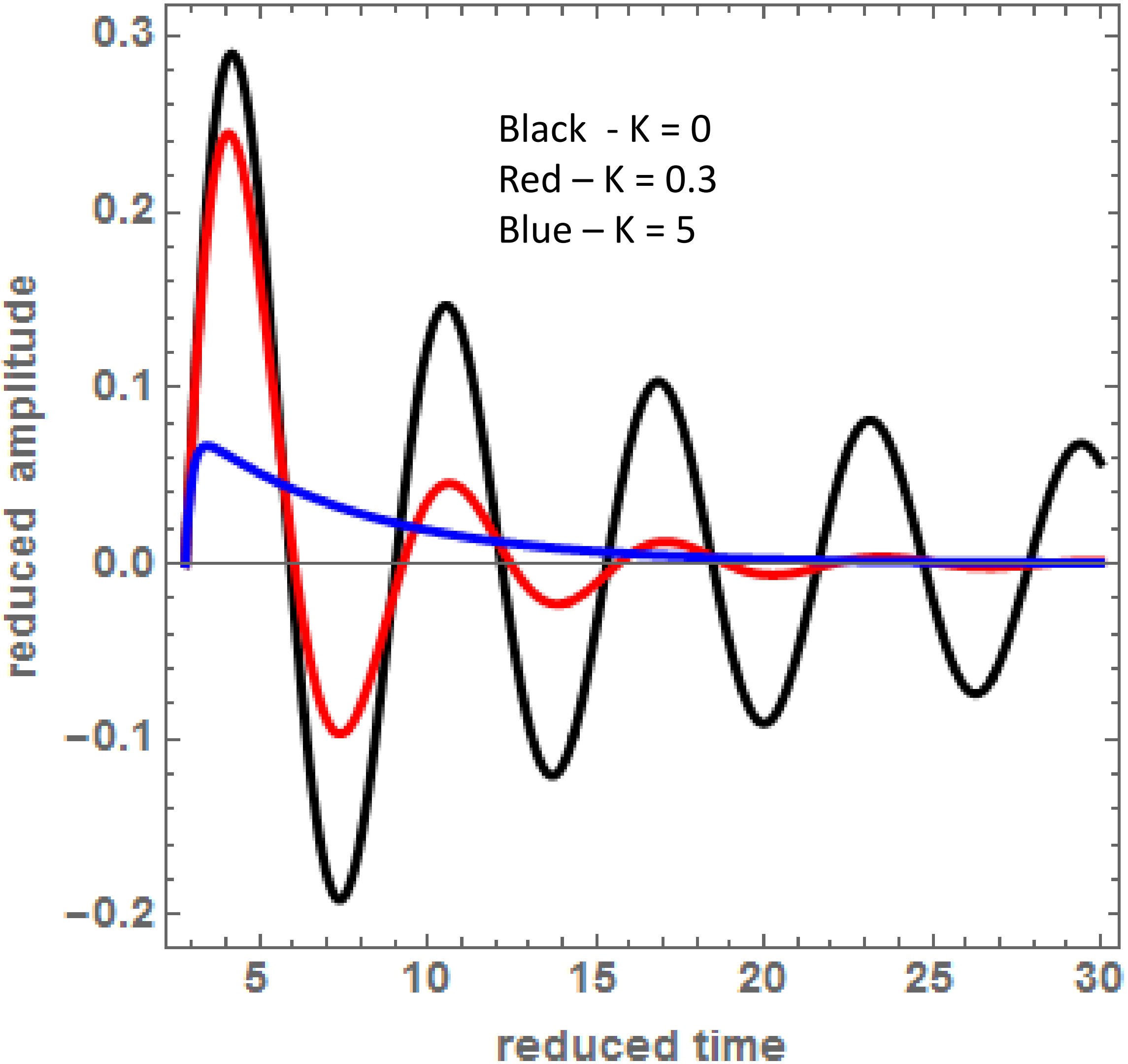
$$y''(x) + \frac{3}{2}\frac{y'(x)}{x} - y(x)\left[1 - y^2(x)\right] = 0$$





Left – Evolution of the field amplitude as a function of time – minimum is reached at x_0 = 20.7245

Right – Evolution of the field kinetic energy – at minimum, $W_\phi = 0.877V_0$



When oscillations begin, particles are produced and a new damping factor must be included in the field equation

$$\Delta\ddot{\phi} + (3H + \Gamma_\phi)\Delta\dot{\phi} + M_\phi^2 \Delta\phi = 0$$

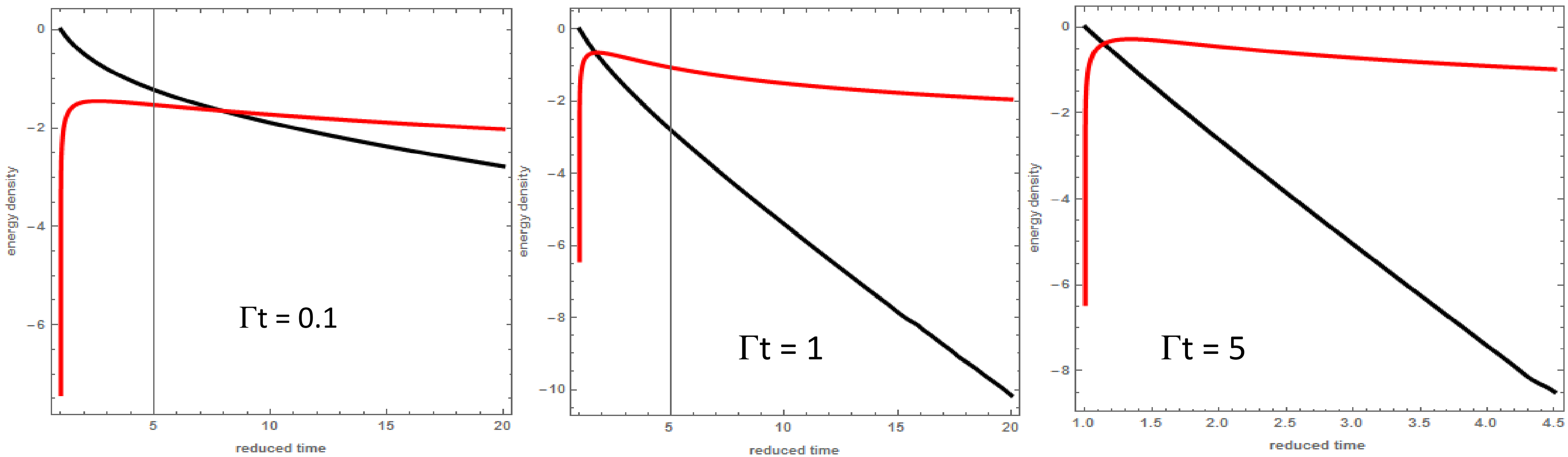
Redefine variables to have a dimensionless equation

$$y = (\phi - \phi_0)/\phi_0 = \Delta\phi/\phi_0$$

$$y'' + \left(\frac{3}{2x} + K\right) y' + y = 0$$

$$with \quad K = \Gamma_\phi / M_\phi$$

When K ≥ 4 decay rate is faster than the expansion rate and the kinetic energy is rapidly converted into particles



Evolution of the energy densities of field (black curve) and dark matter (red curve)

Conversion efficiencies

$$\begin{cases} \Gamma_\phi t_u = 5 \rightarrow \rho_{dm} = 0.469 W_\phi \\ \Gamma_\phi t_u = 1 \rightarrow \rho_{dm} = 0.240 W_\phi \\ \Gamma_\phi t_u = 0.1 \;\; \rho_{dm} = 0.0319 W_\phi \end{cases}$$

$$t_u(z = 10^7) \approx 3.1 \times 10^5 s$$

The kinetic energy of the oscillating field at the potential minimum

$$W_\phi = \frac{1}{2}\Delta\dot{\phi}^2 = V_0\left[y'(x_0)\right]^2 = 0.877V_0$$

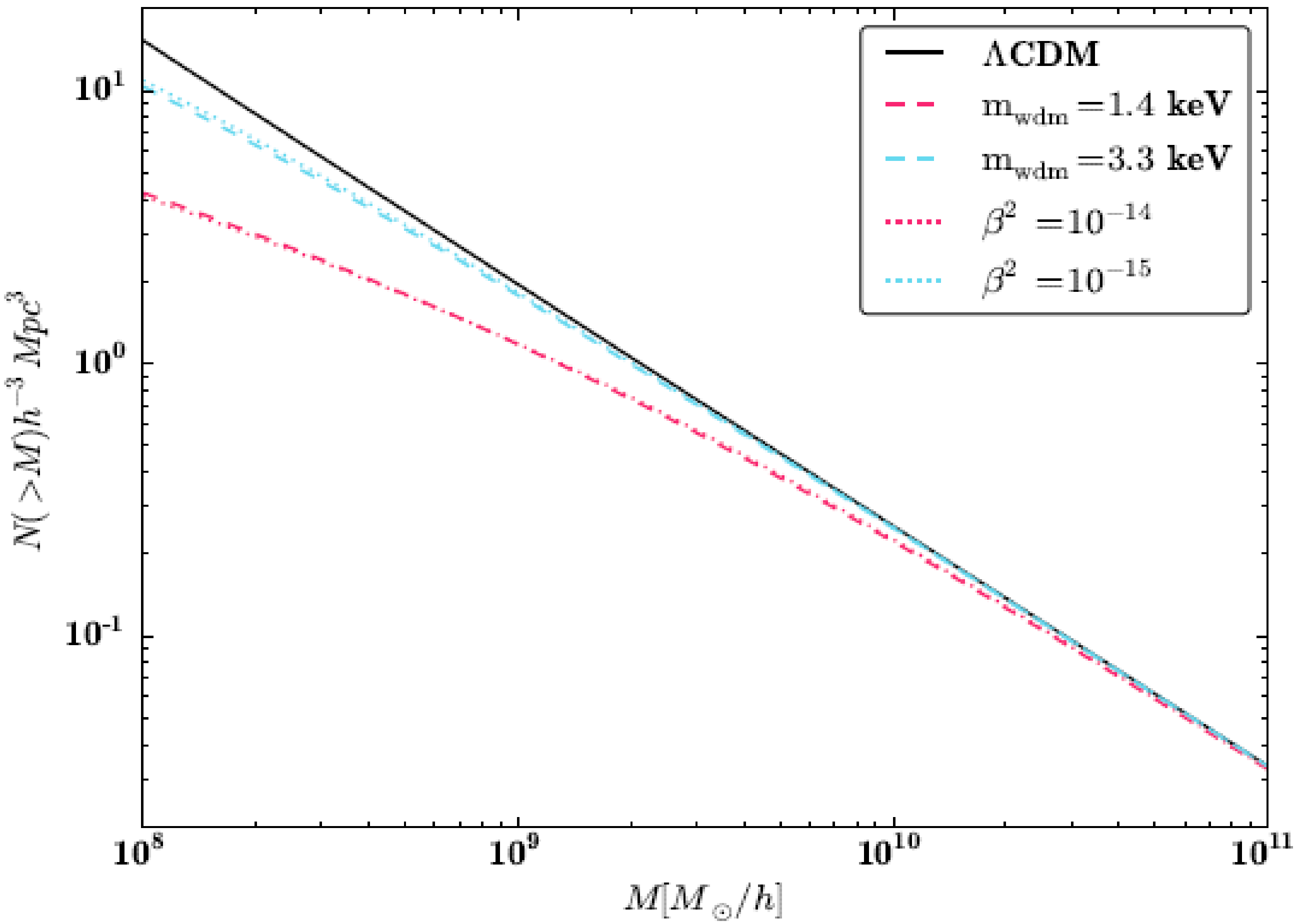
DM energy density at the transition point $\longrightarrow$ $\rho_{dm} \approx \frac{3c^2H_0^2\Omega_{dm}}{8\pi G}(1+z_t)^3 \approx 2\times10^{12}\, ergcm^{-3}$

Consistency demands that the radiation energy density be larger

$$\varepsilon_r = \varepsilon_0\left(\frac{g_t}{g_0}\right)^{4/3}(1+z_t)^4 \approx 4.2\times10^{15}\, ergcm^{-3}$$

Amplitude of the potential $\longrightarrow$ $V_0^{1/4} \simeq 3.9\times10^{-7}\, GeV$

Amplitude required for the inflaton $\longrightarrow 10^{15}$ GeV



Predicted integrated mass spectrum of halos

Summary

- Velocity dispersion effects due to DM particles issued from SS models fix the minimum mass scale of halos, which is of the order of 10^{-6} Mo
- Velocity dispersion effects comparable to those produced by WDM or IDM can be mimicked in a "late forming scenario", in which ~ keV DM particles appear around $z \sim 10^7$ (just after nucleosynthesis)
- DM particles with masses around keV introduce a cutoff in the linear power spectrum of matter able to reduce the number of low mass halos and alleviate the problem of satellites
- The vacuum state of the original scalar field could be eventually associated with the "cosmological constant" – in this case, it would provide an explanation for the origin of both components of the dark sector