



**Observatoire**  
de la CÔTE d'AZUR

*17 September 2014*

UNIVERSITÀ DELLA CALABRIA



# **Intermittent magnetic structures in plasmas: from MHD to Vlasov turbulence**

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**W. H. Matthaeus, P. Veltri, P. Dmitruk, F. Valentini, A. Greco,  
P. Cassak, M. Shay, F. Califano, V. Carbone, K. Osman, J. Gosling**

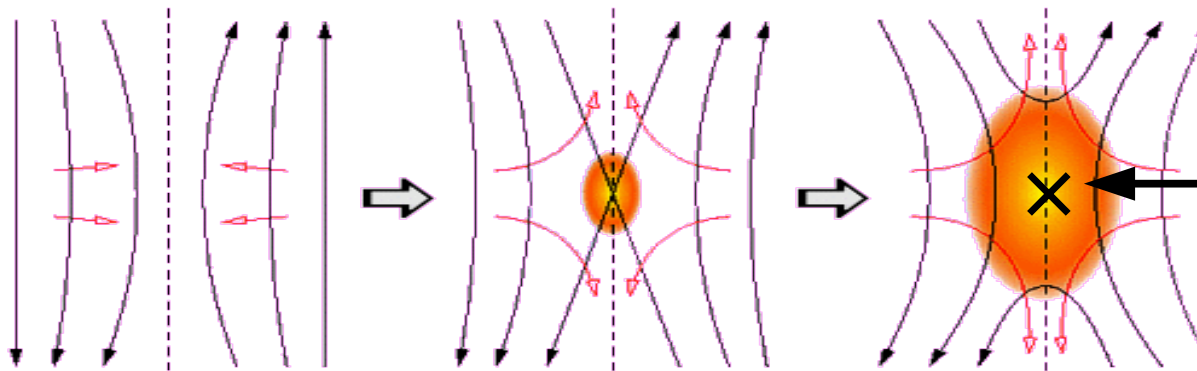
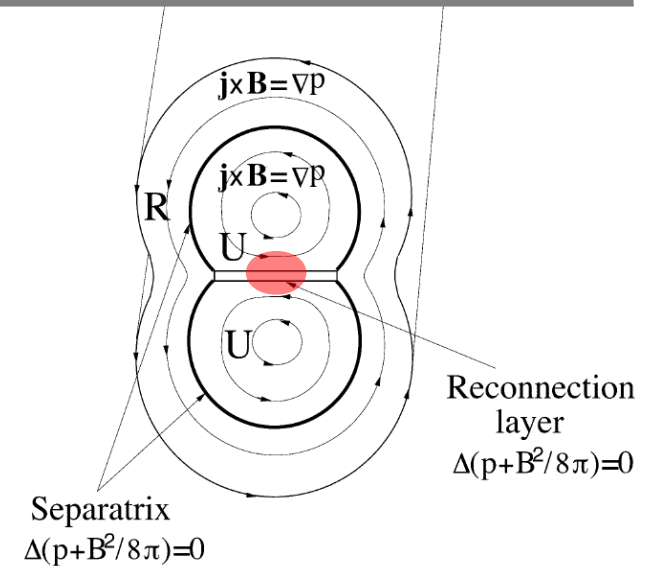
# Outline

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- **Magnetic reconnection & 2D MHD turbulence**
- **From fluid to plasma models**
- **Some “applications”**
- **6D Vlasov simulations**

# Classical Picture of Magnetic Reconnection

- Two bipolar regions come together
- The field flattens “analogous to the flattening of a motor tyre when loaded”
- “...a *thin collision layer of gas is formed*” at the neutral point



Reconnection region at the X-point

$\ell$  system size

$\delta$  thickness

$v_{in(out)}$  in(out)flow velocity

$E$  reconnection rate

$B$  upstream magnetic field

$\eta$  resistivity

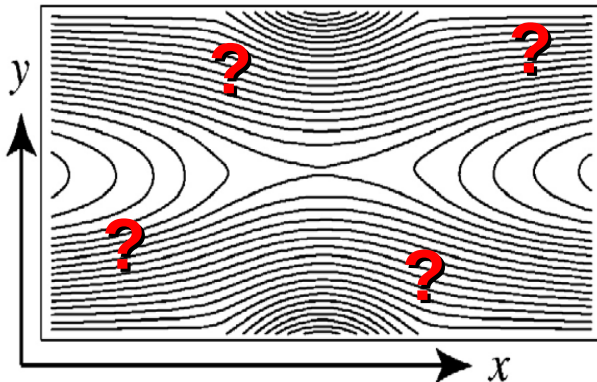
$$\frac{v_{in}}{v_{out}} \sim \frac{\delta}{\ell} \quad \& \quad E \sim \sqrt{\frac{\eta B^3}{\ell}}$$

**P. A. Sweet (1958);  
E. N. Parker (1957)**

# Standard Description of Reconnection

## The *orthodox* procedure

- \* Initially highly ordered, large scale magnetic field
- \* Special well-known boundary conditions
- \* The process can be driven by mechanical pressure supplied by open boundaries, or magnetic flux injected from a conducting wall
- \* Small initial perturbation in the center of the box, with *the right k-vector*
- \* The nonlinear regime is then achieved -> well-known growth rates



\* ***very limited dynamic***

\* ***rarely observed in nature since plasma is generally turbulent***

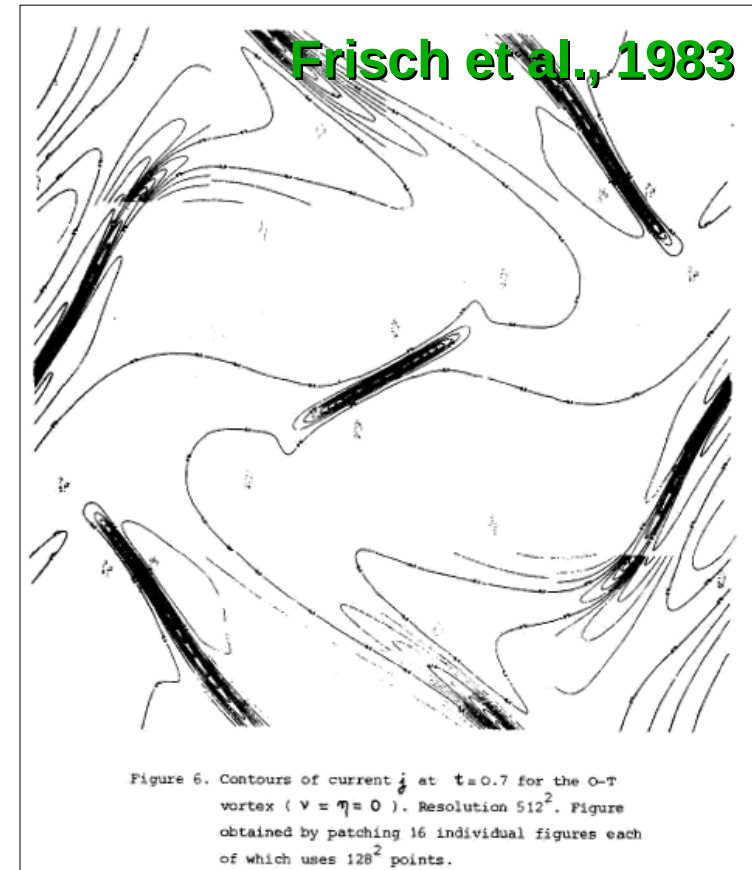


# An Alternative Description ...

## \* Turbulence

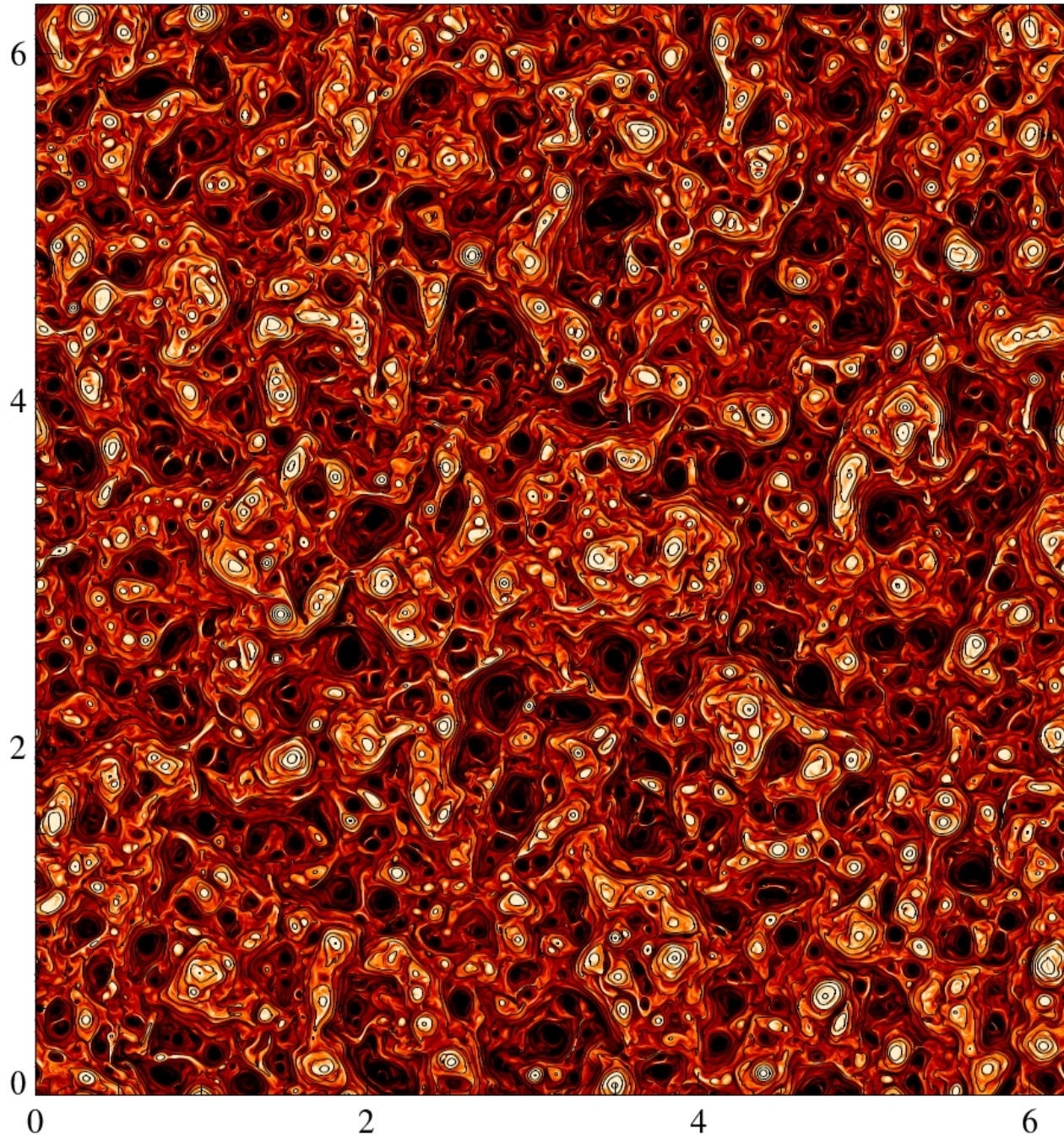
Matthaeus & Montgomery, *Ann. N.Y. Acad. Sci.* (1980);  
V. Carbone et al., *Phys. Fluids* (1990)

U. Frisch, A. Pouquet, P.-L. Sulem and M. Meneguzzi,  
*Journal de Mecanique Theorique et Appliquee* (1983)



**Is possible that reconnection develops in turbulence?**  
**If yes, which are the statistical properties of reconnection in turbulence?**

## 2D MHD



$$\frac{\partial \omega}{\partial t} = -(\mathbf{v} \cdot \nabla) \omega + (\mathbf{b} \cdot \nabla) j + R_v^{-1} \nabla^2 \omega$$

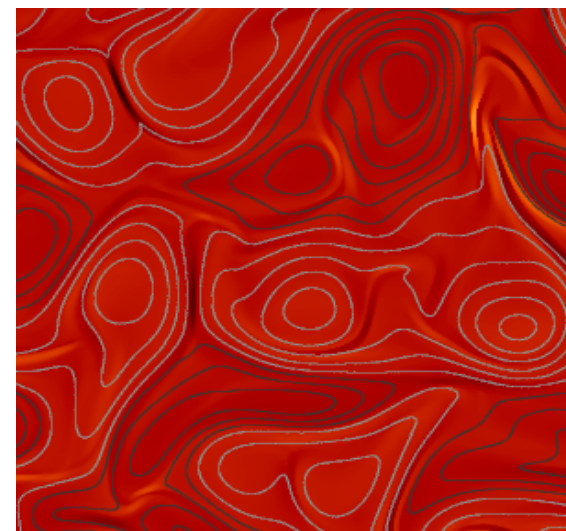
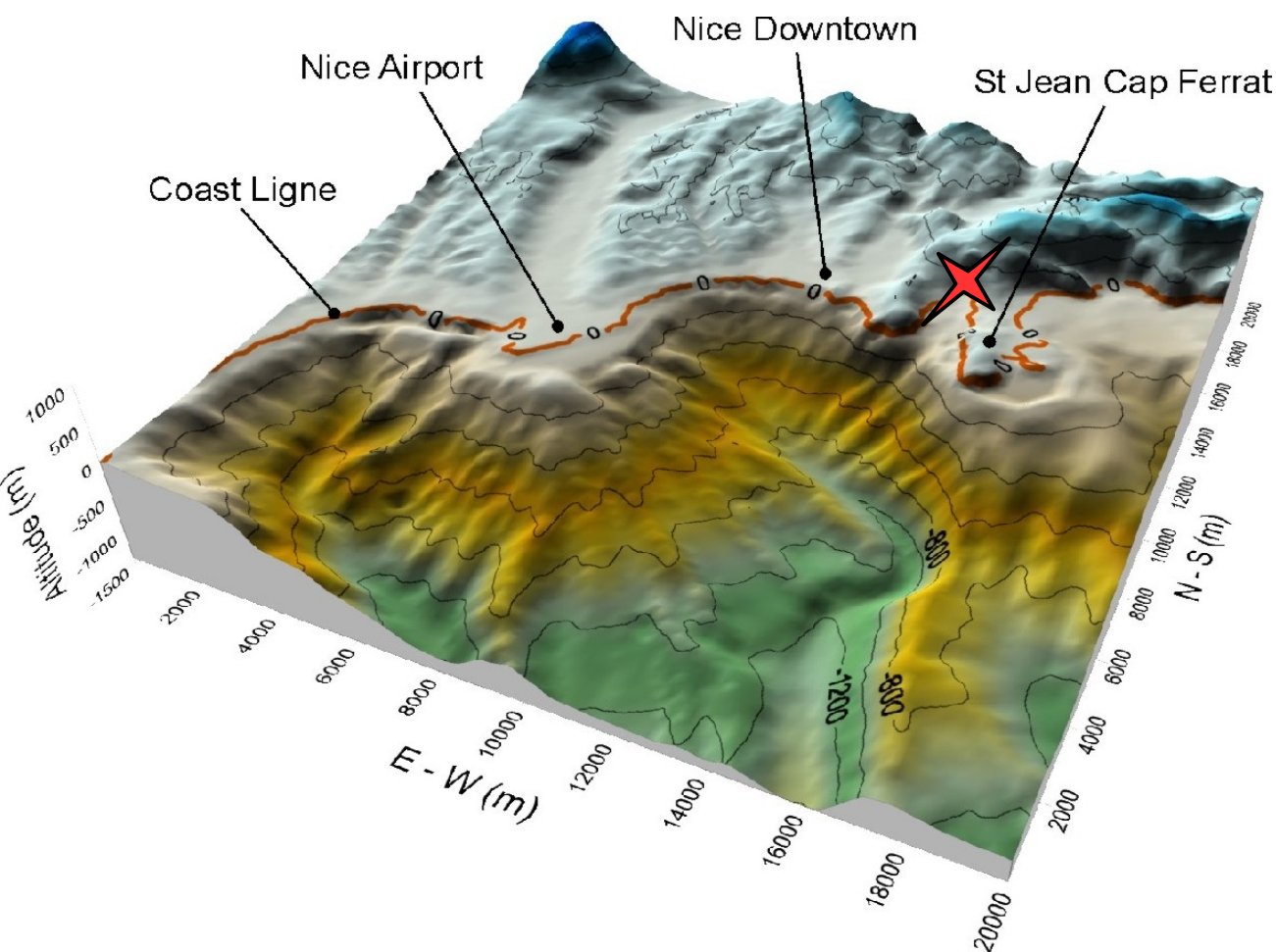
$$\frac{\partial a}{\partial t} = -(\mathbf{v} \cdot \nabla) a + R_\mu^{-1} \nabla^2 a$$

- dealiased pseudo-spectral code
- $16384^2$  mesh points
- $R_v = R_\mu = 10000$

**Where is  
reconnection?**



# Some “topography”...

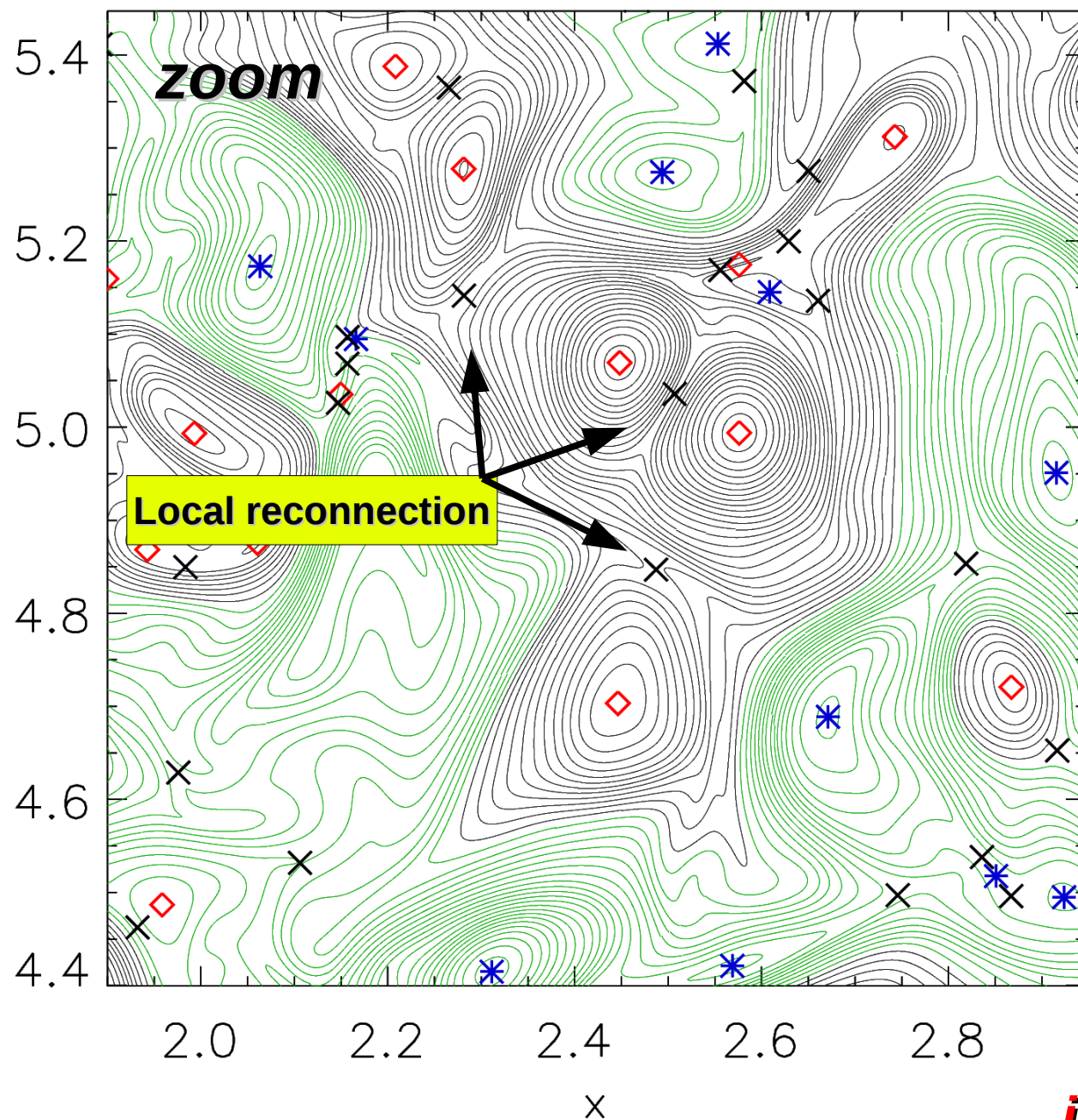


**Critical points of the  
magnetic potential “ $a$ ”**

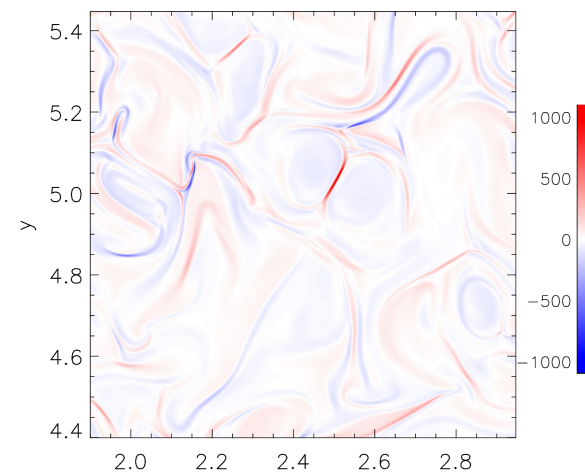
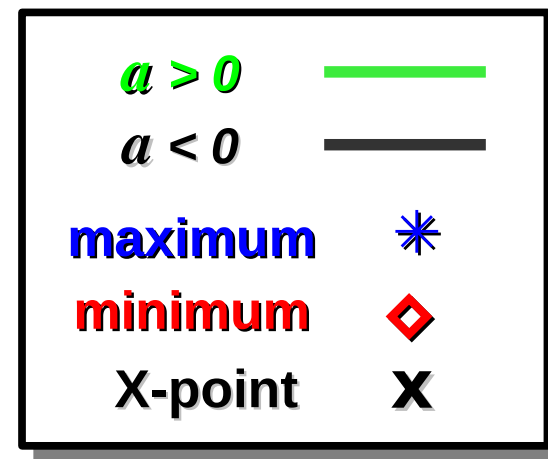
$$H_{ij}^{[a]}(\mathbf{x}) = \frac{\partial_{ij}^2 a}{\partial x_i \partial x_j},$$

where  $\nabla a = 0$

# Critical Points in Turbulence



Magnetic potential  $a$   
and critical points

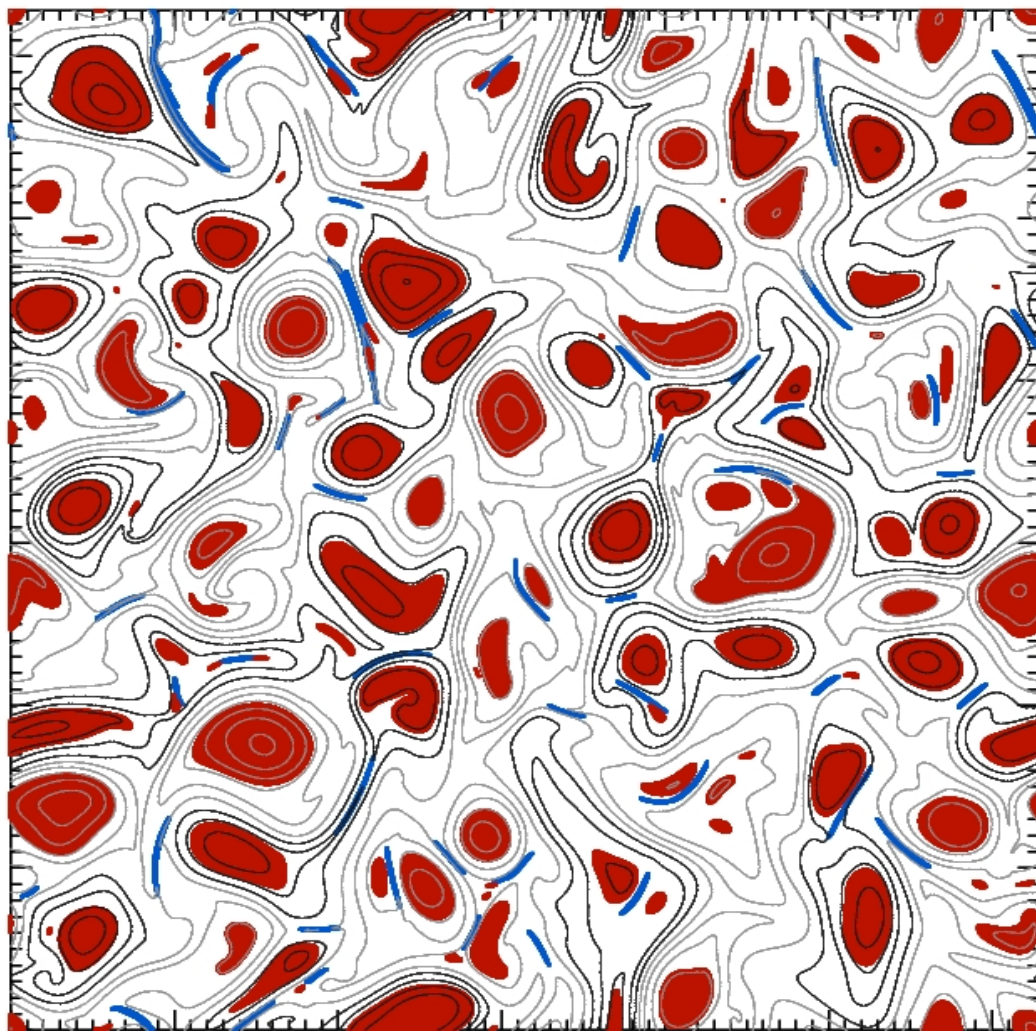


$j$  is intermittently intense



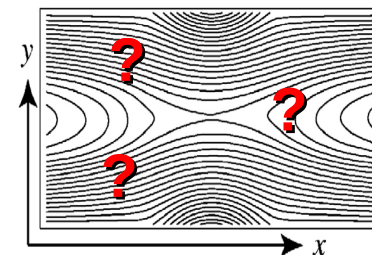
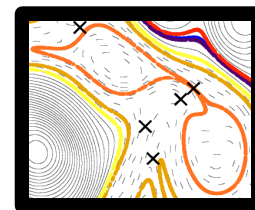
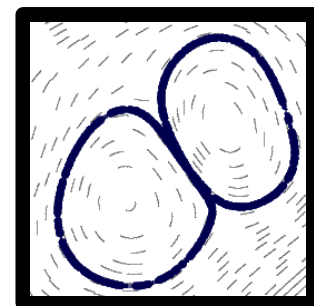
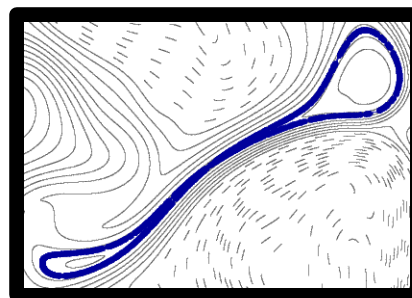
# Islands & Sheets

**current sheets = BLUE regions**  
**magnetic islands = RED regions**



*automated algorithm*

**Boundary conditions?**  
**Symmetry?**



**Reconnecting islands are  
different in size and energy:  
asymmetric reconnection?**

# Distribution of Reconnection Rates

Rate of change of the magnetic flux:

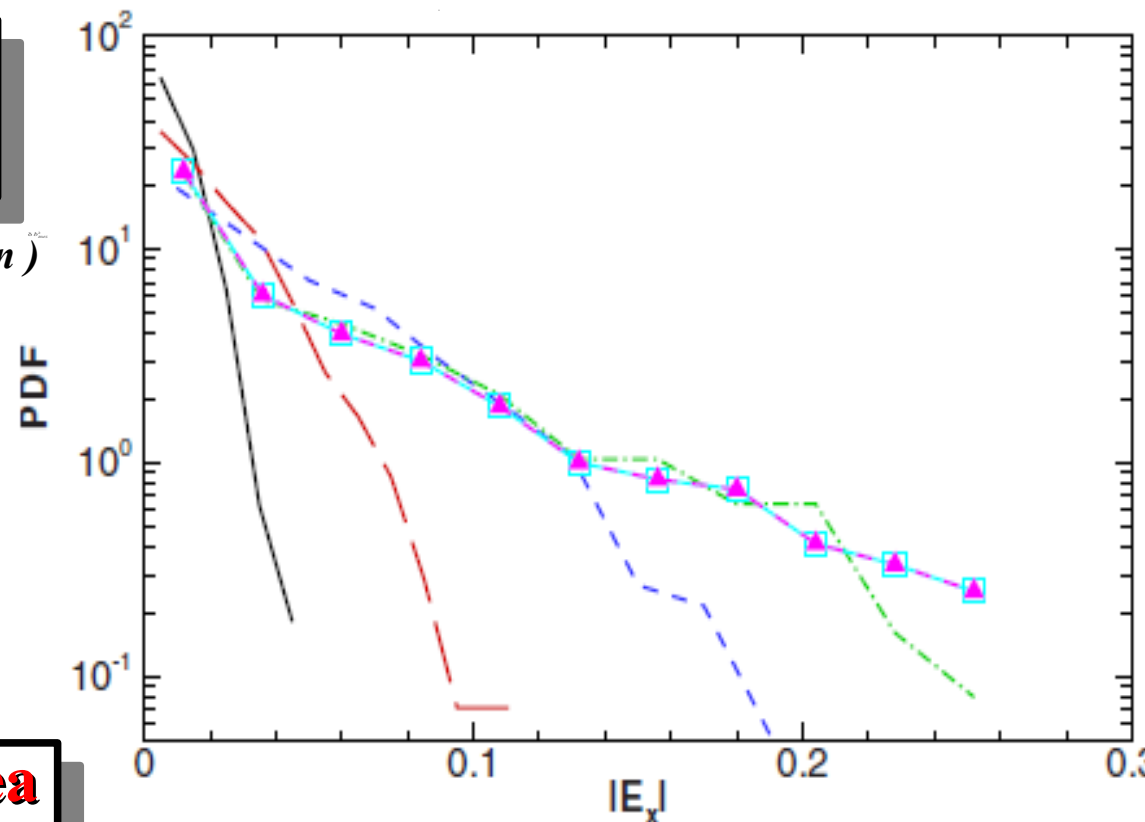
$$\dot{a} = R_{\mu}^{-1} \nabla^2 a \big|_{\times\text{-point}} = -E_{\times}$$

(normalized to the root mean-square magnetic fluctuation)

**Reconnection rates are broadly distributed**

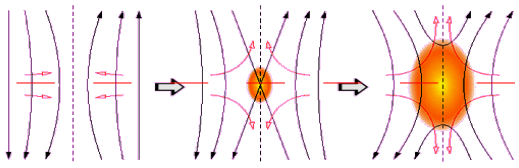
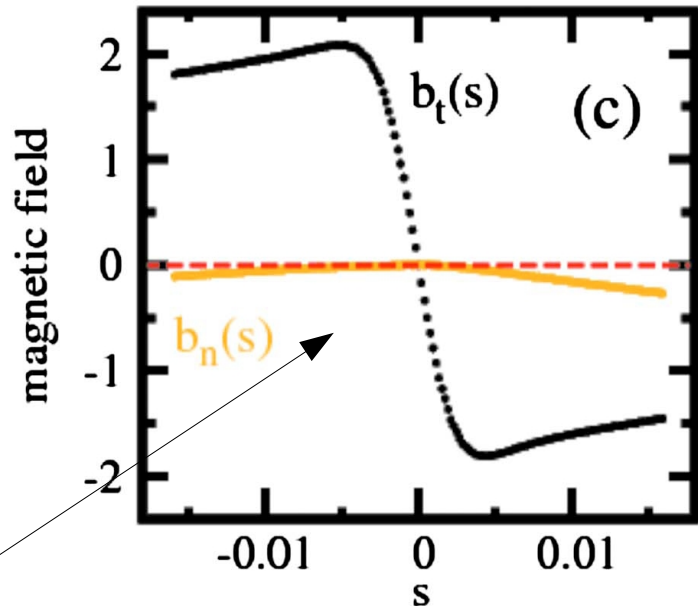
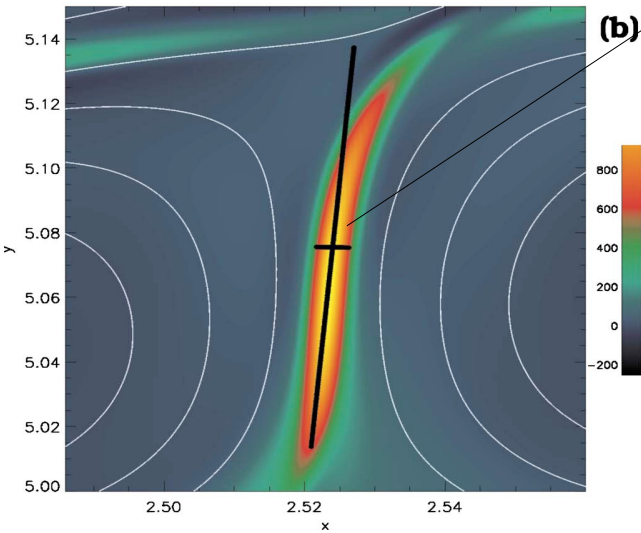
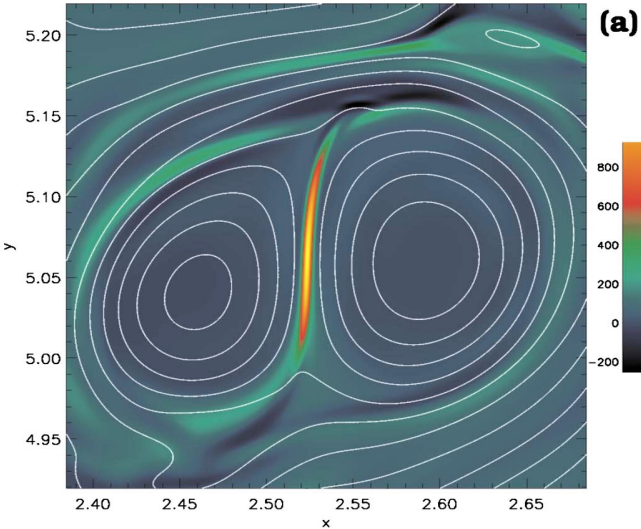
**Turbulence can be viewed as a sea of reconnecting islands with different reconnection rates**

**reconnection rates up to ~0.3**

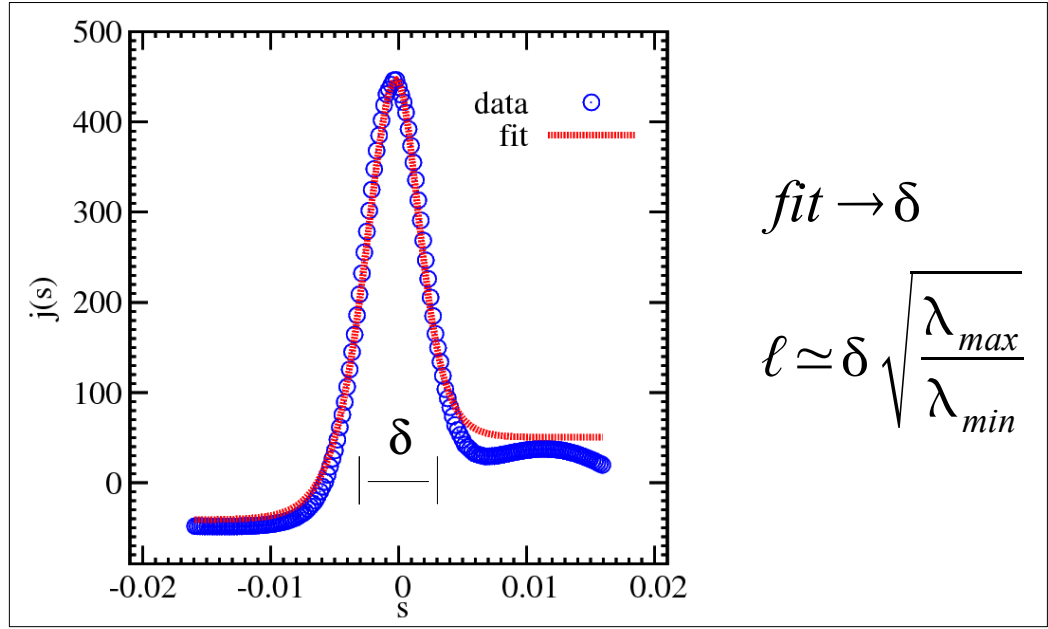


# Dimensions of the Diffusion Region

*Another zoom-in*

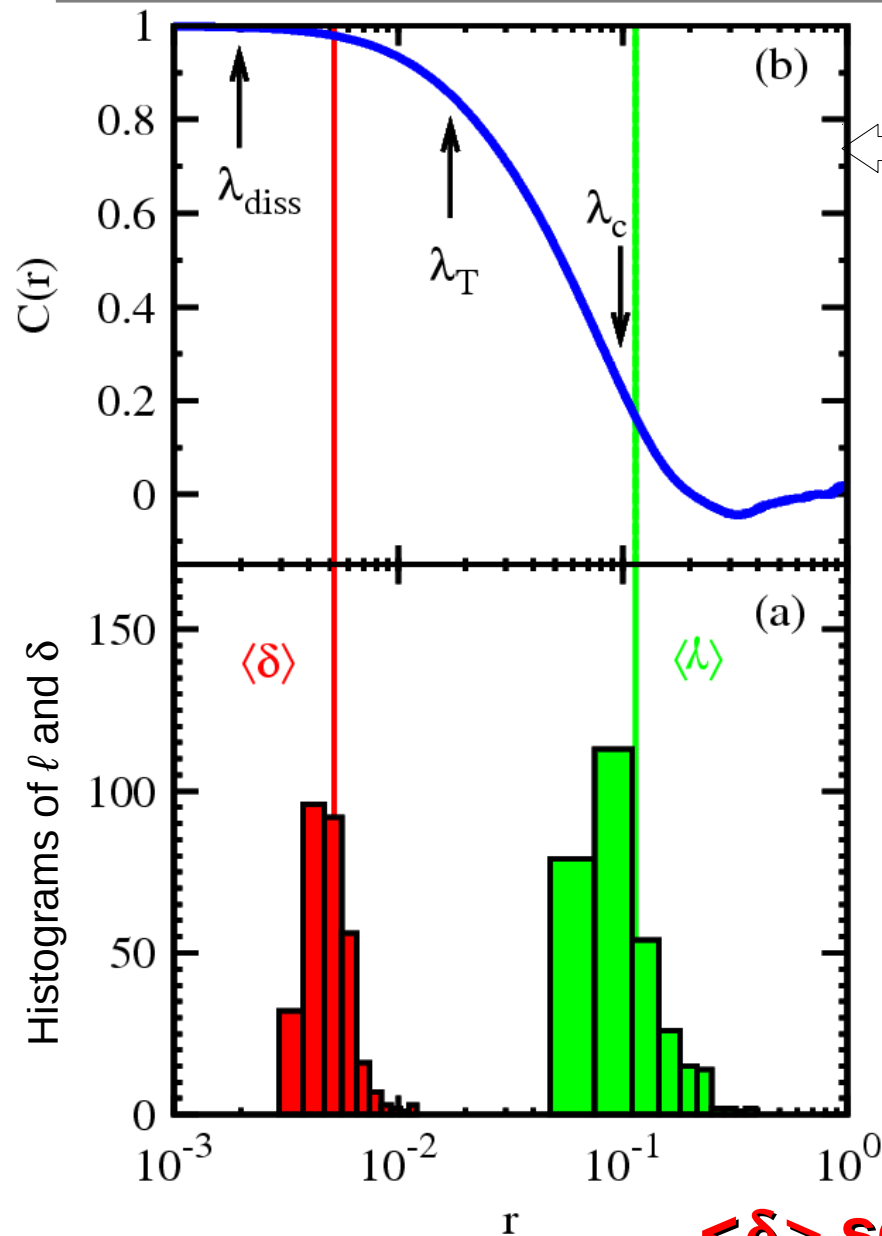


characteristic lengths





# Link between Reconnection and Turbulence



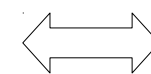
$$C(r) = \frac{\langle \mathbf{b}(\mathbf{x} + \mathbf{r}) \cdot \mathbf{b}(\mathbf{x}) \rangle}{\langle b^2 \rangle}$$

Characteristic lengths in turbulence and in reconnection:

$$\lambda_c = \int C(r) dr$$

$$\lambda_T = \sqrt{\frac{\langle b^2 \rangle}{\langle j^2 \rangle}}$$

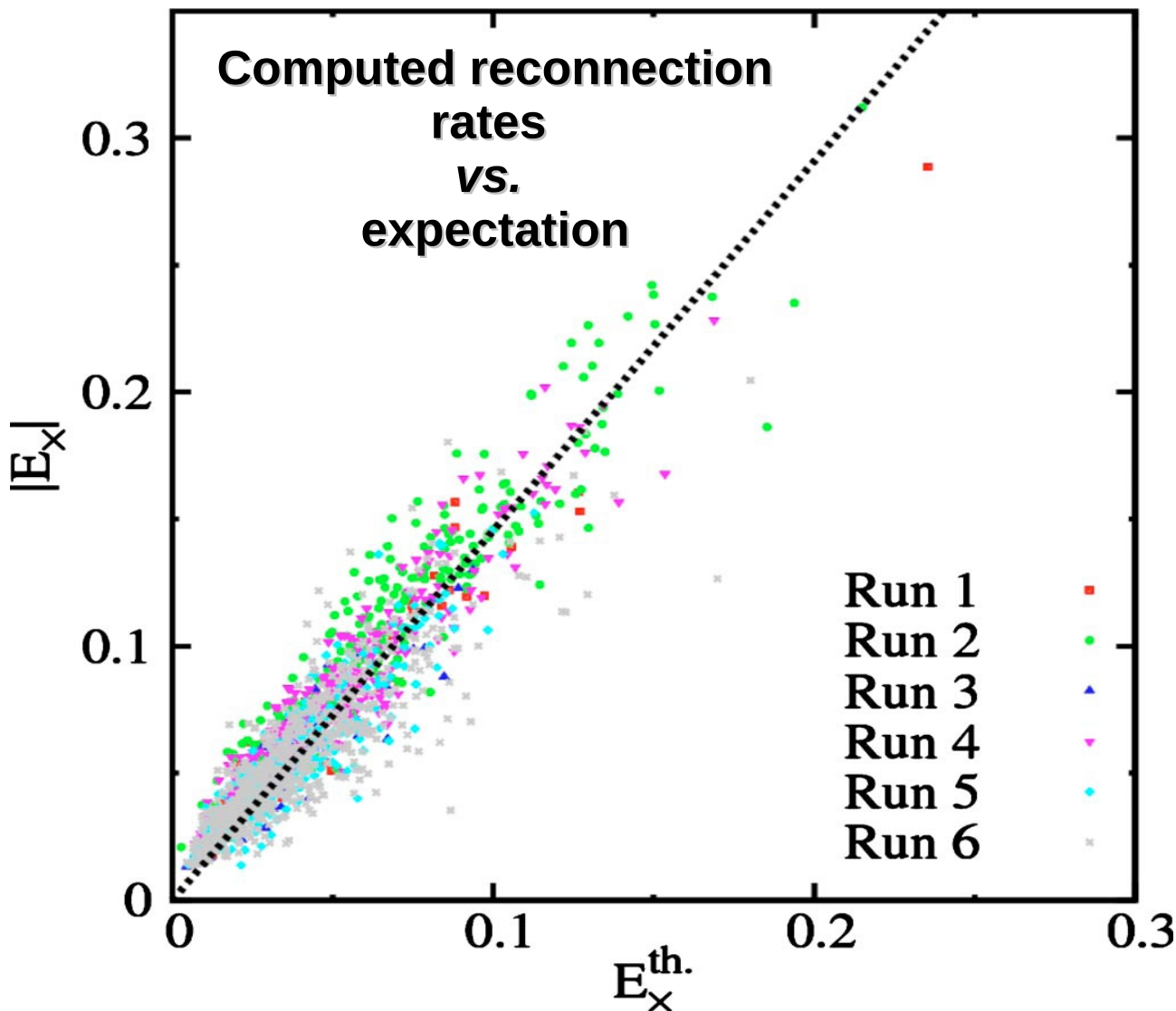
$$\lambda_{diss} = R_\mu^{-\frac{1}{2}} \langle j^2 \rangle^{-\frac{1}{4}}$$



$\ell, \delta$

**$\langle \delta \rangle$  scales with the dissipation length**  
 **$\langle \ell \rangle$  is on the order of the correlation length**

# Reconnection Rate in Turbulence



## Asymmetric reconnection

$$E_{\times}^{th.} = \sqrt{\frac{b_1^{3/2} b_2^{3/2}}{R_{\mu} \ell}} \quad b_i \equiv \text{upstream fields}$$

P. Cassak & M. Shay, Phys. Plasmas 2007; Borovsky & Hesse, Phys. Plasmas (2007); Birn et al., Phys Plasmas (2008); Pritchett, JGR (2008).

**Reconnection rates are consistent with a modified Sweet-Parker model**

**Turbulence provides locally the parameters that determine the Sweet Parker reconnection rate: the lengths and local magnetic field strengths.**

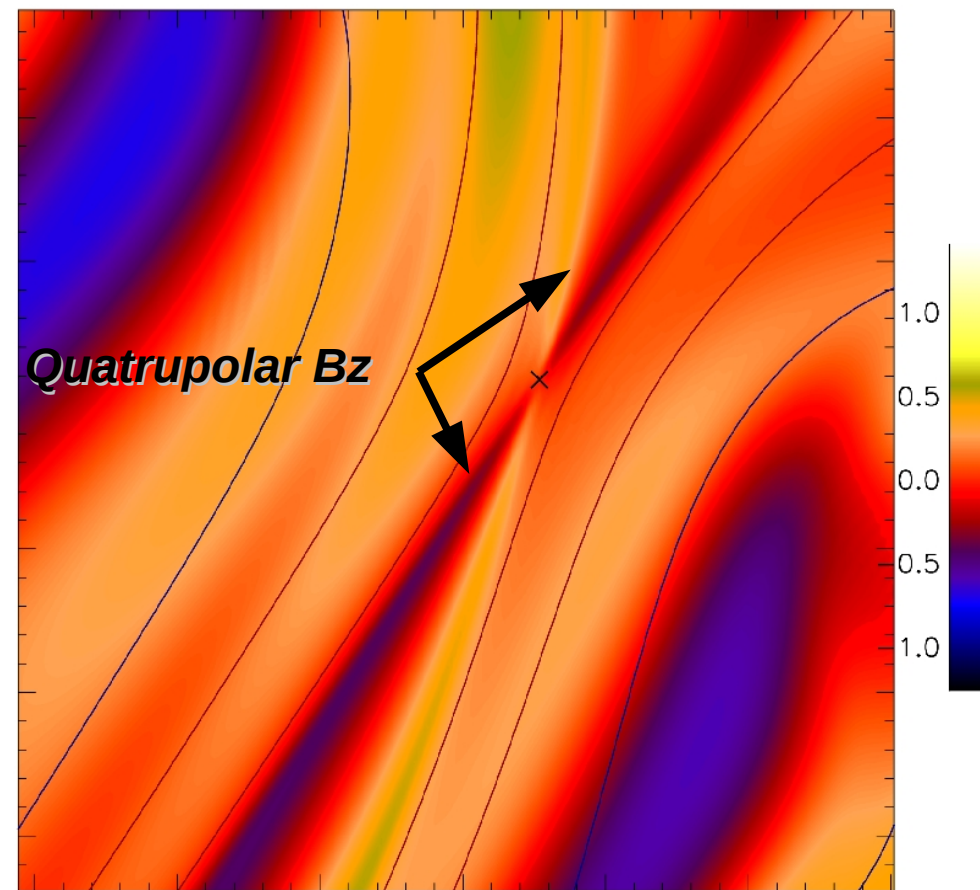
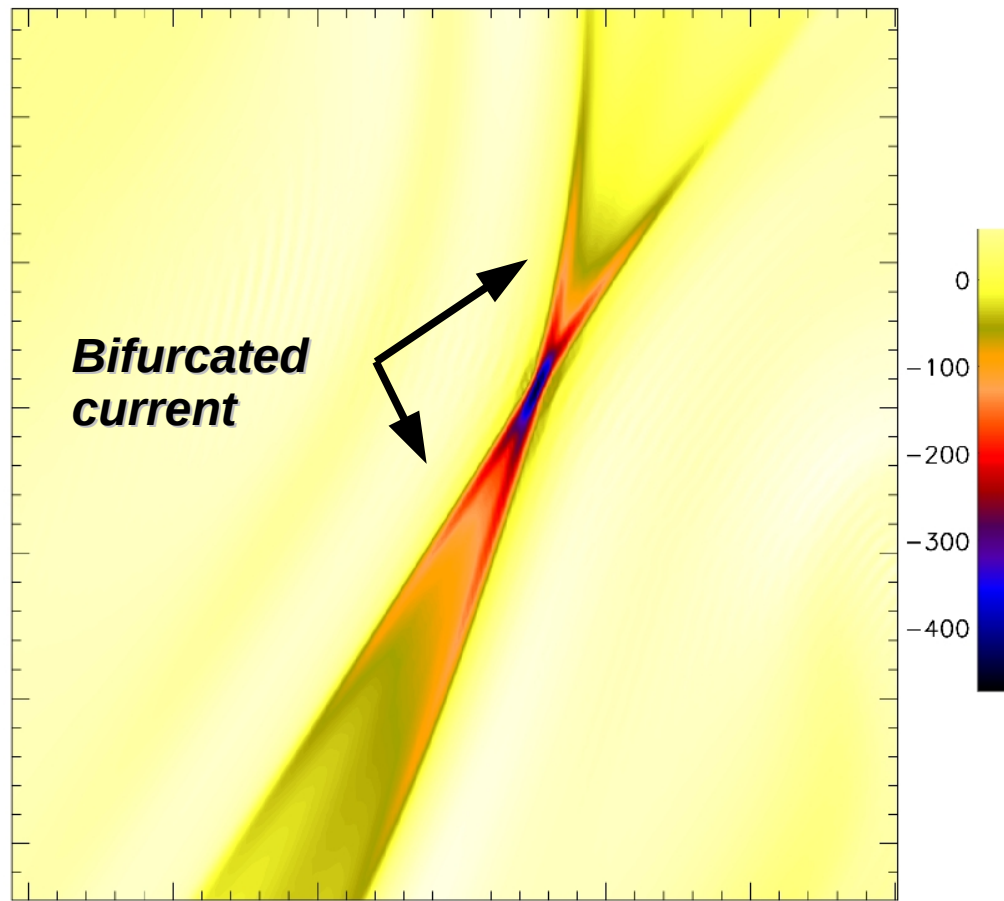
Servidio et al., Phys Rev. Lett. (2009).

# Hall Magnetohydrodynamics

$$\frac{\partial \mathbf{v}}{\partial t} = -(\mathbf{v} \cdot \nabla) \mathbf{v} + \mathbf{j} \times \mathbf{b} - \nabla P + R_v^{-1} \nabla^2 \mathbf{v}$$

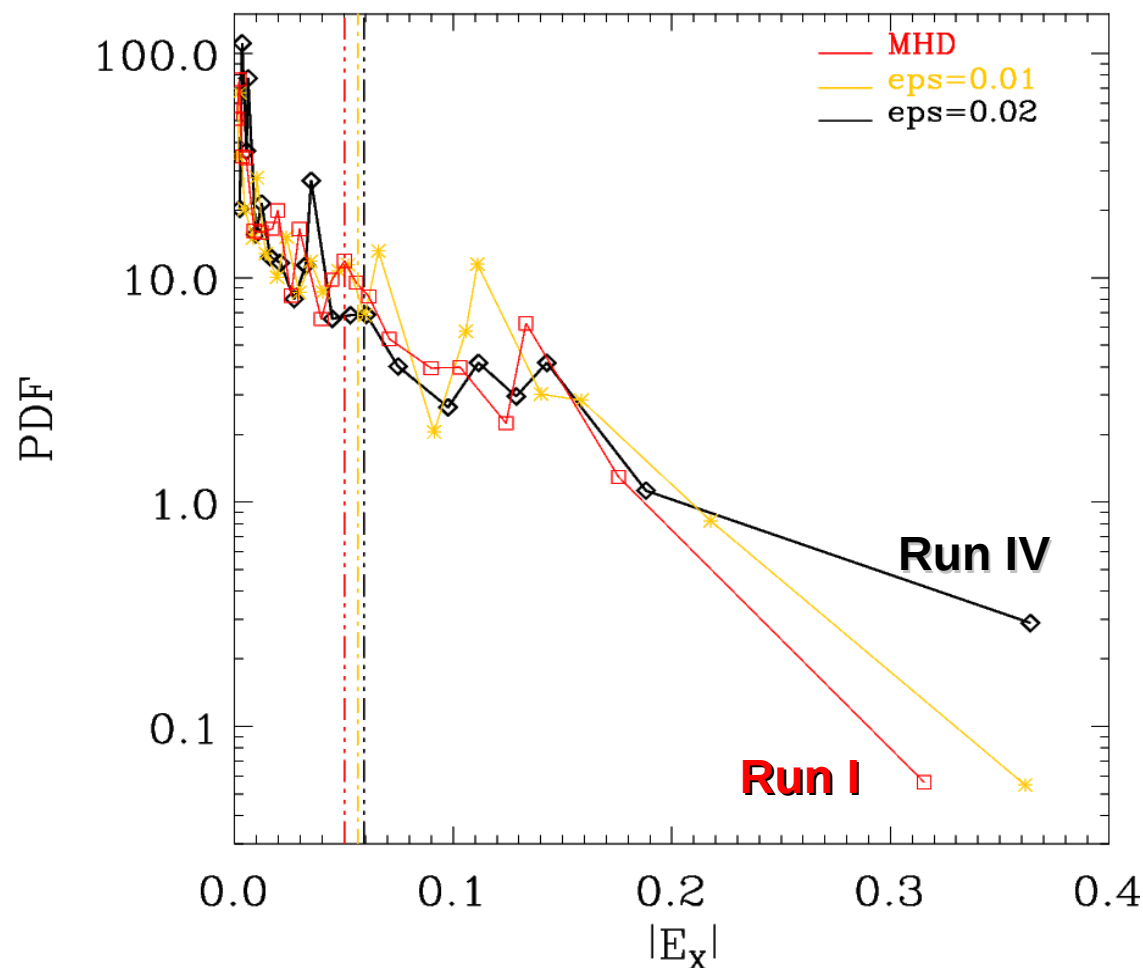
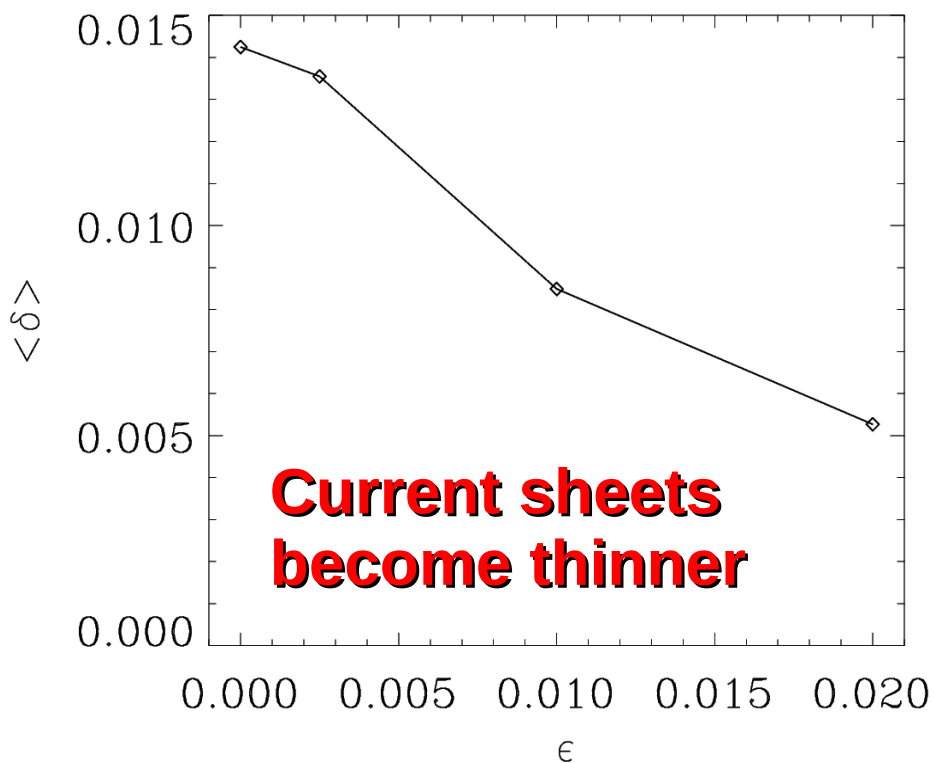
$$\frac{\partial \mathbf{b}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{b}) + \epsilon_H \nabla \times (\mathbf{j} \times \mathbf{b}) + R_\mu^{-1} \nabla^2 \mathbf{b}$$

- *pseudo-spectral*
- **2.5D**
- $8192^2$
- $R_v = R_\mu = 2000$



# Reconnection in Hall MHD Turbulence

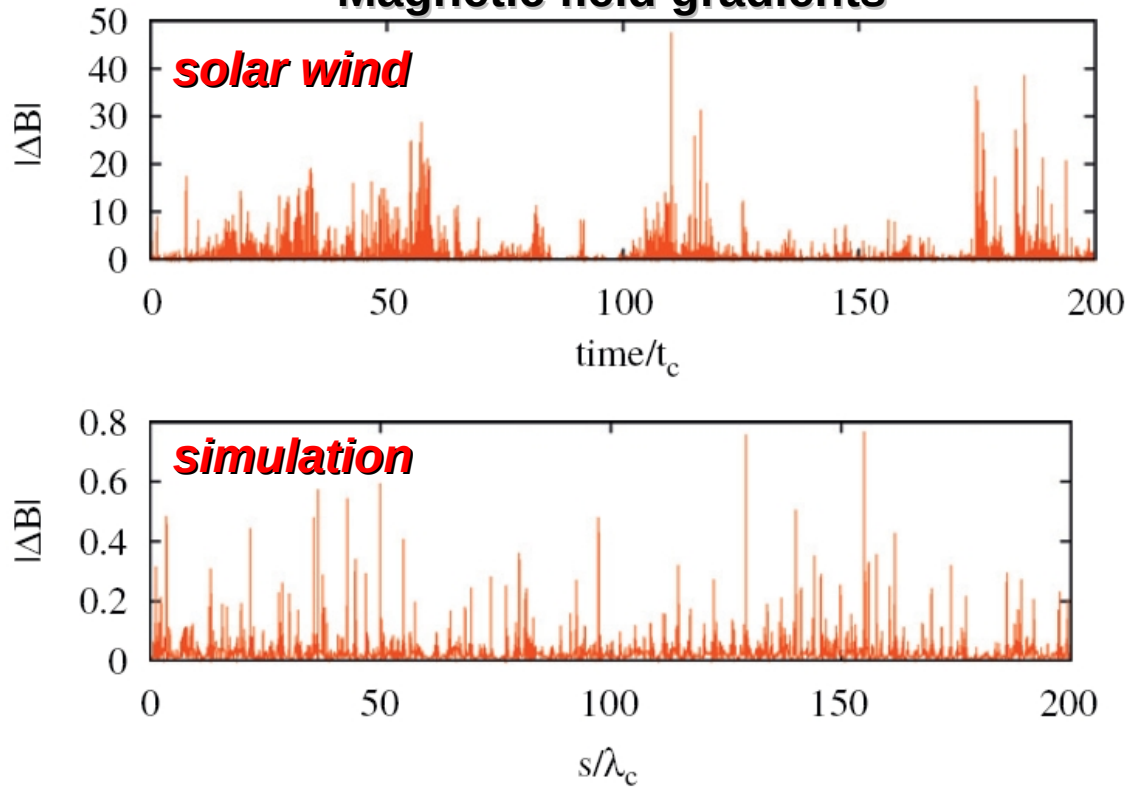
Run	$\epsilon_H$
I	0.0 ( <i>MHD</i> )
II	0.0025
III	0.01
IV	0.02



**The Hall electric field increases the reconnection rate in turbulence**

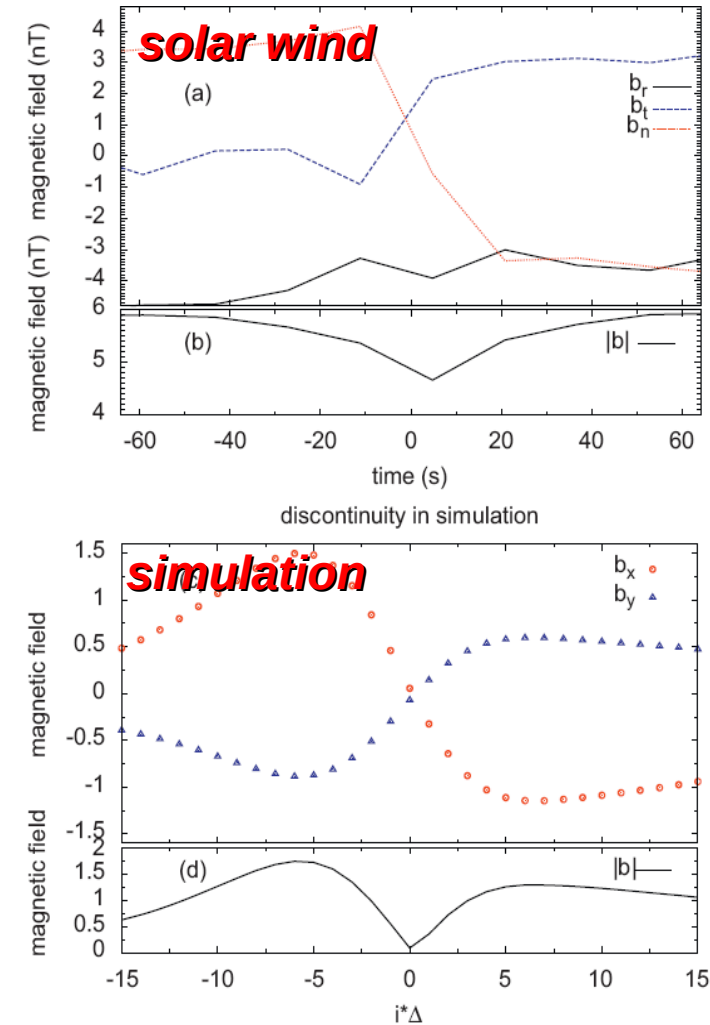
# Solar Wind: Discontinuities & Reconnection

Magnetic field gradients



**Magnetic discontinuities, commonly observed in the turbulent solar wind, may be local reconnection events**

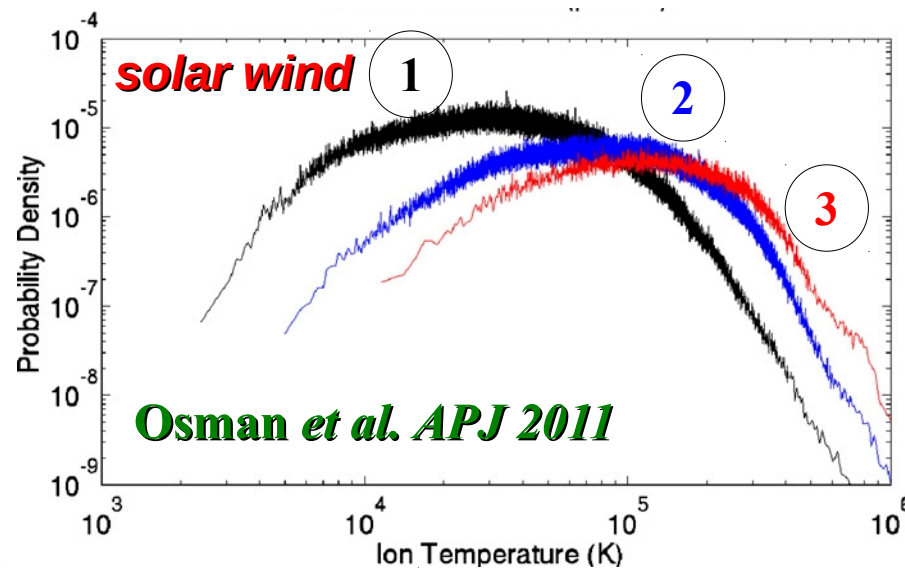
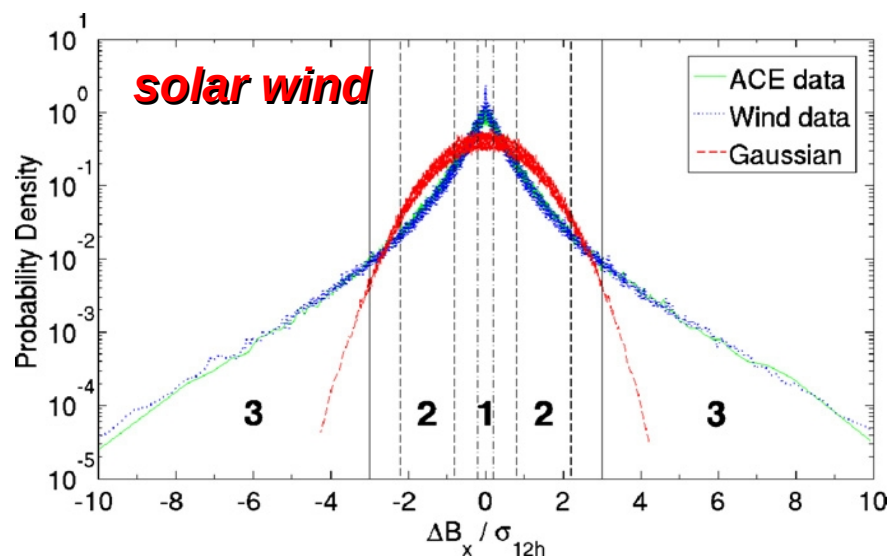
Magnetic discontinuity



A. Greco, S. Servidio, Matthaeus..., Geophys. Rev. Lett. 2008; Astrophys J. 2009; Planet. Space Sci. 2010, Phys Rev. E 2009, Astrophys J. Lett 2011, Journal Geophys. Res. 2011; Phys. Rev. Lett. 2014



# ...Discontinuities, Reconnection & Heating



Greco et al. GRL 2008, APJ 2009;  
Sorriso-Valvo et al., GRL 1999;  
Bruno & Carbone, Liv. Rev. 2005;  
Gosling & A. Szabo JGR 2008

**Intermittent magnetic gradients,  
that may be local reconnection  
events, are sites of enhanced heating**

PRL 112, 215002 (2014)

PHYSICAL REVIEW LETTERS

week ending  
30 MAY 2014

## Magnetic Reconnection and Intermittent Turbulence in the Solar Wind

K. T. Osman,<sup>1,\*</sup> W. H. Matthaeus,<sup>2</sup> J. T. Gosling,<sup>3</sup> A. Greco,<sup>4</sup> S. Servidio,<sup>4</sup> B. Hnat,<sup>1</sup> S. C. Chapman,<sup>1,5,6</sup> and T. D. Phan<sup>7</sup>



**...and from fluid-like models of a plasma  
we explore now “Vlasov turbulence”**

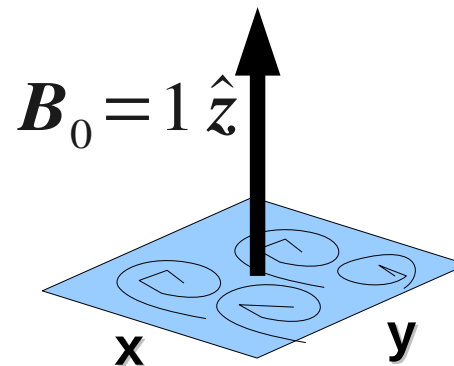
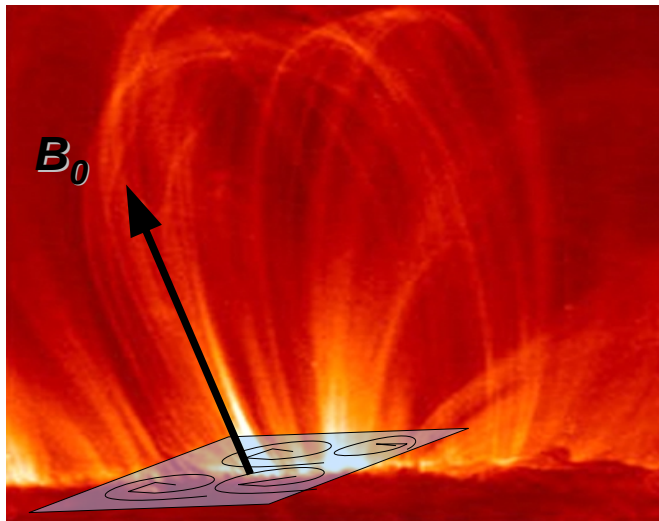


# Hybrid Vlasov-Maxwell

$f(\mathbf{x}, \mathbf{v}) = f(x, y, v_x, v_y, v_z)$  *proton velocity distribution function*

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_{\mathbf{v}} f = 0$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \longrightarrow \mathbf{E} = -\mathbf{u} \times \mathbf{B} + \frac{1}{n} \mathbf{j} \times \mathbf{B} - \frac{1}{n} \nabla P_e + \eta \mathbf{j}$$



**NOISE-FREE!**

**Valentini et al., J. Comp. Phys.  
2007, PRL 2010, PRL 2011**

- **Kinetic ions, fluid electrons**
- **Eulerian model**
- **2D in space + 3V in the velocity space**

**Some parameters ...**

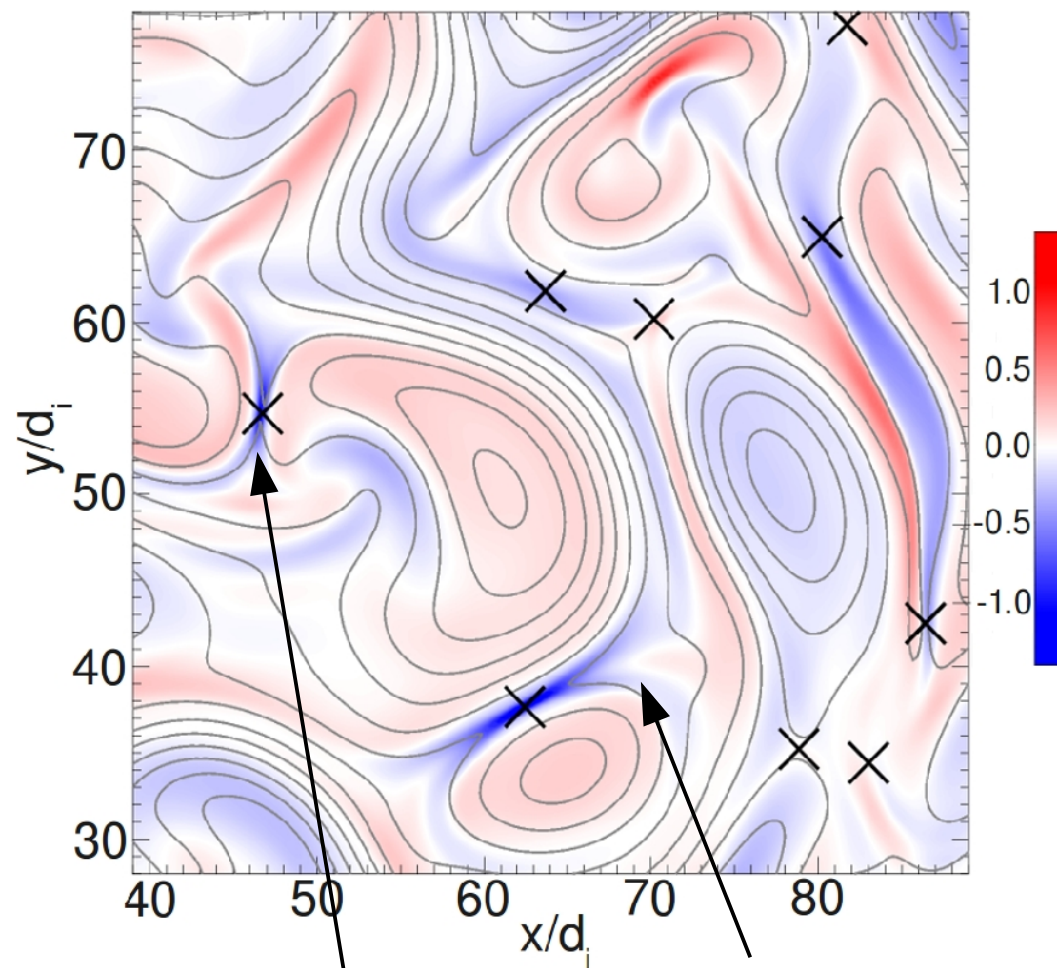
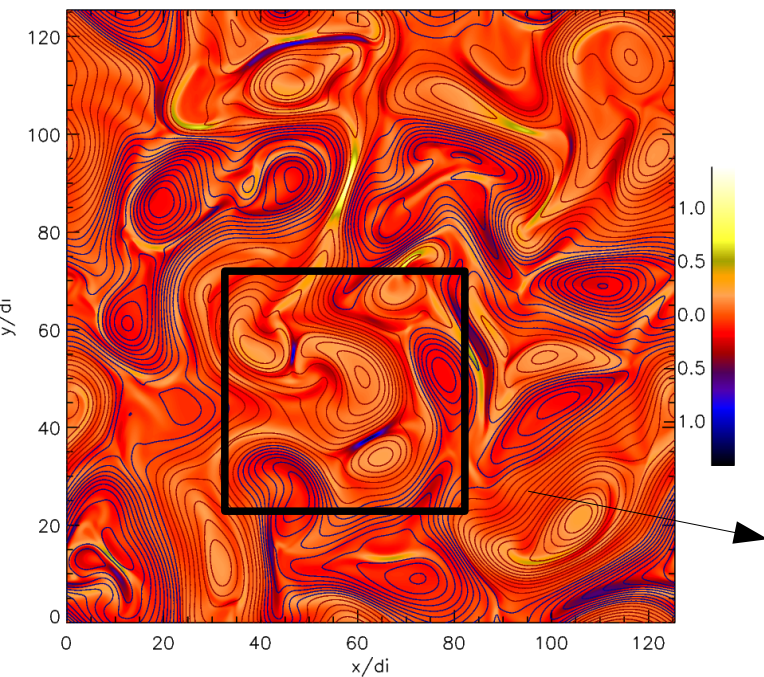
$$L_0 = 2\pi\alpha d_i, B_0 = 1 \hat{e}_z, T_e/T_i = 1,$$

$$\eta = 1.7 \times 10^{-2}, v_{\max} = \pm 5 v_{ti},$$

$$N_x = N_y = 512^2, N_v = 81^3 \rightarrow 3.5 \times 10^{10} \text{ points}$$

# Local Reconnection Events

Current density  $j_z$  (colors) + magnetic potential

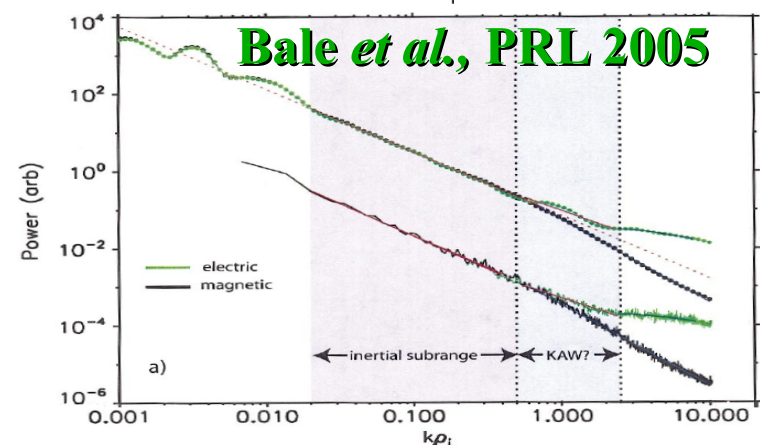
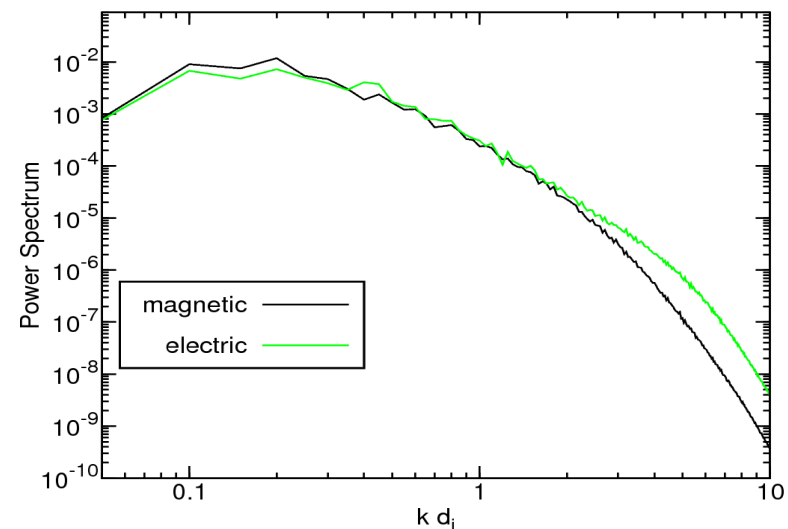
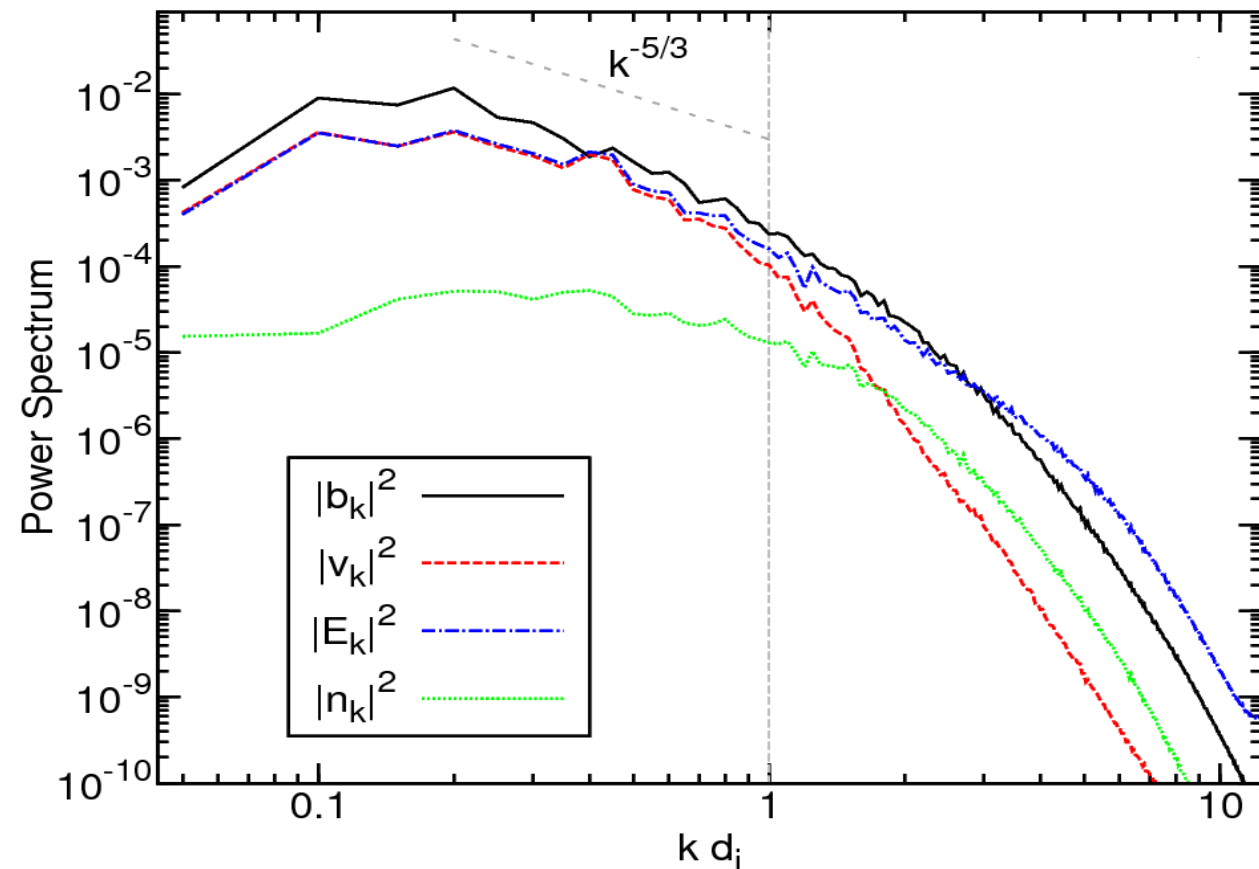


Servidio *et al.* PRL 2009  
Drake *et al.* APJ 2010.

**Bifurcation (Hall effect)**

**Thickness ~ few ion skin depths**

# Power spectra

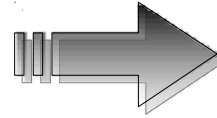


- Large scale Alfvénic correlations
- Kolmogorov-like spectrum
- Low compressibility (density fluct. 8%)
- Intense electric activity at small scales
- Steepening of the magnetic spectrum at  $k d_i \sim 1$

**...several features  
commonly observed  
in space plasmas!**

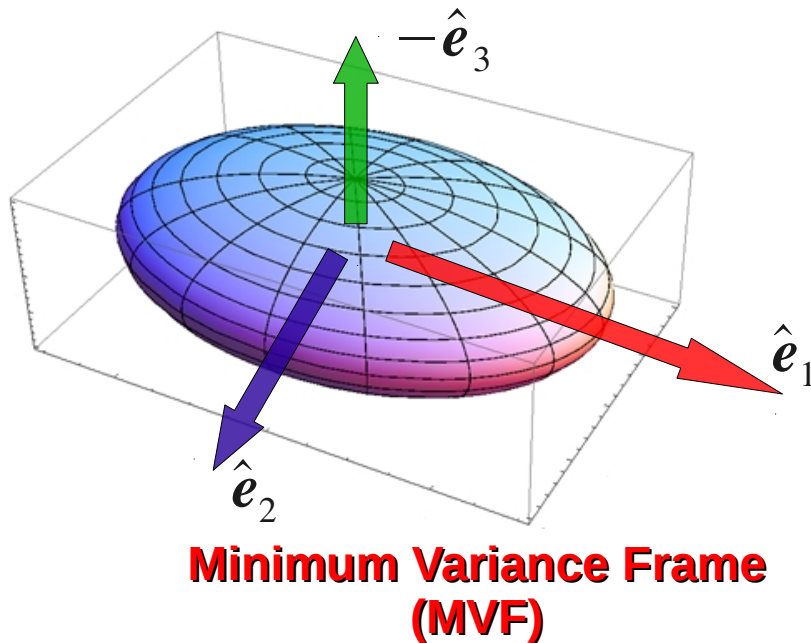
# A Measure of Temperature-Anisotropy

The velocity distribution function  $f$  may exhibit strong deformations in the velocity space



How to properly measure these distortions?

Assuming  $f$  as an ellipsoid:



$$A_{ij}(\mathbf{x}) = \frac{1}{n} \int (v_i - \langle v_i \rangle)(v_j - \langle v_j \rangle) f d^3 v$$

**Eigenvalues  
(temperatures)**

$$\lambda_1 > \lambda_2 > \lambda_3$$

**Eigenvectors**

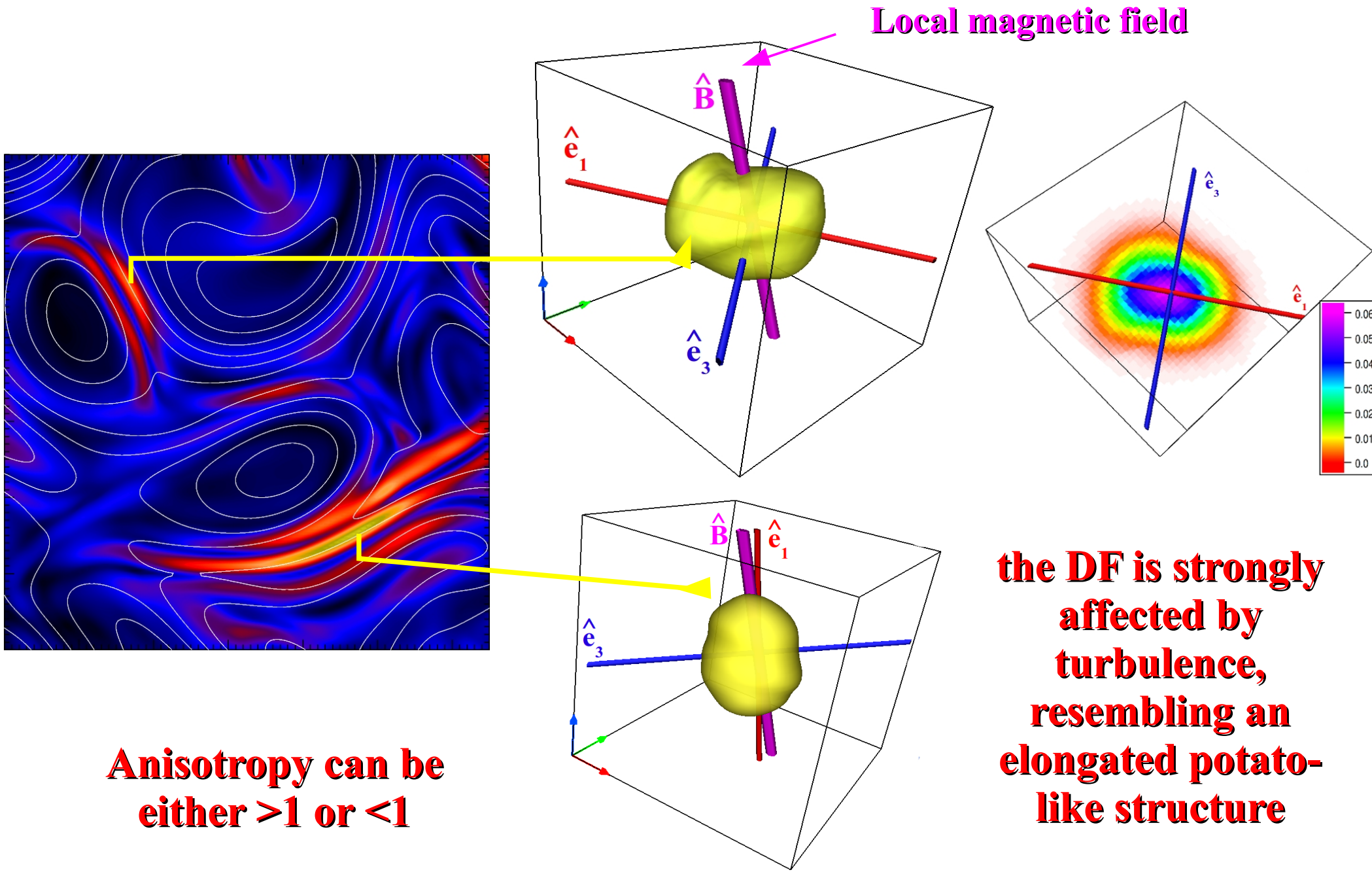
$$\hat{\mathbf{e}}_1 \quad \hat{\mathbf{e}}_2 \quad \hat{\mathbf{e}}_3$$

Note: for a Maxwellian  $\lambda_1 = \lambda_2 = \lambda_3 = 1$

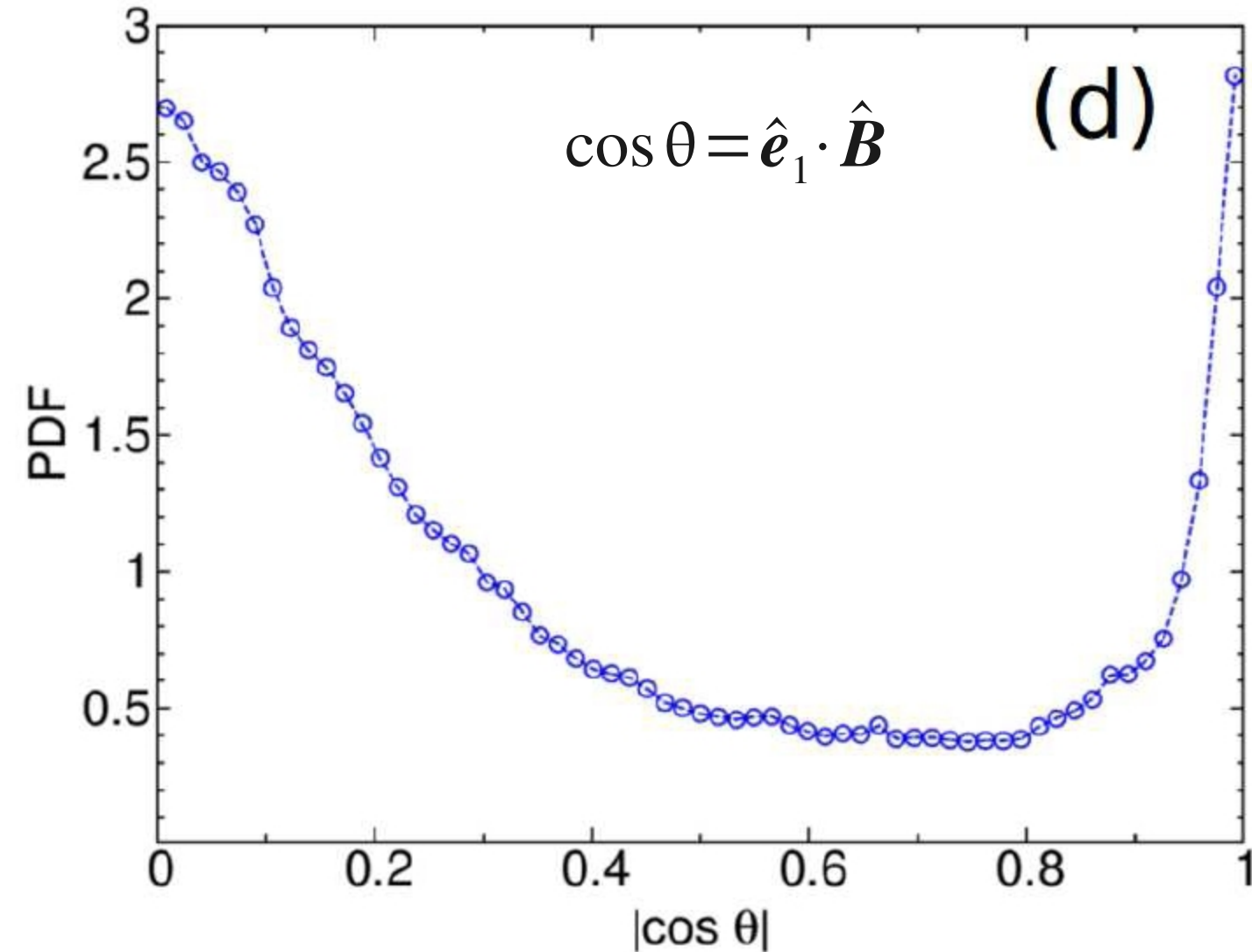
(Maximum) Temperature anisotropy  $\equiv \lambda_1 / \lambda_3$



# Distribution Functions in Turbulence



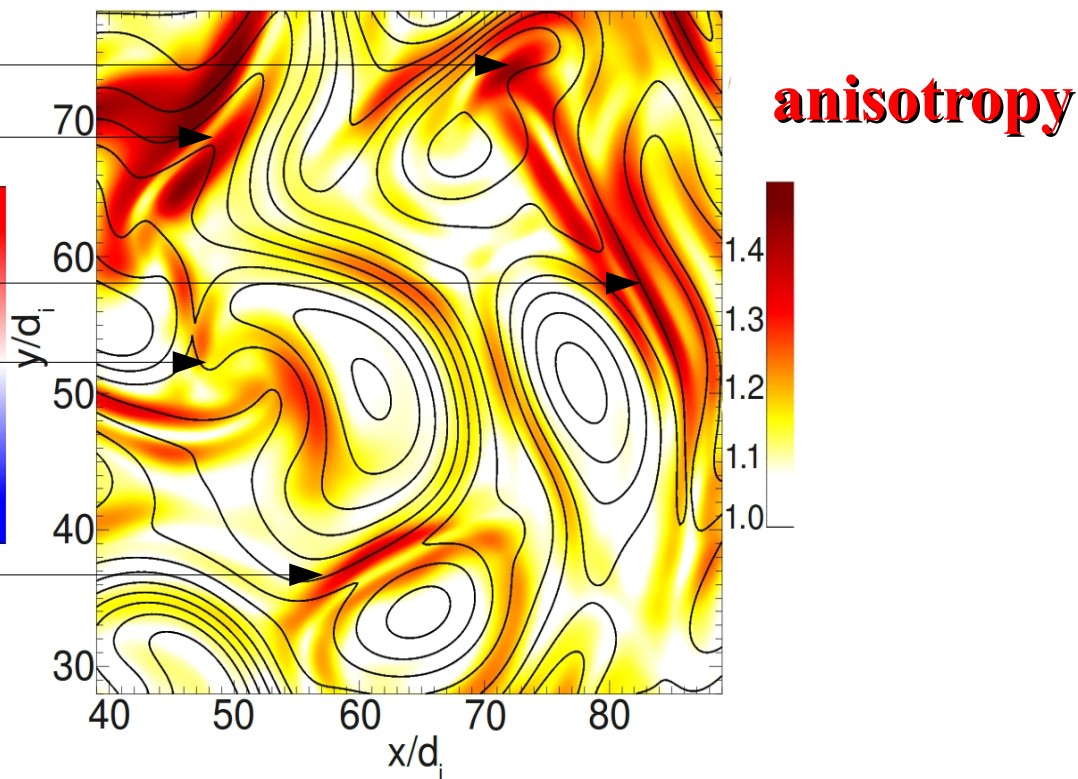
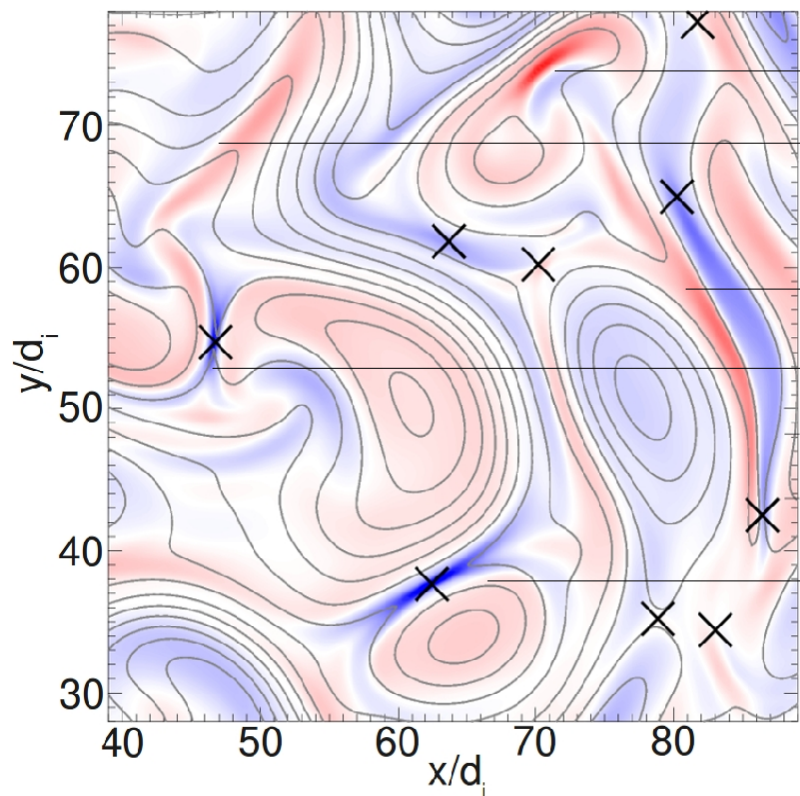
# Temperature Anisotropy & the Magnetic Field



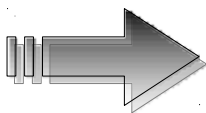
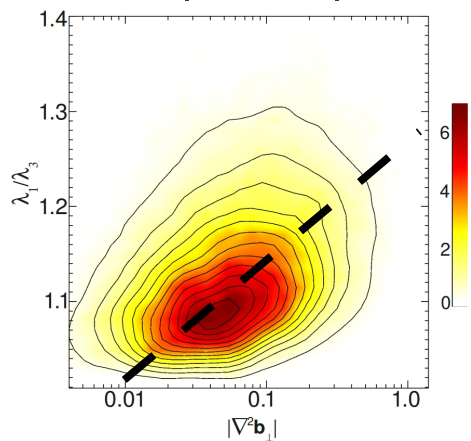
**mainly  $e_1$  can be both along or across  $B$ , but, because of turbulence, a broad distribution of angles is observed.**

**Note: If  $e_1$  and  $B$  were spatially random and uncorrelated,  $\text{PDF}(|\cos \theta|) \sim \text{const. (=1)}$**

# Where these “patches” are located?



$$T_{\perp}/T_{\parallel} \propto |\nabla j_z| \quad (\equiv |\nabla^2 b_{\perp}|)$$



streams of kinetic effects (anisotropy, skewness and Kurtosis) are adjacent to reconnecting current sheets.  
In a fluid model these would correspond to regions where collisional dissipation takes place. Here cyclotron and/or Landau resonances may be at work.



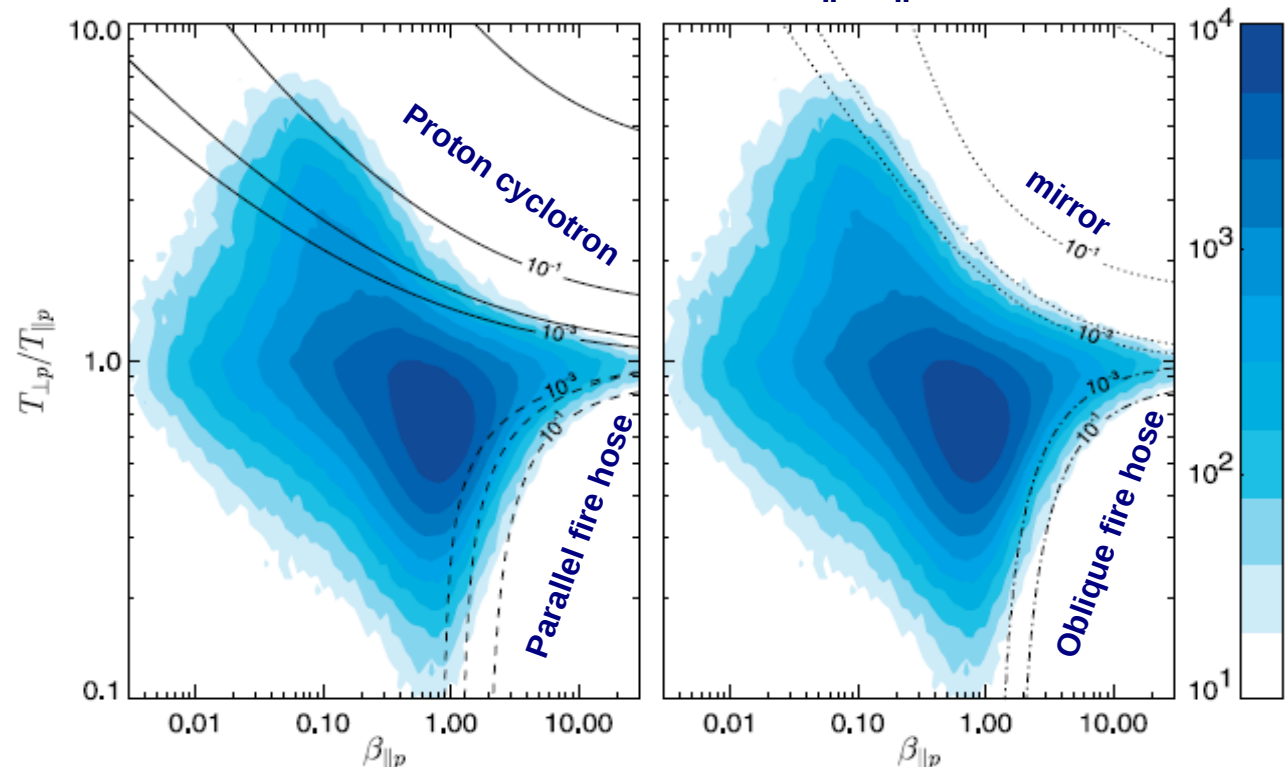
# Applications: Solar Wind

$T_{\parallel}$  and  $T_{\perp} \equiv$

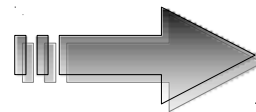
parallel and perpendicular  
proton temperatures with  
respect to the ambient  $B$

Hellinger *et al.* GRL (2006);  
Kasper *et al.* JGR (2006);  
Kasper *et al.*, (2002)

Distribution PDF(  $T_{\perp}/T_{\parallel}$ ,  $\beta_{\parallel}$  )



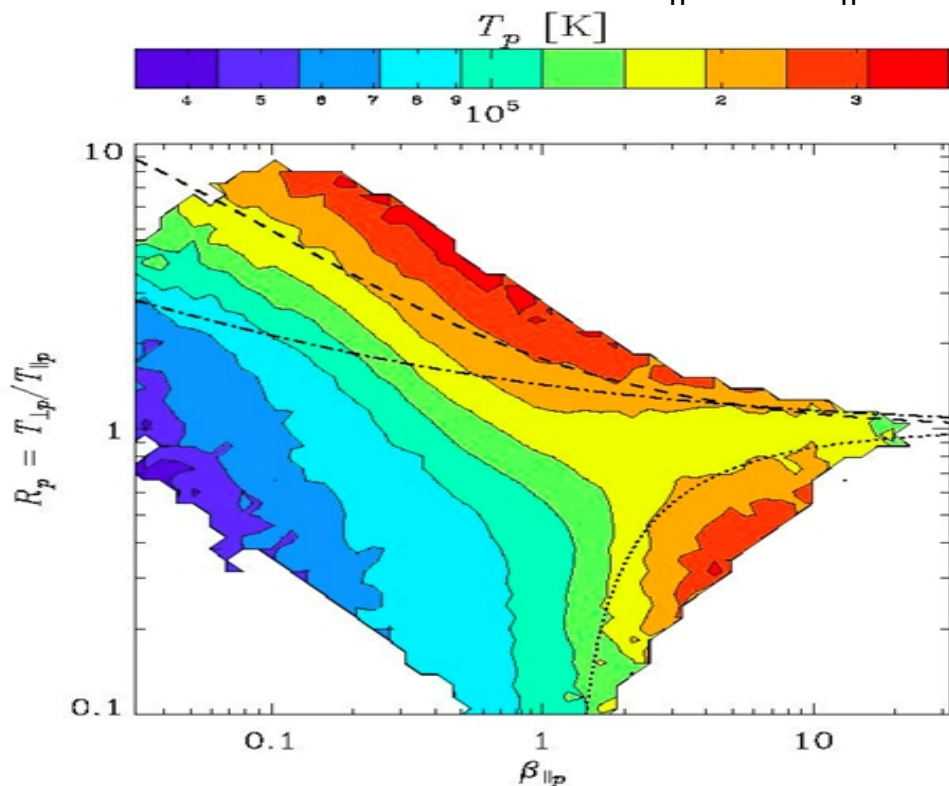
**Kinetic instabilities  
influence the solar wind**



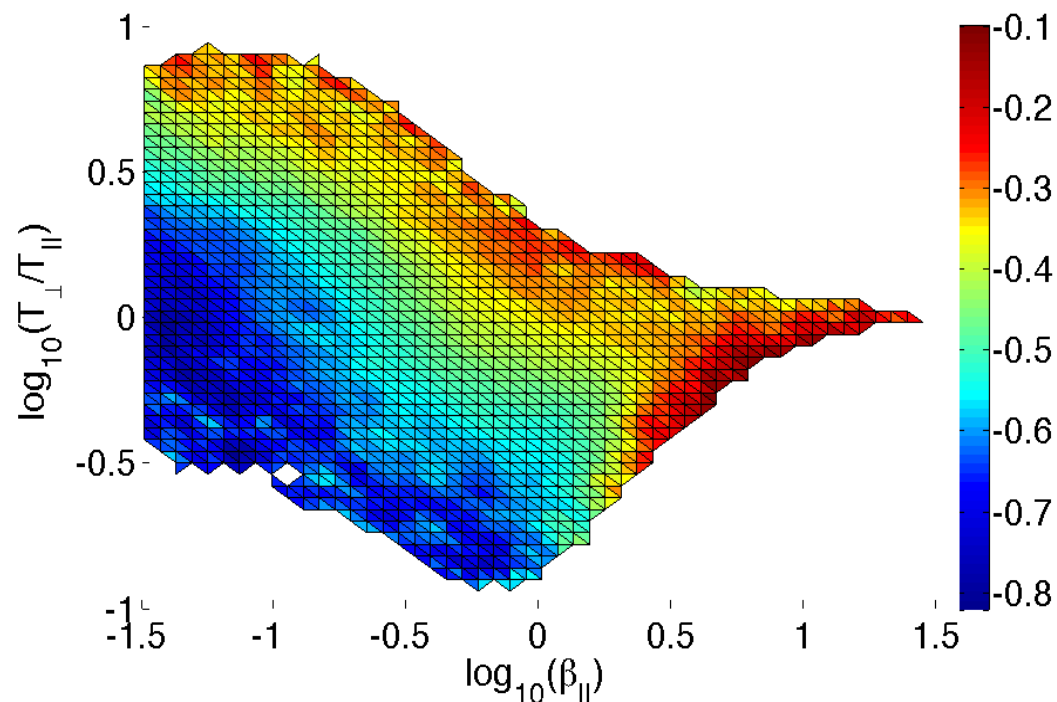
**nonlinear kinetic processes may  
locally occur in turbulence!**

# Non-homogeneous Effects ?

Temperature on  $(\beta_{\parallel}, T_{\perp}/T_{\parallel})$



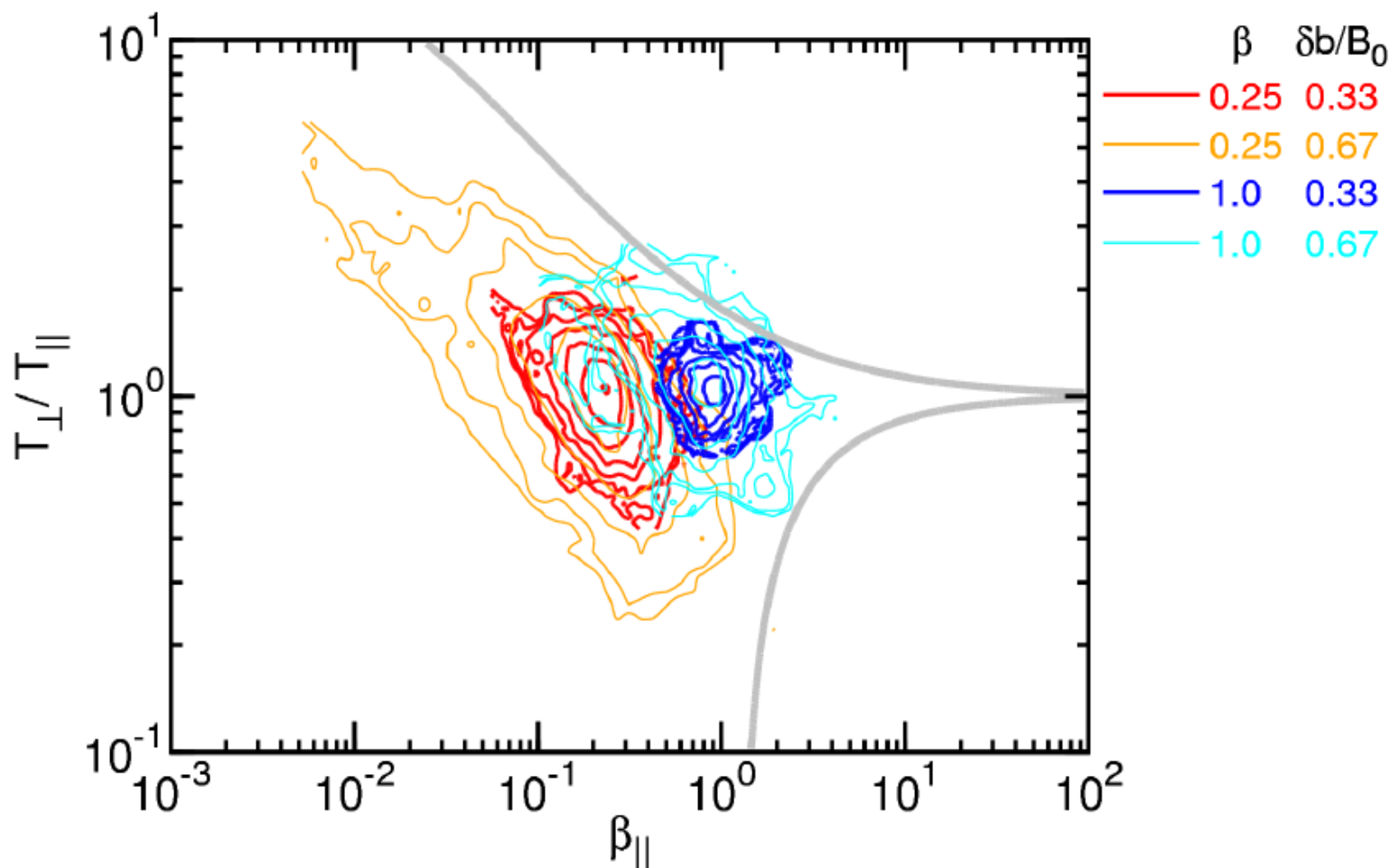
$$PVI = \frac{|\Delta B|}{\sqrt{\langle |\Delta B|^2 \rangle}} \quad \text{on } (\beta_{\parallel}, T_{\perp}/T_{\parallel})$$



Osman et al., PRL 2012

**The solar wind near the thresholds is hotter, and shows higher concentrations of current sheets**

# Vlasov Simulation(s)

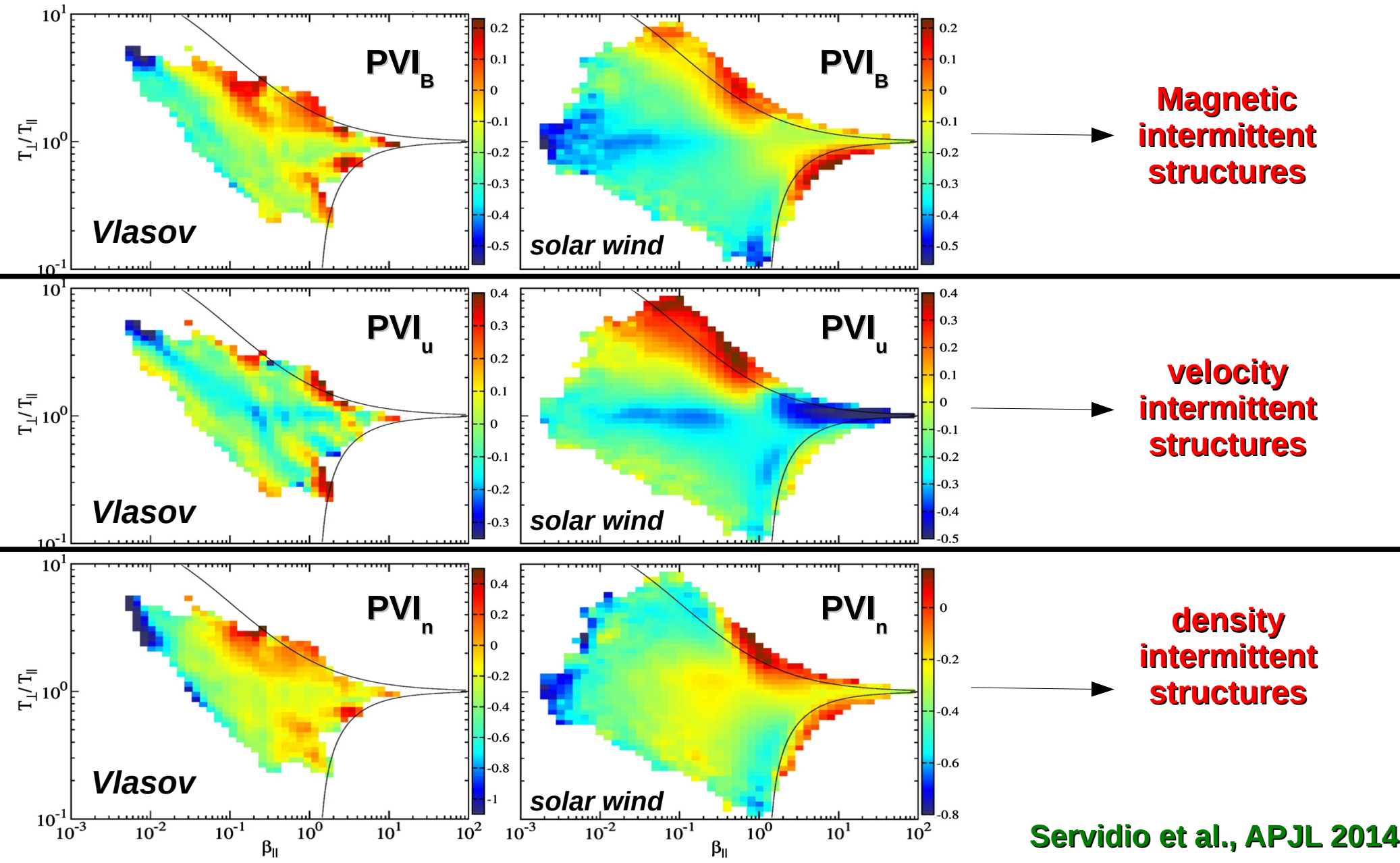


**By varying parameters such as the level of fluctuations and the average plasma beta, Vlasov simulations “explore” distinct regions of anisotropy plane**

# Vlasov vs. Solar wind

Ensemble of simulations

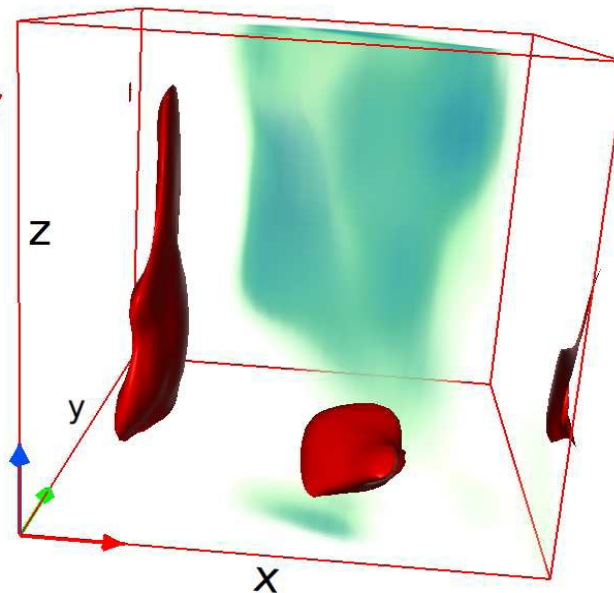
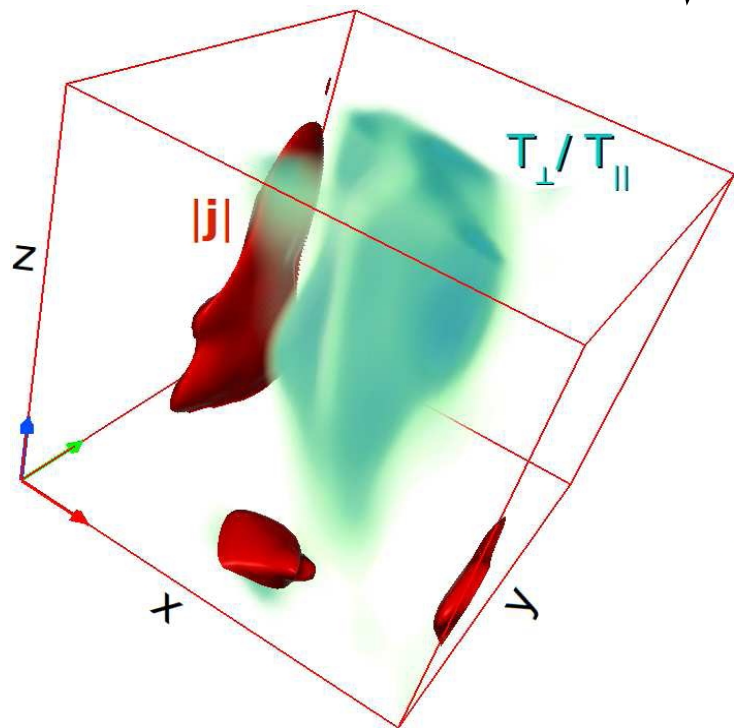
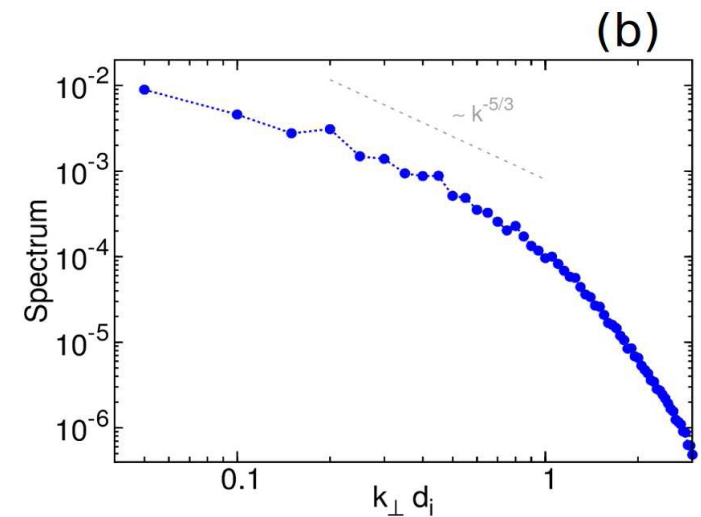
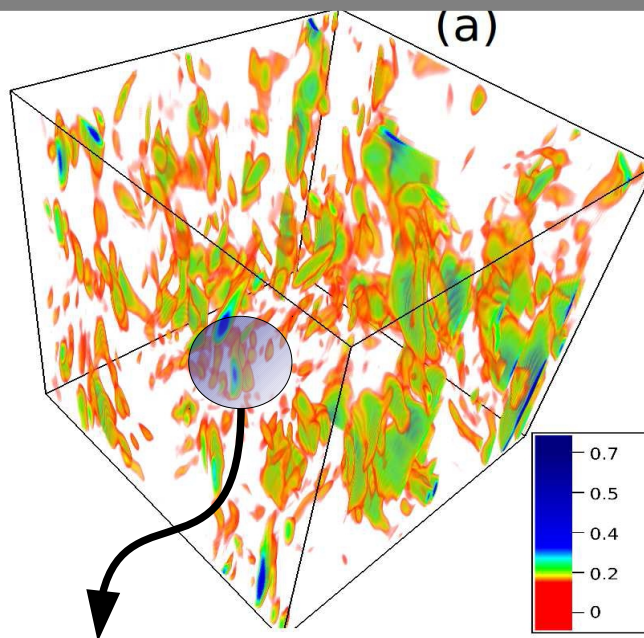
10 years of solar wind





# “6D” Vlasov

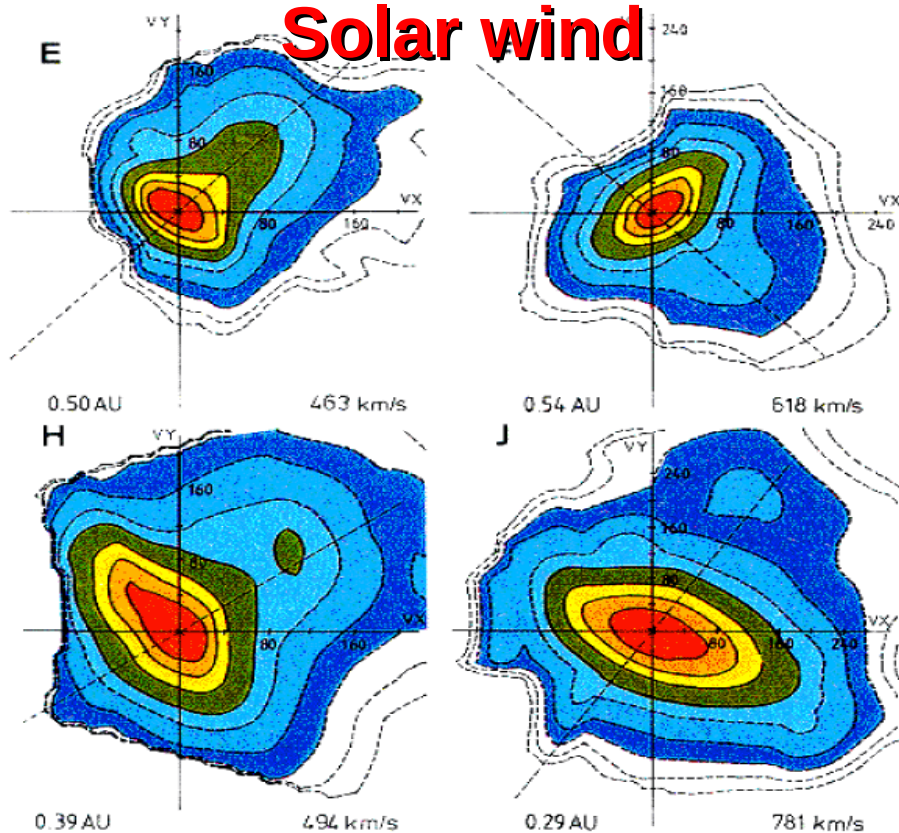
**Intermittent,  
anisotropic Vlasov  
turbulence...**



**Patterns of  
temperature  
anisotropy are near  
current sheets**

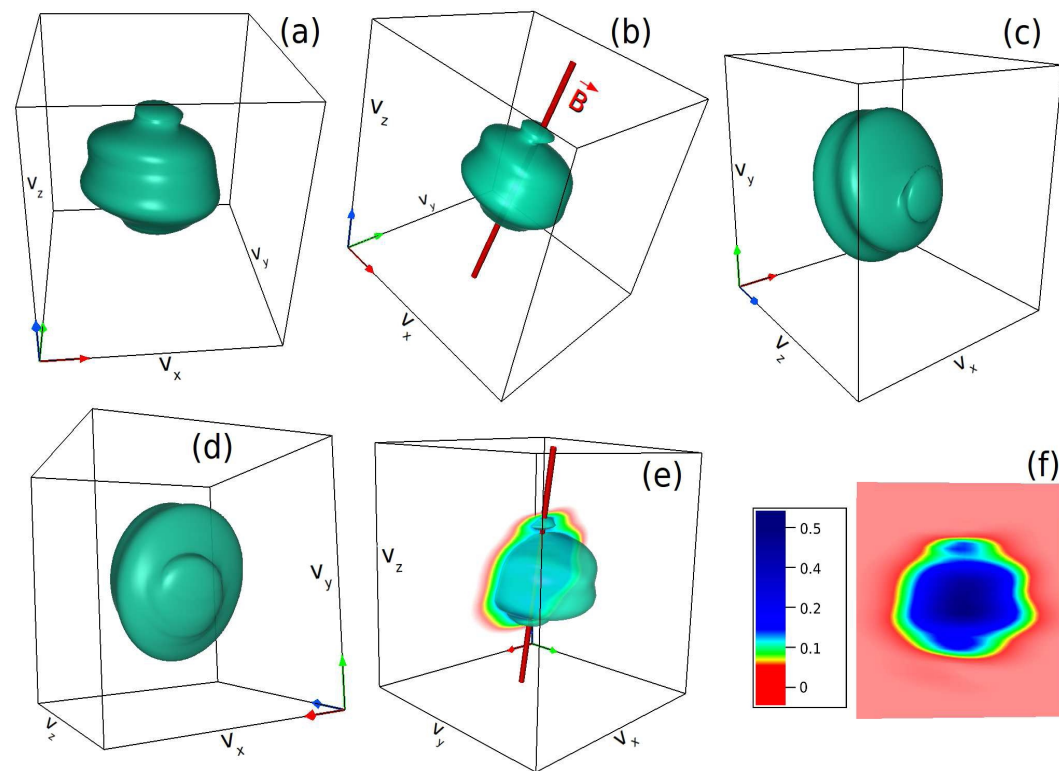
# 6D Vlasov: proton beams

**Solar wind**



**beams, anisotropy, and  
strong non-gyrotropic  
modulations**

**Vlasov**



**Landau resonances can be  
locally excited:**

$$\omega - k_{\parallel} v_{\parallel} - n \Omega_{ci} = 0$$

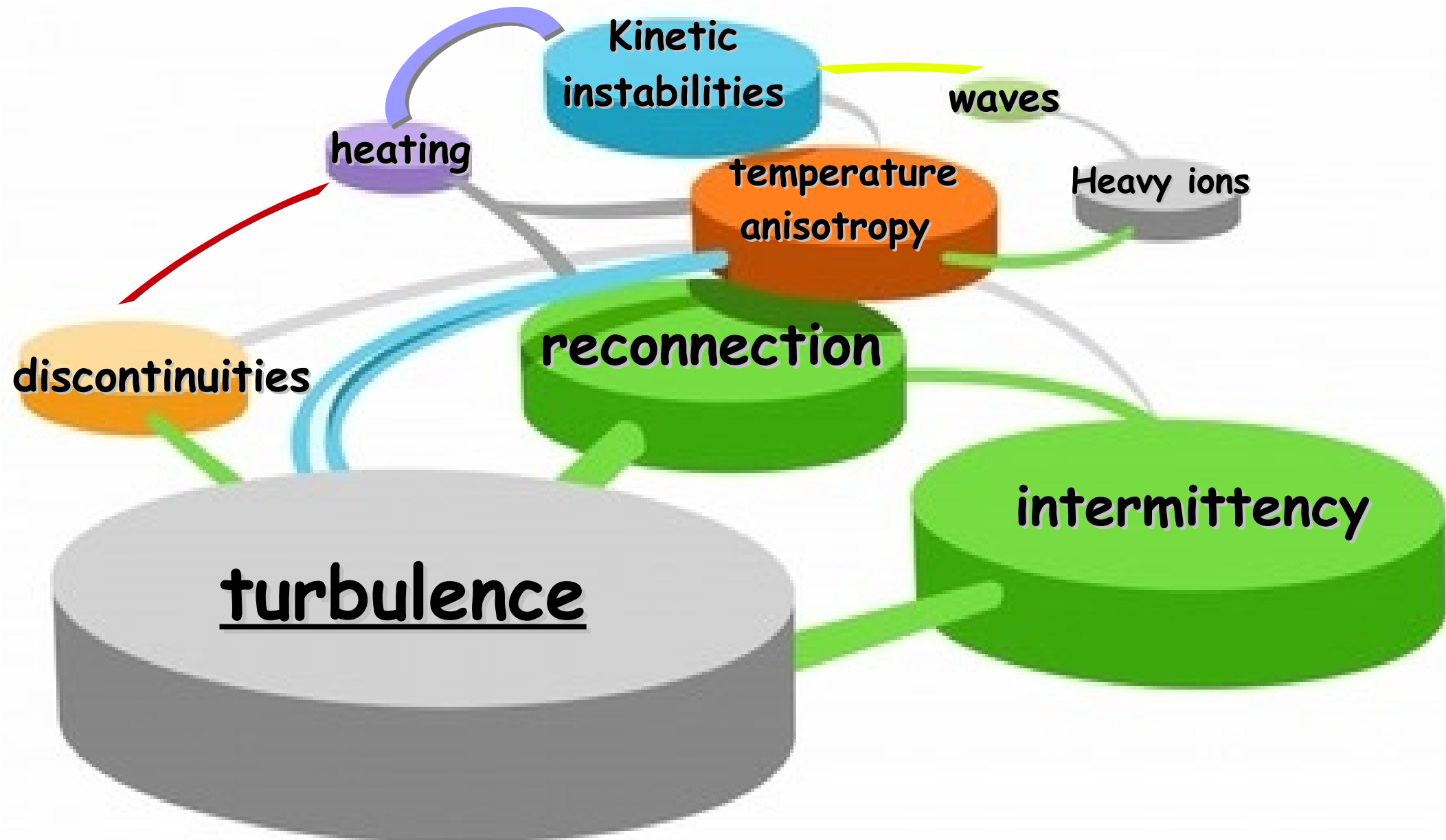
## Conclusions

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- **MHD turbulence provides a broad range of reconnection rates**
- **Reconnection rates can be determined statistically in terms of measurable correlation, Taylor, and dissipation scales.**
- **Hall effect enhances the reconnection process in turbulence**
- **Hybrid-Vlasov simulations confirm the above results and moreover show that kinetic effects are stronger nearby reconnection events**
- **Temperature anisotropy is higher in regions of strong magnetic stress, and in velocity and density gradients**

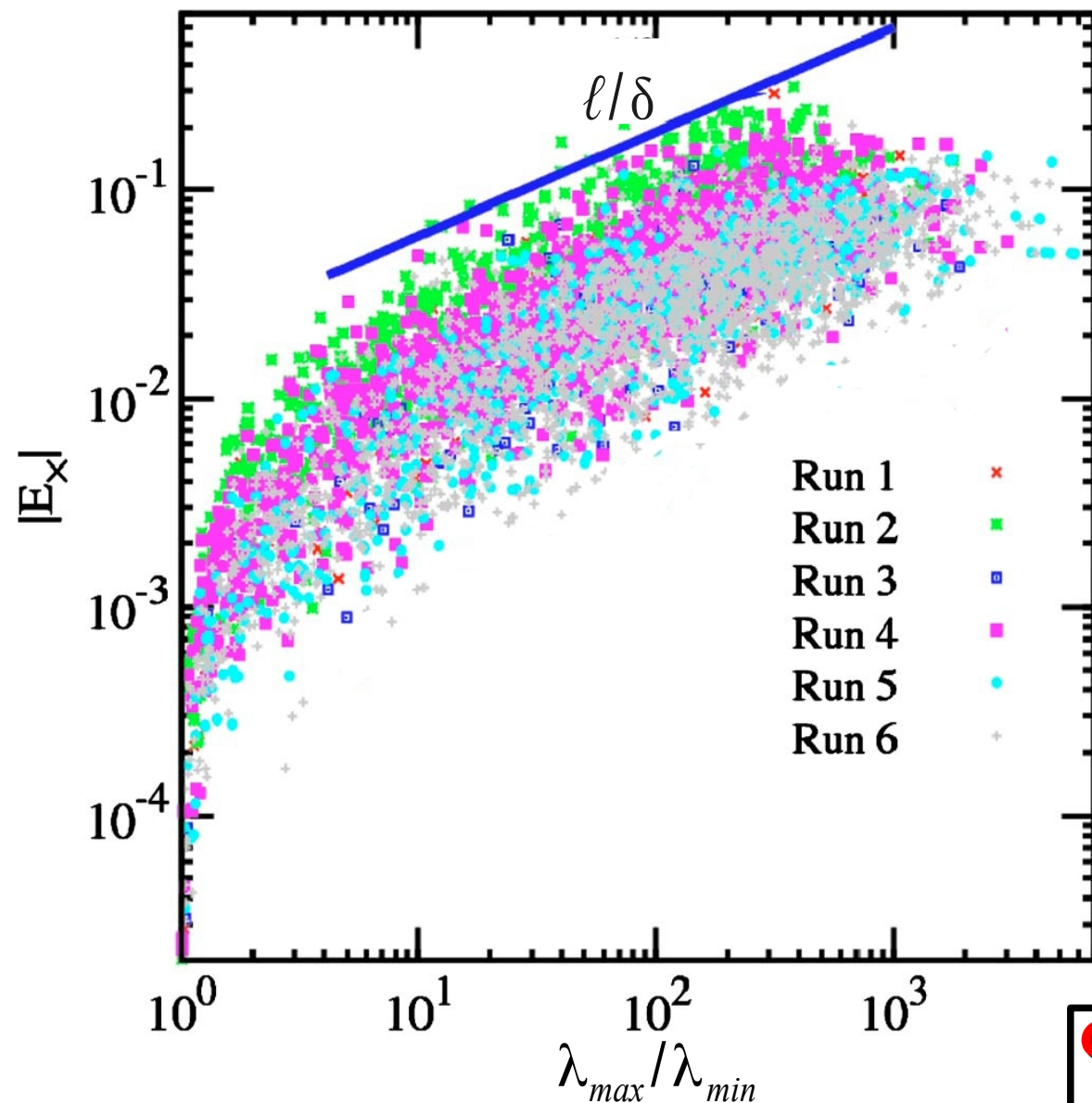


# A “simple” sketch



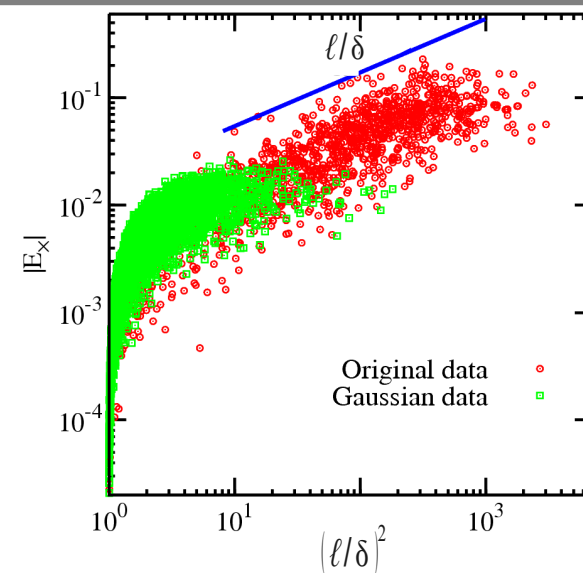
Extra slide

# Dependency on the Geometry



**Fastest reconnection rates depend on the geometry of the diffusion layer**

$$E_x \sim \frac{\ell}{\delta} \quad \begin{array}{l} \ell \equiv \text{elongation} \\ \delta \equiv \text{width} \end{array}$$

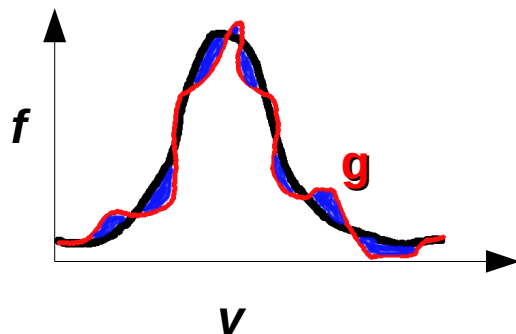


**Gaussian fields have much slower reconnection rate:  
Turbulence enhances reconnection**

Extra slide

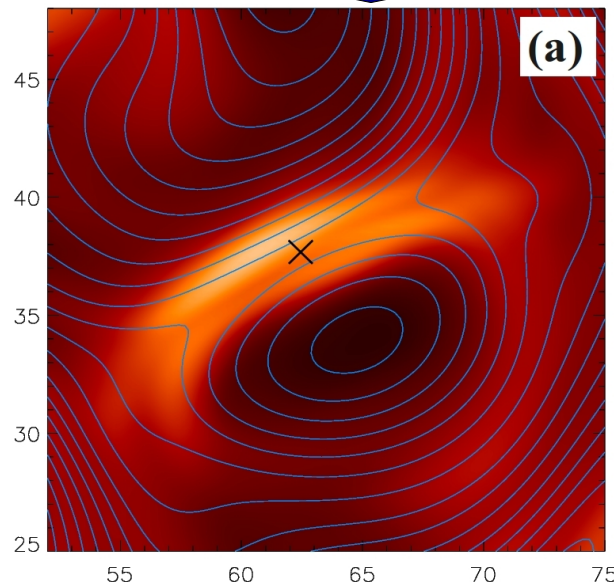
# Quantifying kinetic effects ...

$$\epsilon(x) = \frac{1}{n} \sqrt{\int (f - g)^2 d^3 v}$$

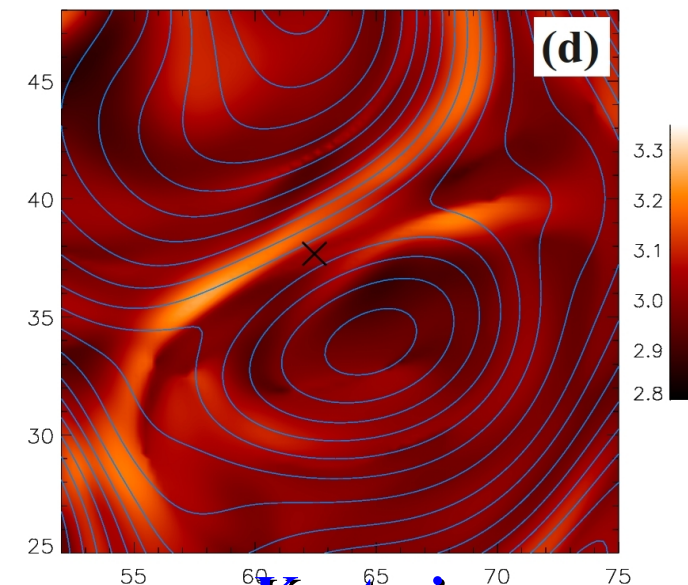
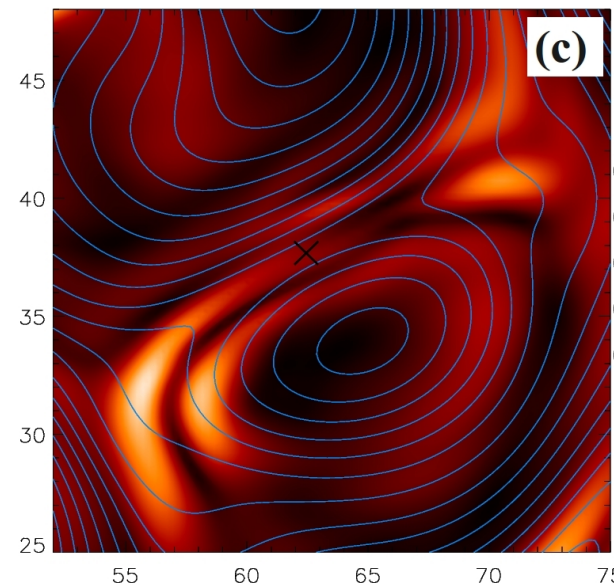
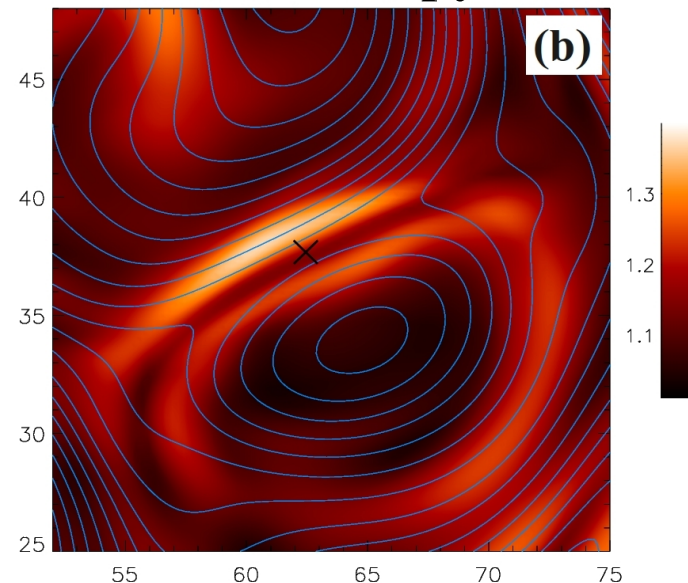


*g is the Maxwellian associated to f (with same bulk velocity, density, and (isotropic) temperature.)*

**Skewness**



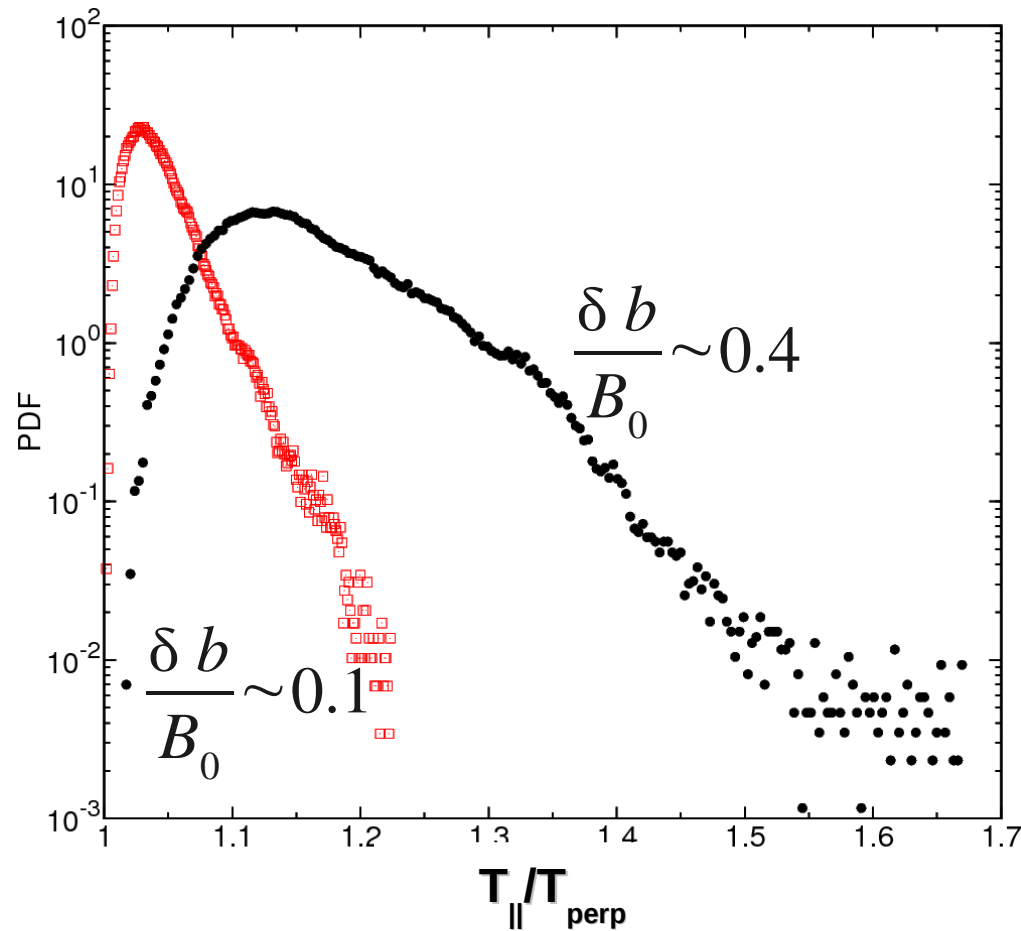
**Anisotropy**



**Kurtosis**

Extra slide

# Anisotropy & Reconnection



**Turbulence enhances the  
level of local anisotropy  
nearby reconnection sites**

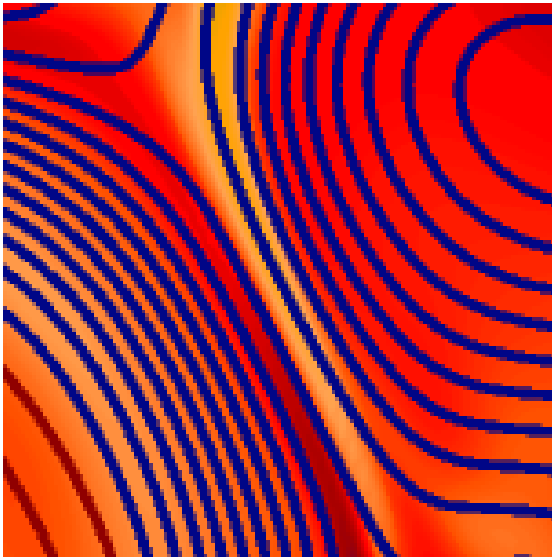


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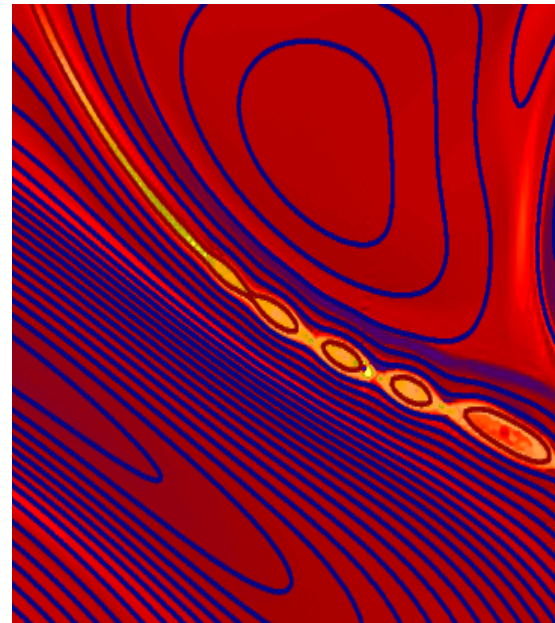
## Note on the Numerics

Generally simulators, tempted by the “high Reynolds number dream”, may use too low resistivity...

$$k_{\max} \sim N_{\text{mesh points}}$$
$$k_{\text{diss}} \sim \eta^{-1/2}$$



$$k_{\max}/k_{\text{diss}} > 3$$



$$k_{\max}/k_{\text{diss}} < 1$$

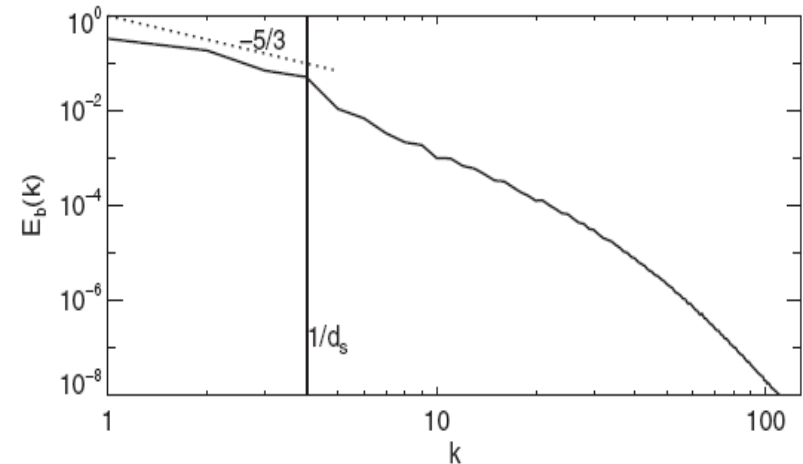
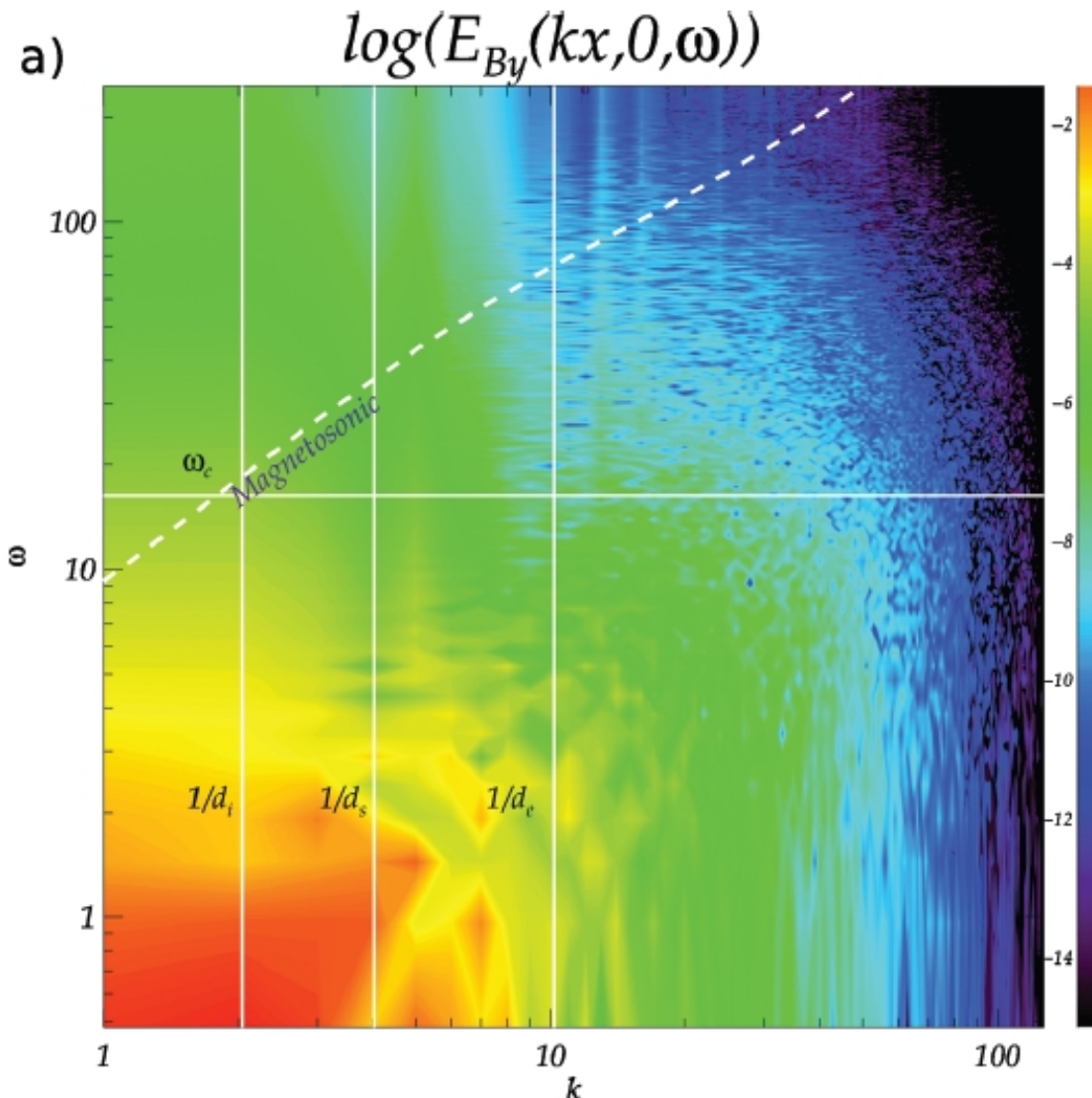
**Reconnection events in turbulent regime are strongly affected by “Gaussianization” of small scales!!!**



Extra slide

# Waves in turbulence?...

Driven simulations of kinetic  
turbulence using Hybrid PIC code



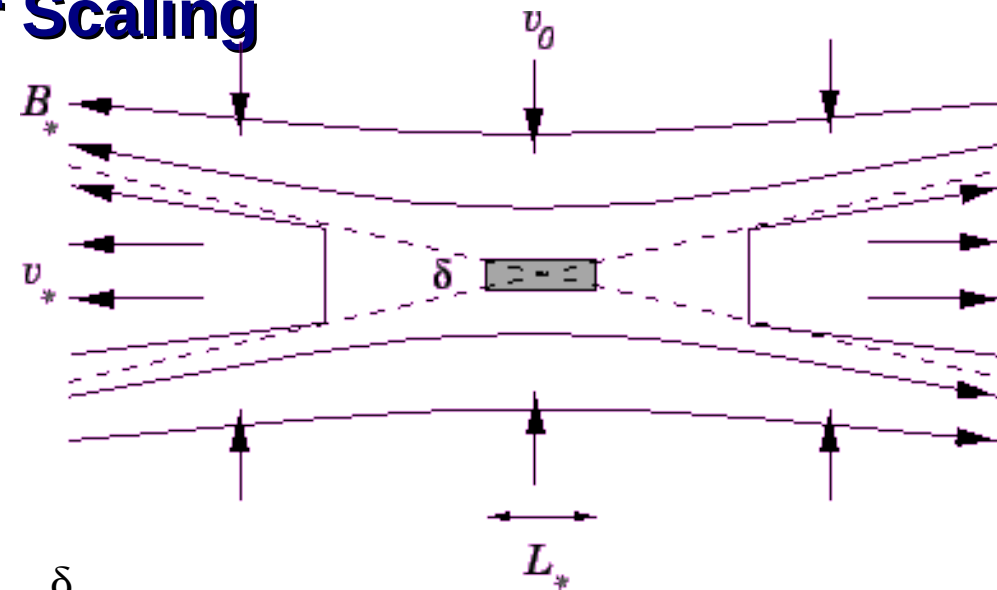
**The  $k$ - $\omega$  spectra  
show a complete  
absence of waves in  
turbulence**

**Parashar et al. Phys.  
Plasmas 2010, 2011**

# Theory of magnetic reconnection

## Sweet-Parker Scaling

A steady-state is reached when field lines convect into the collisional layer at the same rate that they are annihilated



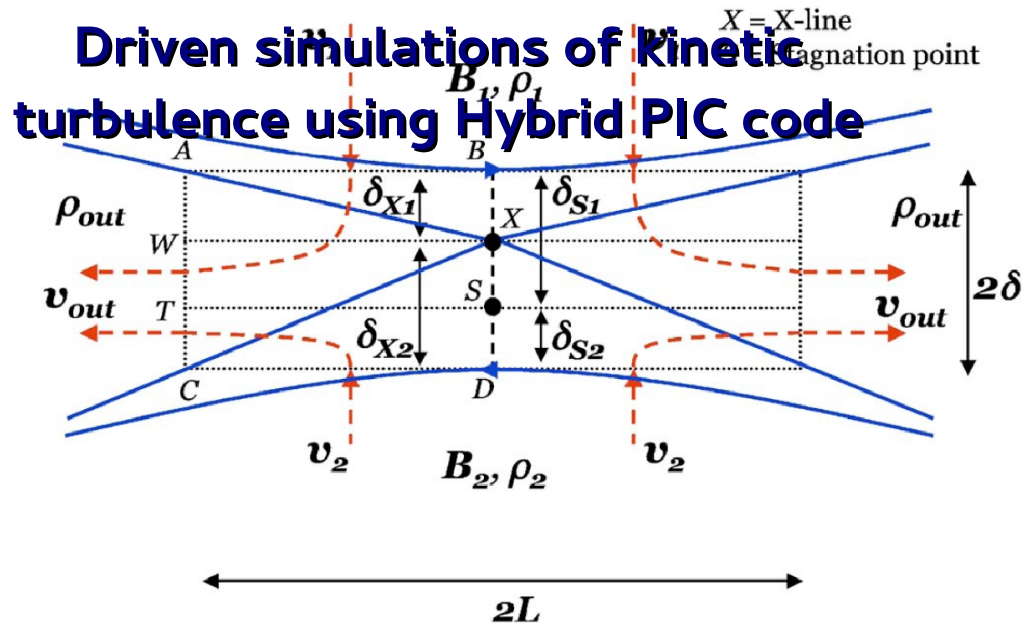
- 1) Continuity / mass conservation  $\Rightarrow v_{in} = \frac{\delta}{L} v_{out}$
- 2) The pressure available for squeezing the fluid out is magnetic ( $B^2/8\pi$ ), so, from energy considerations, the outflow speed is:  $\Rightarrow v_{out} = \frac{B}{\sqrt{4\pi\rho}} = c_A$
- 3) Continuity of the electric field between x point and the upstream region  $\Rightarrow v_{in} = \frac{\eta c^2}{4\pi\delta}$

$$\frac{\delta}{L} \sim \frac{v_{in}}{v_{out}} \sim \frac{c E}{B c_A} \sim \sqrt{\frac{\eta c^2}{4\pi c_A L}} \sim S^{-1/2}$$

$L$  is a free parameter which scales with the system size. Is approximately the size of the flux tube

Extra slide

# Asymmetric Reconnection



**P. Cassak & M. Shay,**  
**Phys. Plasmas 2007**

A Sweet-Parker-type scaling analysis for asymmetric, anti-parallel, reconnection has been recently proposed by Cassak & Shay (2007):

$$E_{\times}^{th.} = \sqrt{\frac{b_1^{3/2} b_2^{3/2}}{R_{\mu} \ell}}$$

**Borovsky & Hesse, Phys. Plasmas (2007); Birn *et al.*, Phys Plasmas (2008); Pritchett, JGR (2008).**

Extra slide

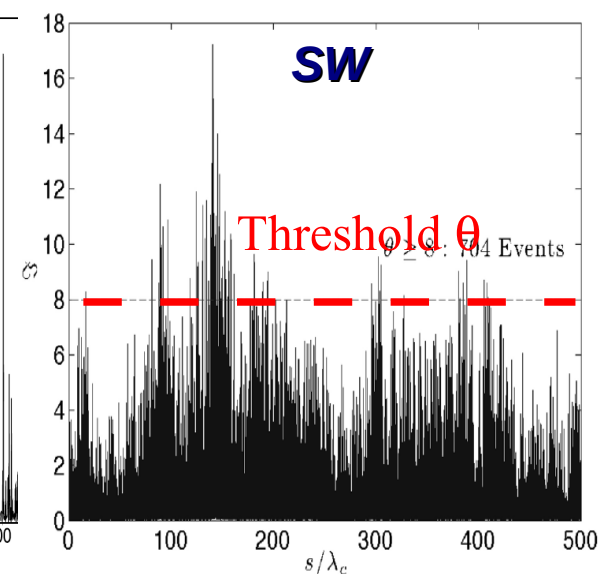
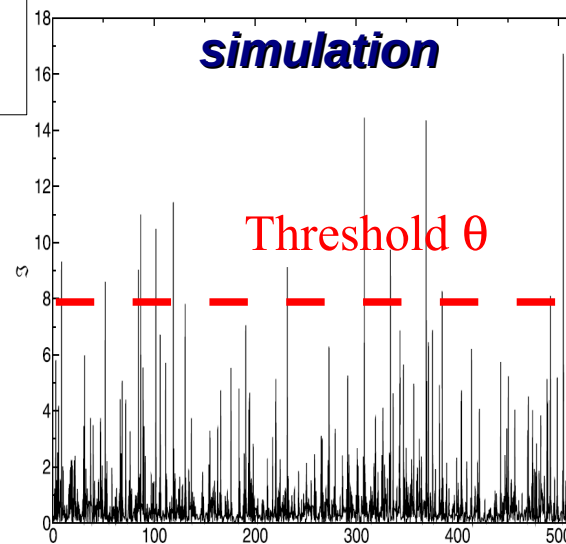
# A method for identifying discontinuities

The Partial Variance of Increments (PVI)

$$\Delta \mathbf{B}(s, \Delta s) = \mathbf{B}(s + \Delta s) - \mathbf{B}(s)$$



$$\mathfrak{I}(\Delta s, \ell, s) = \frac{|\Delta \mathbf{B}(s, \Delta s)|}{\sqrt{\langle |\Delta \mathbf{B}(s, \Delta s)|^2 \rangle_\ell}}$$



**For each threshold  $\theta$ , a number of discontinuities can be localized and “counted”**